CALIFORNIA STATE COLLEGE
AT PALOS VERDES

B Hegel, G.
2942 Hegel's science of logic
E5
H5
v.1

Date Due

PRINTED IN U.S.A.

MUIRHEAD LIBRARY OF PHILOSOPHY

An admirable statement of the aims of the Library of Philosophy was provided by the first editor, the late Professor J. H. Muirhead, in his description of the original programme printed in Erdmann's *History of Philosophy* under the date 1890. This was slightly modified in subsequent volumes to take the form of the following statement:

'The Muirhead Library of Philosophy was designed as a contribution to the History of Modern Philosophy under the heads: first of different Schools of Thought—Sensationalist, Realist, Idealist, Intuitivist; secondly of different Subjects—Psychology, Ethics, Aesthetics, Political Philosophy, Theology. While much had been done in England in tracing the course of evolution in nature, history, economics, morals and religion, little had been done in tracing the development of thought on these subjects. Yet "the evolution of opinion is part of the whole evolution".

'By the co-operation of different writers in carrying out this plan it was hoped that a thoroughness and completeness of treatment, otherwise unattainable, might be secured. It was believed also that from writers mainly British and American fuller consideration of English Philosophy than it had hitherto received might be looked for. In the earlier series of books containing, among others, Bosanquet's *History of Aesthetic*, Pfleiderer's *Rational Theology since Kant*, Albee's *History of English Utilitarianism*, Bonar's *Philosophy and Political Economy*, Brett's *History of Psychology*, Ritchie's *Natural Rights*, these objects were to a large extent effected.

'In the meantime original work of a high order was being produced both in England and America by such writers as Bradley, Stout, Bertrand Russell, Baldwin, Urban, Montague, and others, and a new interest in foreign works, German, French and Italian, which had either become classical or were attracting public attention, had developed. The scope of the Library thus became extended into something more international, and it is entering on the fifth decade of its existence in the hope that it may contribute to that mutual understanding between countries which is so pressing a need of the present time.'

The need which Professor Muirhead stressed is no less pressing today, and few will deny that philosophy has much to do with enabling us to meet it, although no one, least of all Muirhead himself, would regard that as the sole, or even the main, object of philosophy. As Professor Muirhead continues to lend the distinction of his name to the Library of Philosophy it seemed not inappropriate to allow him to recall us to these aims in his own words. The emphasis on the history of thought also seemed to me very timely; and the number of important works promised for the Library in the near future augur well for the continued fulfilment, in this and other ways, of the expectations of the original editor.

H. D. LEWIS

MUIRHEAD LIBRARY OF PHILOSOPHY

General Editor: H. D. Lewis

Professor of History and Philosophy of Religion in the University of London

The Analysis of Mind. By Bertrand Russell. 8th Impression.

Analytic Psychology. By G. F. Stout. 2 Vols. 5th Impression.

Coleridge as Philosopher. By J. H. Muirhead. 2nd Impression.

Contemporary American Philosophy. Edited by G. P. Adams and W. P. Montague.

Contemporary British Philosophy. Edited by J. H. Muirhead.

Contemporary Indian Philosophy. Edited by Radhakrishnan and J. H. Muirhead.

Contemporary British Philosophy. Third Series. Edited by H. D. Lewis. 2nd Impression

Development of Theology Since Kant. By O. Pfleiderer.

Dialogues on Metaphysics. By Nicholas Malebranche. Translated by Morris Ginsburg.

Ethics. By Nicolai Hartmann. Translated by Stanton Coit. 3 Vols.

The Good Will: A Study in the Coherence Theory of Goodness. By H. J. Paton.

Hegel: A Re-Examination. By J. N. Findlay.

Hegel's Science of Logic. Translated by W. H. Johnston and L. G. Struthers. 2 Vols. 3rd Impression.

History of Æsthetic. By B. Bosanquet. 4th Edition. 5th Impression.

History of English Utilitarianism. By E. Albee. 2nd Impression

History of Psychology. By G. S. Brett. Edited by R. S. Peters. Abridged one-volume edition.

Human Knowledge. By Bertrand Russell. 3rd Impression.

A Hundred Years of British Philosophy. By Rudolf Metz. Translated by J. W. Harvey, T. E. Jessop, Henry Sturt. 2nd Impression.

Ideas: A General Introduction to Pure Phenomenology. By Edmund Husserl. Translated by W. R. Boyce Gibson. 3rd Impression.

Indian Philosophy. By Radhakrishnan. 2 Vols. Revised 2nd Edition.

The Intelligible World Metaphysics and Value. By W. M. Urban.

Introduction to Mathematical Philosophy. By Bertrand Russell. 2nd Edition. 10th Impression.

Kant's First Critique. By H. W. Cassirer.

Kant's Metaphysic of Experience. By H. J. Paton. 2nd Impression.

Know Thyself. By Bernardino Varisco. Translated by Guglielmo Salvadori.

Language and Reality. By Wilbur Marshall Urban. 2nd Impression

Matter and Memory. By Henri Bergson. Translated by N. M. Paul and W. S. Palmer. 6th Impression.

Modern Philosophy. By Guido de Ruggiero. Translated by A. Howard Hannay and R. G. Collingwood.

The Modern Predicament. By H. J. Paton. 2nd Impression

Moral Sense. By James Bonar.
Natural Rights. By D. G. Ritchie. 3rd Edition. 5th Impression.
Nature, Mind and Modern Science. By E. Harris.
The Nature of Thought. By Brand Blanshard. 2nd Impression.
On Selfhood and Godhood. By C. A. Campbell.
Personality and Reality. By E. J. Turner.
The Phenomenology of Mind. By G. W. F. Hegel. Translated by Sir James Baillie. Revised 2nd Edition. 4th Impression.
Philosophical Papers. By G. E. Moore.
Philosophy and Political Economy. By J. Bonar. 4th Impression.
Philosophy of Whitehead. By W. Mays.
The Platonic Tradition in Anglo-Saxon Philosophy. By J. H. Muirhead.
The Principal Upanisads. By Radhakrishnan.
The Problems of Perception. By R. J. Hirst.
Some Main Problems of Philosophy. By G. E. Moore.
Time and Free Will. By Henri Bergson. Translated by F. G. Pogson. 7th Impression.
The Ways of Knowing: or The Methods of Philosophy. By W. P. Montague. 5th Impression.

The Muirhead Library of Philosophy
EDITED BY H. D. LEWIS

SCIENCE OF LOGIC

HEGEL'S

SCIENCE OF LOGIC

TRANSLATED BY
W. H. JOHNSTON, B.A., AND
L. G. STRUTHERS, M.A.

★

With an Introductory Preface by
Viscount Haldane of Cloan
K.T. P.C. O.M. F.R.S.

VOLUME ONE

LONDON: GEORGE ALLEN & UNWIN LTD
NEW YORK: THE MACMILLAN COMPANY

FIRST PUBLISHED IN ENGLISH IN 1929
SECOND IMPRESSION 1951
THIRD IMPRESSION 1961

This book is copyright under the Berne Convention. Apart from any fair dealing for the purpose of private study, research, criticism or review, as permitted under the Copyright Act, 1956, *no portion may be reproduced by any process without written permission. Enquiries should be addressed to the publisher.*

First published in German 1812–1816 under the title "Wissenschaft der Logik."

The greater part of this translation was made from the edition published in 1923 by Felix Meiner, Leipzig, edited by Dr. Georg Lasson, of Berlin.

B
2942
E5
H5
v. 1

Printed in Great Britain by Photolitho
UNWIN BROTHERS LIMITED
WOKING AND LONDON

CALIFORNIA STATE COLLEGE

THIS TRANSLATION IS

DEDICATED TO THE MEMORY OF

FABIAN PEMBER REEVES

1895–1917

INTRODUCTORY PREFACE

THE *Wissenschaft der Logik* is one of the most important of Hegel's works. It was first written when he was at Nürnberg in 1812. He was engaged in revising it for a second edition at the time of his death, in the end of 1831. It now appears in an English form, carefully rendered for the benefit of students in this country, in a fashion which shows that the translators have not only reproduced the words in our own language, but have interpreted Hegel's meaning fully.

Hegel, who at one time dominated the study of philosophy, is not at present much read. People have gone back to Kant. But the very reasons which led them back to Kant have led some of the keenest inquirers on to Hegel. The revision of current ideas which was effected by the latter is just as close as that brought about by Kant. It follows the same line but follows it further.

Hegel is very difficult to read for those who have not first of all mastered the conception of the Critical Philosophy. But if they have mastered this conception and have also mastered the general outlook of the Hegelian system in its amendment, Hegel is comparatively easy to read. There is in his exposition a principle closely worked out, and the principle when once grasped makes his language readily intelligible. If it is asked why then, since 1850 at any rate, his writings have been a good deal neglected, the answer is not difficult to give. It is only by the multitude that they have been laid aside. The principle has penetrated in this country into the attitude and methods of some of the later thinkers who have been greatest among us, men like Green, Caird, Bradley and Bosanquet. These have indeed hardly been Hegelians. It was not probable that a system which was given to the world a century since should serve the world sufficiently to-day. But its broad principle has profoundly moved the English thinkers to whom I have referred, and I might add instances of the same kind from the United States and from other countries, as well as from Germany.

Not the less is it true that there has been a reaction against the Hegelian outlook. First of all its author wrote as though his teaching culminated in the exposition of an Absolute. I do not think that Hegel ever meant his Absolute to be accepted as more than an ideal, to be worked towards but not to be regarded as capable of description excepting in abstract terms which were therefore insufficient for the concrete ideas of human beings. In the second place he has been held responsible for an idea of the State as a super-person. He has even been accused of having been the indirect but potent cause of the late war. A little study of his writings shows this to be a gross exaggeration. He held that Society was no mechanical aggregate of isolated individuals, and that there were identities in thinking and willing which united Society into what was more real and of a higher order than such a mere aggregate. As for the State, he declared that whether it was to be monarchical or republican was a question with which philosophy had no concern.

But a more serious criticism on Hegel has been made from another side. He attached high importance to the application of his principle to the whole field of experience. This principle he applied to the current results of the physical and natural science of his time. Although his reading was vast he was not a trained man of science. He had too little knowledge to be sufficiently critical of the results reached a century ago, at a time when people were carried away by such brilliant aberrations as Goethe's attack on Newton's explanation of colour. Along with other great thinkers of his time he committed himself to some views largely accepted early in last century but now wholly superseded. The outcome of this was that after his death there was a great revulsion against certain of the views expressed in his *Philosophy of Nature*.

Now although some of Hegel's opinions on scientific subjects were wrong and are now wholly superseded, none of the criticisms made upon them affected the validity of his fundamental principle. The fierceness of the attacks on him was partly at least due to his personality. He was a hard dry man. He had, notwithstanding what has been said by people who have not mastered his system, nothing of the mystic in his composition. He exercised at Berlin, where he was professor for many years, more influence with the Prussian Ministry than was desirable in one who had better have held himself aloof from these things. In short, though worshipped as a teacher, he was far from being universally liked as a man. Schopenhauer's testimony against him is far too bitter to be reliable, but that it should have been given wide publicity is indicative of much that was felt.

It was therefore to an audience not wholly indisposed to listen that the critics of Hegel addressed themselves after he was dead and his reign was over. In reality what they attacked most were doctrines that were not vital to the system. But public opinion had become suspicious of attempts to rationalize on high principles of metaphysics the fruits which they held not unjustly that they were receiving from observation and experiment alone, and men were not then conscious how impossible it is to avoid the recognition in these deliverances of factors due to the unavoidably moulding influence of reflection.

Then, too, the system had become too much of an effort after a complete and all-embracing account of the Universe, of God as well as Men and things, to inspire confidence in those who belonged to a later generation with different tendencies. The day for any attempt at a display of the "Absolute" was gone by.

Notwithstanding all these things the greatness of Hegel's real contribution to the history of thought seems to me to remain intact. His method is what he himself declared to be the important thing; the results might well, he himself said, assume a form different from that they took in his own hands. What have to be grasped are therefore the ground-principle and the method. We need not be deterred by the feeling that we are no longer interested in the doctrine of the Absolute which engrossed attention in the early part of last century; nor by Hegel's political tendencies, whatever they were, and they were diverse; nor by the insufficiency of the scientific teaching which was that of his time. For none of these things detracts from the principle underlying his method, a method which is applicable to the subject-matter of our general experience as it is interpreted to-day just as much as it was applicable to that subject-matter in the shape which was current in the period in which he wrote.

What then was the principle underlying Hegel's method?

He starts from the fact of human experience, differently interpreted as it may well be in different times and by different individuals, but in substance analogous for those who possess the brain and senses that are common to mankind. Experience is the actual and the actual is experience. We really, therefore, make self-conscious man our point of departure. He is the "That" which we have to interpret into the "What." The result of doing so is exhibited in his first great book, the *Phenomenology of Mind*, which appeared in 1807, before the present book. There we encounter the actual. We find that mind moulds its experience, and that mind and its object alike are no static entities, independent of each other, but consist in a process which is dynamic, in that each distinction made turns out to involve further distinctions ahead. Reflection does not stand still when it makes the generalizations which it is of its character to make; it always implies and leads on to further generalizations which in the end turn out to be themselves abstract and relatively insufficient. Its procedure is in this sense dialectical, and the ideal of knowledge is a whole towards which knowledge is always by its very nature striving, but which it can never, limited as it is by our station in nature, reach as an immediate object of concrete experience. Such a whole can only be rendered in the abstractions of reflection. It cannot be directly apprehended.

Still experience can be sifted out by thought. It is the actual, but it is actual only in so far as thought *sets* feeling in it and so gives it meaning for us. Into every form of experience reflection thus enters, not as constituting it *ab extra*, but as inherent in its very nature. The universal and the particular factors which thus belong to the constitution of actual experience are not separate entities. Neither exists apart from the other, nor is either reducible to the other. What is concrete is also universal, and what is universal has reality only in the concrete form of the actual. What reflection can do is to disengage by analysis of what is before it in experience the notions or categories which give experience its meaning and therefore its reality. For to have no meaning is to be wholly unreal. The meaning of things must be found, but the nature and relations of the categories which enter into and constitute their significance can only be reached by a sustained process of investigation. As reflection is dynamic and has not static nor self-contained notions to deal with, the development of the activity of thought as a system may thus turn out to be a process of self-development, and this is just what Hegel finds.

The principle of his idealism is accordingly very different from that of Kant. The latter also found categories or notions that entered into the constitution of experience. But these he got at empirically, by a scrutiny of the forms of judgment as stated in the current logic which dealt with subjective thinking. Each of his categories was self-contained and stood by itself as though it were a particular form of subjective thought, detachable from and applied to a raw material independent of it, which indicated as its origin the existence in some form of things in themselves real apart from any knowledge of them. For Kant mind was also real apart from its activity in judgment and the forms of space and time into which it moulded the matter that came within its grasp. For him experience was thus made up of separable factors, capable of being treated independently of each other, and the result was an idealism which was in truth subjective.

Hegel laid hold of the principle which Kant had expressed when the latter said that we must search for the categories which mould our subjective experience and give it reality with the aid of the other elements which he brings in. But Hegel was far from content with the narrow view which his predecessor took of the number and nature of the categories. For Hegel they were not forms which the mind imposed on something from without it. It was true that by themselves they could not give anything actual, for they were abstract universals intelligible only to reflection. Nor were they to be found by themselves in the objects of experience. For these so far as real were always individual. The individual is indeed unique, just this and none other in the universe. But what is individual is real only in so far as there is recognized the universal aspect which reflection discloses in it. This is indeed only disclosed as inseparable from a particularity which is indefinable and is only there in the individual form in which thought sets it. The particular as such, bare sensation for instance, is only actual in so far as it is recognized as my sensation or as analogous to it, and it is thus that reflection must enter into the constitution of our experience. Universal and particular conceived in isolation from each other are mere abstractions from an experience which can be real only in an individual form.

But as all that is significant in the actual is the work done by reflection in setting particulars in individual aspects that are due to the abstract universals of reflection, it is obvious that to ascertain the character and system of these universals is a matter of great importance for philosophy. We can study these only as disclosed in experience itself, an experience that is always changing, and has orders or grades. It is this abstract side of experience with which metaphysics is concerned. Its form varies according to the phases of the world with which we are dealing. Mathematics and physics deal with certain phases of experience which are distinguished and made definite by the characters of the categories which these phases present. In biology, ethics, art and religion we are confronted with other categories of different orders. The investigation of the characters of these varying categories and of their relation to each other is the subject of Hegel's *Encyclopaedia*.

The *Encyclopaedia* contains the system in its full scope. The first part is directed to Logic, really to what is properly Metaphysics, for the abstract notions with which it is concerned lie at the foundations of all experience. But they are not the only forms in which that experience is presented. In the "Philosophy of Nature" to which the second part is devoted, he sets out his view of thought as it enters into the fashioning of the external world, a world directly apprehended as a not-self. This world, of which we take ourselves to be immediately aware, is a form which is essential in our experience, apart from which it has no reality nor significance. In the Logic the categories are set out as abstractions in their relations to each other. They form an entirety which he calls the Idea, an entirety in which every phase, from mere being onwards, has significance only as a stage or level in the whole. Every category, every determination which mind makes, is dynamic. It is an activity in which form develops itself out of form in a process where negation or determination is the characteristic feature. The system of these abstract categories constitutes what he calls the "Idea," a system in which every feature implies and involves all the others. It is this complete system of notional categories that has its logical counterpart and complement in Nature. It is a mistake to suppose that Hegel deduces Nature from his categories. Thought for him does not make things. It is exemplified in Nature in the form of externality. But mind in this abstract form is not yet actual. It only becomes so in a logical development later than both Logic and its other, Nature, in which both become actual for the first time in the concrete experience in which they are real only as moments, separable solely through reflection. The categories as realized in this latest form can be traced just as they can in the other form to which they give the aspect of externality. It is the same single process throughout. Nature and thought imply each other, but neither creates the other.

In the third part of the system, in the individual objects of our experience, we find these objects considered as actual. Here the self-evolution of thought exhibits itself in the shapes which the moulding influence of reflection gives to the contents of our self-consciousness. The shapes are more concrete than those of the Logic, with its system of abstract notions. They are more nearly of the nature of self-conscious knowledge than they could have been had the antithetical phase of externality not been there for the self to distinguish itself from. But in all cases it is thought in some form that we are concerned with. Subject and object must be distinguished by us, but they fall within a larger entirety. The investigation of the content of self-consciousness in the "Philosophy of Mind," the third part of the *Encyclopaedia*, is the investigation of actual experience and the bringing to light of the logical constituents implied in it. It is a question throughout of disentangling and setting out categories. What is not reached nor exhausted is just that moment of the particular, itself an abstraction but one that is inevitable, which is ever present in finite experience but which cannot be even defined, for to define it would be to bring it under some category and so to turn it into a universal. Our thought does not create things. It finds itself in them. They belong to our experience, which we have not to explain because it is the starting-point of which we cannot rid ourselves.

There is for Hegel an ideal towards which we are ever striving, although it is only in abstract reflection and consequently in general terms that we can ever approach it. He holds that he has to assume such an ideal as the completion of his system. That is how he comes to the idea of what is absolute.

The system, then, first of all is concentrated on the character of the categories, not isolated from each other in a table, as with Kant, but as the limitless self-determination of knowledge in an abstract form. Hegel then examines the antithesis to itself which this self-determination implies in order to attain to the status of self-conscious mind. The antithesis is not that of the indefinably particular to a universal. It is no creation *ab extra*. It is the system of universals reproducing itself in the form of what for mind is apparent externality to itself, of object as set over against subject. The two phases are actual and cease to be unreal abstractions only in mind itself, which is the degree below which actual experience cannot be attained. But here as elsewhere in the self-determination of thought the activity of the movement of reflection in categories is present in the same fashion, only at a level which is higher than in the stages which so far as logic is concerned are earlier. We now reach Mind with its self-development towards an ideal form in which the antithesis between universal and particular and between subject and object has been superseded, and both belong to the whole in which the distinction between them will, like all other distinctions, be found to be one that is due only to thought itself. In that way and in that way only are we impelled towards the ideal of an Absolute outside which nothing has any meaning and within which all falls, an Absolute which we cannot envisage in itself.

This is the principle, and the disclosure of the self-arranging relation through the activity of thought is the method. We have travelled further than Kant. The criticism of knowledge by itself has taken a new shape. Behind knowledge we cannot go. There is no standard of truth save in its own process.

Were this investigation of the forms of thought only a general and superficial one, the student of philosophy might prefer to pass it by as being uncertain and highly speculative. But the investigation is so thorough and is carried into detail with such care and illuminating illustration that it is not safe to neglect it. For if the method be a true one it seems to solve difficulties with which Kant refused even to try to cope by means of the restricted instrument which was all that he had in his hands. It is not too much to say that no student of philosophy can be sure of the ground he is treading unless he has made the effort required to follow out what Hegel claims to tell him. For the task is to get a clear conception of how thought enters into every form of experience in ways that determine the character of that experience in its differing shapes and grades and orders. Hegel may or may not have succeeded in giving a complete account of how this task is to be performed, but at least he has realized fully what it implies, and has warned us to scrutinize carefully the categories which we employ, often with little regard to the question whether they are appropriate to their subject-matter.

HALDANE

EDITOR'S NOTE

It may seem a matter of surprise that the "Larger Logic" which has been described as the "Bible of Hegelianism" should now, for the first time, more than a century after its publication, be translated into English. It would seem to be a case of the good being the enemy of the better. In 1865 appeared J. Hutchison Stirling's *Secret of Hegel* in which was included "a translation from the Complete Logic of the whole first section, Quality" and "a Summary of the second section, Quantity." This was followed in 1874 by William Wallace's translation of the "lesser Logic" of the *Encyclopaedia* with the accompanying *Prolegomena* republished in separate volumes in 1892 and 1894 respectively. With these present aids British students, unfamiliar with German, have hitherto been fain to be content. In America the history of the larger Logic has been different though up to the present time, so far as publication is concerned, without more fortunate result. So early as 1858 a group of scholars in St. Louis, Missouri, became acquainted with the book of which Henry C. Brockmeyer, a German who had come while quite a boy to America, executed unaided a complete translation into English. This was copied out by William T. Harris, subjected to revision by some of Brockmeyer's friends, and again before his death revised by Brockmeyer himself, but left by him unpublished.

Some years ago the Editor of the Library of Philosophy was approached by the surviving friends of Henry Brockmeyer with a view to the publication in that series of the translation which he had left. It was to be accompanied with a short biography of the translator, and to partake of the character of a tribute to his memory both as a philosopher and as Governor of the State of Missouri. As it seemed inappropriate to have a volume of this kind included in a series devoted to the pure study of philosophy, it was impossible to accept this offer, and as there seemed no immediate prospect of the American translation coming out, the Editor felt himself free to make an arrangement with the present translators for the publication of a work begun some years before by distinguished Cambridge scholars both of whom have since died. He wishes, however, to dissociate himself from any spirit of rivalry with the American translation. Each must stand on its own merits. Whatever Brockmeyer's may possess, its romantic history and the distinguished names with which it also is associated ought to assure it a welcome on this as well as on the other side of the Atlantic.

TRANSLATORS' PREFACE

This translation is made from the Fourth Edition of the *Wissenschaft der Logik*, issued in 1923 by Dr. Georg Lasson of Berlin. The book was first published at Nuremberg in three parts, which appeared respectively in 1812, 1813, and 1816. The second edition was printed in 1833, and the third in 1841. No English version of this book has hitherto been published. The translation made by Wallace and published under the title *The Logic of Hegel* in 1874 is a translation of the section on Logic in Hegel's later book the *Encyklopädie der philosophischen Wissenschaften.* Wallace's book has been long and deservedly popular; but at the same time a need has often been felt for an English version of the earlier, fuller, and more authoritative Logic, the corner-stone of the Hegelian system, upon which estimates of Hegel must always ultimately be based.

The translation was begun, at the suggestion of the late Dr. McTaggart, by the late Miss Constance Jones (formerly Mistress of Girton). The first fifty pages of the text represent in a finished state the work which she had roughly drafted before her death in 1922. Permission to use this initial work and to incorporate it with their own was kindly granted to the present translators by her sister and literary executrix Miss Parnell Jones; and we, in completing the work from this point, have (like Miss Jones) adhered as closely as possible to the terminology used by McTaggart in his *Commentary on Hegel's Logic* (Cambridge University Press, 1910), departing from it only where the exigencies of translation made it imperative to do so in order to avoid ambiguity. In point of interpretation too our chief guide has been this Commentary, though we have derived assistance also from the *Logique de Hegel* of M. Georges Noël (Paris, Félix Alcan, 1897). It may further be added that there is in McTaggart's book an exact account of the relation of this, the "Greater," Logic to the Logic of the *Encyclopaedia* mentioned above. It thus results that McTaggart's *Commentary* may be used in direct connexion with this translation, and we have therefore felt ourselves absolved from the task of supplying explanatory notes or appendices, in which in any case we could not hope to excel or usefully to supplement the work of McTaggart. We must here also, as former pupils of Dr. McTaggart at Trinity College in Cambridge, express our gratitude for the help and encouragement which he gave us in the early stages of the work, and our deep sense of the irreplaceable loss, in this as in all ways, which we suffered by his untimely death.

Little attempt has been made to impart grace to Hegel's remarkable style. No doubt roughness and obscurity abound; but these qualities are often a necessary consequence of Hegel's peculiar conception of his subject-matter and attitude to it; though indeed we have occasionally been tempted to think that he was rough and obscure from sheer devilry. But our aim throughout has been to represent as exactly as possible in English what Hegel was apparently trying to say in German, regardless of the notorious "Himalayan" severity and the strangeness of phraseology. Without doubt, in many places we have failed to understand the thought perfectly. Moreover, our collaboration has been carried on in circumstances of difficulty during the leisure hours of four years, for half of which we have been widely separated in space across Europe. This, however, though it may be taken as an apology, is not put forward as an excuse for any shortcomings in a work now deliberately published; and it is our hope that we have not so frequently or so completely misrepresented Hegel's meaning as to render our translation entirely useless.

We wish to acknowledge our debt of gratitude to Dr. C. D. Broad of Trinity College, Cambridge, and Professor J. C. Burkill of the University of Liverpool, for valuable help in the notes on the Mathematical Infinite and the Differential Calculus; to Dr. G. Lasson, of Berlin, for assistance in elucidating obscurities in terminology and text; and to Mr. R. L. Collett, M.A., Fellow and Assistant Secretary of the Institute of Chemistry of Great Britain and Ireland, for aid in many passages requiring detailed knowledge of chemistry. We are also under obligation to Professor J. H. Muirhead, General Editor of the Library of Philosophy, and to Messrs. George Allen and Unwin's Philosophical Reader, for numerous helpful suggestions throughout.

The publication of this work was made possible by grants of money made by the Council of Trinity College, Cambridge, and by certain other Benefactors, our gratitude to whom we take this opportunity of expressing.

L. G. S.
W. H. J.

August 1928

TABLE OF CONTENTS

PAGE
INTRODUCTORY PREFACE 7
EDITOR'S NOTE 17
TRANSLATORS' PREFACE 19
TABLE OF CATEGORIES . . . . *facing page* 24
GLOSSARY 25

VOLUME I: OBJECTIVE LOGIC

PREFACE TO THE FIRST EDITION 33
PREFACE TO THE SECOND EDITION 39
INTRODUCTION:
General Concept of Logic 53
General Classification of Logic 70

BOOK ONE

THE DOCTRINE OF BEING

WITH WHAT MUST THE SCIENCE BEGIN? 79
GENERAL CLASSIFICATION OF BEING 91

SECTION ONE

DETERMINATENESS (QUALITY) . . . 93
CHAPTER
I. BEING 94
A—Being 94
B—Nothing 94
C—Becoming 95
1. Unity of Being and Nothing 95
Observation 1. The Opposition of Being and Nothing in Imagination 95
Observation 2. The Inadequacy of the Expression "Unity" or "Identity of Being and Nothing" 102
Observation 3. The Isolation of these Abstractions . . . 106
Observation 4. Incomprehensible Nature of the Beginning . 116
2. Moments of Becoming: Arising and Passing Away . . . 118
3. Transcendence of Becoming 118
Observation: The Expression "to transcend" 119

CHAPTER PAGE
II. DETERMINATE BEING 121
A—DETERMINATE BEING AS SUCH 121
(a) Determinate Being in General 121
(b) Quality 123
Observation: Quality and Negation 124
(c) Something 127
B—FINITUDE 129
(a) Something and an Other 129
(b) Determination, Modification, and Limit 135
(c) Finitude 141
(α) The Immediacy of Finitude 142
(β) Barrier and Ought 144
Observation: Ought 146
(γ) Transition of Finite into Infinite 149
C—INFINITY 150
(a) Infinity in General 150
(b) Reciprocal Determination of Finite and Infinite . . . 151
(c) Affirmative Infinity 156
Transition 163
Observation 1. Infinite Progress 164
Observation 2. Idealism 168

III. BEING FOR SELF 170
A—BEING FOR SELF AS SUCH 170
(a) Being Determinate and Being for Self 171
(b) Being for One 172
Observation 173
(c) One 176
B—THE ONE AND THE MANY 177
(a) The One in Itself 178
(b) The One and the Void 178
Observation: The Atomistic Principle 179
(c) Many Ones. Repulsion 180
Observation: Leibniz's Monad 182
C—REPULSION AND ATTRACTION 183
(a) Exclusion of the One 183
Observation: Proposition of the Unity of the One and the Many 185
(b) The one One of Attraction 186
(c) The Relation of Repulsion and Attraction 187
Observation: Kant's Construction of Matter out of Attractive and Repulsive Force 192

SECTION TWO

PAGE
MAGNITUDE (QUANTITY) 198

Observation 199

CHAPTER
I. QUANTITY 201

A—PURE QUANTITY 201
Observation 1. The Concept of Pure Quantity 202
Observation 2. Kant's Antinomy of the Indivisibility and Infinite Divisibility of Time, Space, and Matter 204

B—CONTINUOUS AND DISCRETE MAGNITUDE 213
Observation: The Customary Distinction between these Magnitudes 214

C—LIMITATION OF QUANTITY 215

II. QUANTUM 217

A—NUMBER 217
Observation 1. The Species of Calculation in Arithmetic. Kant's Synthetic Propositions *a priori* of Intuition 219
Observation 2. Employment of Numerical Determinations to express Philosophic Concepts 227

B—EXTENSIVE AND INTENSIVE QUANTUM 232
(*a*) Their Difference 232
(*b*) Identity of Extensive and Intensive Magnitude 235
Observation 1. Examples of this Identity 236
Observation 2. Kant's Application of Degree-Determination to the Existence of the Soul 239
(*c*) The Alteration of Quantum 240

C—THE QUANTITATIVE INFINITY 241
(*a*) Its Notion 241
(*b*) The Quantitative Infinite Progress 242
Observation 1. High Esteem enjoyed by the Progress to Infinity 244
Observation 2. Kant's Antinomy of the Temporal and Spatial Limitedness—and Unlimitedness—of the World . . 249
(*c*) The Infinity of Quantum 253
Observation 1. The Conceptual Determinateness of the Mathematical Infinite 256
Observation 2. The Purpose of the Differential Calculus deduced from its Application 291
Observation 3. Further Forms connected with the Qualitative Determinateness of Magnitude 321

CHAPTER PAGE
III. THE QUANTITATIVE RATIO 333
A—The Direct Ratio 334
B—The Inverse Ratio 336
C—The Ratio of Powers 340
Observation 343

SECTION THREE

MEASURE

MEASURE 345
I. THE SPECIFIC QUANTITY 351
A—The Specific Quantum 351
B—Specifying Measure 355
(*a*) The Rule 355
(*b*) The Specifying Measure 355
Observation 357
(*c*) Relation of Both Sides as Qualities 357
Observation 360
C—Being for Self in Measure 362
II. REAL MEASURE 367
A—The Relation of Stable Measures 368
(*a*) Union of Two Measures 368
(*b*) Measure as a Series of Measure-Relations 371
(*c*) Elective Affinity 374
Observation: Berthollet on Chemical Elective Affinity and Berzelius's Theory on this Subject 376
B—Nodal Line of Measure-Relations 386
Observation: Examples of such Nodal Lines; *natura non facit saltum* 388
C—The Measureless 391
III. THE BECOMING OF ESSENCE 394
A—The Absolute Indifference 394
B—Indifference as Inverse Relation of its Factors . . . 395
Observation: On Centripetal and Centrifugal Force . . 399
C—Transition to Essence 403

GLOSSARY

OF SOME PRINCIPAL RECURRENT EQUIVALENTS USED IN THIS TRANSLATION

NOTE.—Words used to translate the names of categories, or in a technical sense, are spelt with initial capitals.

1. GERMAN—ENGLISH

an sich	in itself, implicitly
Anderssein	otherness
Anderswerden	other-becoming
Anschauung	intuition
Ansichsein[1]	Being-in-Self
Anstoss	resistance-principle
Anzahl	amount
aufheben	transcend, cancel
auflösen	dissolve, resolve
Aussereinandersein	juxtaposition
begreifen	conceive, form a Notion of
Begriff	concept, Notion
bei sich	with itself
beschränken	restrict
Bestimmtheit	determinateness
Bestimmtsein	determinedness
Bestimmtwerden	becoming-determinate
Bestimmung	determination
Beziehung	relation
Beziehung auf sich	self-relation
Dasein	Determinate Being, existence
Dasein, Schluss des (*category*)	Qualitative Syllogism
Dasein, Urteil des (*category*)	Judgment of Inherence
Ding	thing
Einheit	unity, or (as opposed to *Anzahl* = "amount") unit
Einteilung	classification, division
Entschluss	resolution
entstehen	arise
erscheinen	appear
Erscheinende, das	Apparent, the
für sich	for itself, by itself
Gegensatz	opposition
Gegenstand	object

[1] NOTE.—(etwas) ist an sich *is* in itself
ist an ihm *has* in itself

Gesetztsein	positedness
Gleichgültigkeit	indifference
Gleichheit	equality, likeness
Grenze	limit
Grundlage	foundation
hinausgehen	overpass
ideal	ideal
ideell	of ideal nature
in sich	within itself
Inbegriff	sum-total
Indifferenz	Indifference
Insichreflektiertsein	intro-reflectedness
Lehrsatz	proposition, theorem (in Cognition: Proposition)
Mannigfaltigkeit	multifariousness, manifoldness, multiplicity (in qualitative as well as quantitative sense)
Mehrere	heap
Mehrheit	manyness
Menge	multitude,number (in Elective Affinity: amount)
Merkmal	characteristic
Objekt	Object
real	real
realisieren	realize
Realität	reality
reell	of real nature
Reflexion in sich	intro-Reflection
Sache	fact, case
scheinen	seem, show (intransitive)
Schranke	barrier
Seele	(in Life this word is not translated)
Sein	Being
setzen	posit
Streben	tendency
Trieb	impulse
Übergang	transition
übergehen	pass over
untergehen	perish
Unterschied	distinction, difference
Unterschiedenheit	differentiation
Urteil	Judgment. (After the categories of Judgment, and also in the Introduction, Hegel seems frequently, in using this word, to be mindful of its etymological meaning, "fundamental division.")

vergehen	pass away
Verhalten	attitude
Verhältniss	ratio, proportion, relation
Vernunft	reason
Vernünftig	reasonable
Verschiedenheit	variety
Verstand	understanding
verständig	rational
Vielfachheit	muluiplicity
Vielheit	plurality
Voraussetzung	presupposition
Vorstellung	sensuous representation, image, imagination, presentation, idea, general idea, ideation

Note.—This word, which has occasioned great difficulty in translation, commonly stands in Hegel for something between sensuous intuition (*Anschauung*) or perception (*Wahrnehmung*) on the one hand, and Notion (*Begriff*) on the other. It usually means representation before the mind, in sensuous or semi-sensuous form, of some particular or universal. It is applied sometimes to a faculty, sometimes to an activity, and sometimes to the product of an activity. Often it corresponds roughly to the "general idea" of the English empiricists, occasionally to the "image" of modern psychology.

Wahrnehmung	perception
Widerspruch	contradiction
Widerstreit	conflict
Wirklichkeit	actuality

zugrundegehen	perish
zugrundeliegend	fundamental
zusammenfallen	collapse
zusammengehen	coincide

2. ENGLISH—GERMAN

actuality	Wirklichkeit
amount	Anzahl
Apparent, the	Erscheinende, das
appear	erscheinen
arise	entstehen
attitude	Verhalten
barrier	Schranke
becoming-determinate	Bestimmtwerden
Being-in-Self	Ansichsein
by itself	für sich
cancel	aufheben
case	Sache
characteristic	Merkmal
classification	Einteilung
coincide	zusammengehen
collapse	zusammenfallen
conceive	begreifen
concept	Begriff
conflict	Widerstreit
contradiction	Widerspruch
Determinate Being	Dasein
determinateness	Bestimmtheit
determination	Bestimmung
determinedness	Bestimmtsein
difference	Unterschied
differentiation	Unterschiedenheit
dissolve	auflösen
distinction	Unterschied
division	Einteilung
entity	(this word used with an adjective or participle represents a German adjective or participle in the neuter gender used as a noun)
equality	Gleichheit
existence	Existenz, Dasein
Existence	Existenz (*category*)
fact	Sache
for itself	für sich
form a Notion of	begreifen
foundation	Grundlage
fundamental	zugrundeliegend
general idea	Vorstellung

heap	Mehrere
idea	Vorstellung
Idea	Idee
ideal	ideal
ideal nature, of	ideell
ideation	Vorstellung
image	Vorstellung
imagination	Vorstellung
implicitly	an sich
impulse	Trieb
in itself	an sich
indifference	Gleichgültigkeit
Indifference	Indifferenz
individual, the	Einzelne, das
Individual	Individuum (in The Living Individual
Inherence	Inhärenz
Inherence, Judgment of	Urteil des Daseins (*category*)
intro-reflectedness	Insichreflektiertsein
intro-Reflection	Reflexion in sich
intuition	Anschauung
Judgment of Inherence	Urteil des Daseins (*category*)
juxtaposition	Aussereinandersein
likeness	Gleichheit
limit	Grenze
manifoldness	Mannigfaltigkeit
manyness	Mehrheit
multifariousness	Mannigfaltigkeit
multiplicity	Vielfachheit, Mannigfaltigkeit
multitude	Menge
Notion	Begriff
number	Zahl, Menge
object	Gegenstand
Object	Objekt
opposition	Gegensatz
other-becoming	Anderswerden
otherness	Anderssein
overpass	hinausgehen
pass away	vergehen
pass over	übergehen
perception	Wahrnehmung
perish	untergehen, zugrundegehen
plurality	Vielheit
posit	setzen
positedness	Gesetztsein
presentation	Vorstellung
presupposition	Voraussetzung

proportion	Verhältniss
proposition	Satz, Lehrsatz
Qualitative Syllogism	Schluss des Daseins (*category*)
ratio	Verhältniss
rational	verständig, rationell
real	real
real nature, of	reell
reality	Realität
realize	realisieren
reason	Vernunft
reasonable	vernünftig
relation	Beziehung, Verhältniss
resistance-principle	Anstoss
resolution	Entschluss
resolve	auflösen
restrict	beschränken
seem	scheinen
self-relation	Beziehung auf sich
sensuous representation	Vorstellung
show (intransitive)	scheinen
sum-total	Inbegriff
tendency	Streben
theorem	Lehrsatz
thing	Ding
transcend	aufheben
transition	Übergang
understanding	Verstand
unit	Einheit (as opposed to *Anzahl* = "amount")
unity	Einheit
variety	Verschiedenheit
with itself	bei sich
within itself	in sich

NOTE.—The biographical footnotes which will be found in various parts of this work are not part of Hegel's text. They were added by the German Editor, and it seemed worth while to take them up into this translation.

VOLUME I

OBJECTIVE LOGIC

PREFACE TO THE FIRST EDITION

THE complete transformation which philosophical thought has undergone in Germany during the last five-and-twenty years[1] and the loftier outlook upon thought which self-conscious mind has attained in this period, have hitherto had but little influence on the structure of Logic.

That which before this period was called Metaphysic, has been, so to speak, extirpated root and branch, and has disappeared from the ranks of the Sciences. Where could one now catch an echo—where would any echo venture to linger—of the Ontology, the Rational Psychology, the Cosmology, or even the Natural Theology, of former times? Where, for instance, would investigations concerning the Immateriality of the Soul, or Efficient and Final Causes—where would these now arouse any interest? And the other proofs of the existence of God are now brought forward only from an historical standpoint, or with a view to edification and spiritual uplifting. It is the fact that men have lost interest partly in the content of the old Metaphysic, partly in its form—and partly in both content and form. If it is a remarkable thing when a nation finds that its Constitutional Theory, its customary ways of thinking and feeling, its ethical habits and traditional virtues, have become inapplicable, it is certainly not less remarkable when a nation loses its Metaphysic, when the intellect occupying itself with its own pure essence, has no longer any real existence in the thought of the nation.

The exoteric doctrine of Kantian Philosophy that Understanding cannot go beyond Experience, because if so the faculty of cognition would be a merely theoretical intelligence which could by itself produce nothing but idle fancies of the brain—this doctrine has given a scientific justification to the renunciation of Speculative Thought. This popular doctrine was supported by the cry of modern educationalists, voicing the needs of the hard times, which draw men's attention to immediate requirements; it was clamoured that as for knowledge experience is the starting point, so for ability in public and in private life, theoretical insight is actually injurious, while it is practice and technical education which are above all essential, and alone lead to better things. *Philosophy* and crude *Common Sense* playing thus into each other's hands for the downfall of Metaphysic, there was presented the strange spectacle of a cultured people having no Metaphysic—as it were a temple, in all other respects richly ornamented, but lacking its Holy of Holies.—Theology, which in earlier times had been the guardian of speculative mysteries, and of a Metaphysic subordinate to itself, had given up this science in exchange for emotions, for popular practicality, and learned historicity. On the other hand, in correspondence with this change, those lonely souls who were sacrificed by their fellows, and isolated from the world, to the end that contemplation of the eternal and a life dedicated thereto should be maintained—the life and the contemplation being for the sake of an ideal and not of a practical Good—these devotees passed out of existence;—and their passing away may, from another point of view, be regarded as essentially one with the phenomenon of which we have already spoken. And so, when such metaphysical shadows, and such colourless self-concentration of the introspective spirit, had been brushed aside, existence seemed to be transformed into the sunny land of flowers—and, as we know, no flowers are black.

---

[1] This was written in 1812 (Translators' note).

It did not fare quite so ill with Logic as with Metaphysic. The view that by Logic one would learn how to think (the usefulness and hence the purpose of Logic being supposed to consist in this)—which was just as though one were to expect to learn how to digest and how to move, by the study of anatomy and physiology—this prejudice has long ago been exploded, and the spirit of practicality probably intended for Logic a fate no better than that which had fallen to the lot of the sister Science. In spite of this, however, and probably on account of some formal utility, a place among the Sciences was left to Logic, and it was even retained as a subject of public instruction. But this better fate concerned only externals, for the form and content of Logic had remained the same that it had inherited by long tradition—a tradition which in being handed down had become ever more meagre and attenuated; there are no traces in Logic of the new spirit which has arisen both in Learning and in Life. It is, however (let us say it once for all), quite vain to try to retain the forms of an earlier stage of development when the inner structure of spirit has become transformed; these earlier forms are like withered leaves which are pushed off by the new buds already being generated at the roots.

But even in the scientific sphere this ignoring of the universal change is beginning to fail. Imperceptibly the new ideas became familiar even to their opposers, who appropriated them and—though persistently slighting and gainsaying the sources and principles of these ideas—yet had to accept their results, and were unable to evade their influence. The only way in which opposers could give content and positive value to their negative attitude (which was getting to be of ever less and less importance) was by giving in their adherence to the new ways of thinking.

On the other hand, the period of fermentation with which a new creation begins seems to be past. At its first appearance such a period generally wears an aspect of fanatical hostility towards the prevalent systematization of the older principle; it is also, partly, fearful of losing itself in the wilderness of particulars while it shuns the labour required for scientific development, and in its need of such a development grasps, at first, at an empty formalism. The demand for the digestion and development of the material now becomes so much the more pressing. This is a period in the development of an age, as in the development of an individual, when the chief business is to acquire and maintain the principle in its undeveloped intensity. But the higher requirement is that the principle should be elaborated into systematized knowledge.

Still, whatever may have been already done for the cause and for the form of philosophy in other respects, the logical science which is the true content of genuine Metaphysic or pure speculative philosophy has heretofore been very much neglected. What I more exactly understand by this Logic and its standpoint I have set forth provisionally in the Introduction. The necessity of once more beginning this science from the very beginning, the nature of the subject itself, and the absence of previous work which could be used in the projected transformation, should all be taken into account by fair and reasonable judges—even though the labour of many years has not been able to secure for this attempt a nearer approach to perfection. The essential point of view is, that we have to do, altogether, with a new concept of philosophical method. As I have elsewhere recalled[1] Philosophy, since it is to be Ordered Knowledge, cannot borrow its Method from a subordinate science, such as Mathematics, any more than it can rest satisfied with categorical assertions of pure intuition, or use reasonings based on external reflection. But it is the nature of the content and that alone which lives and stirs in philosophic cognition, while it is this very reflection of the content which itself originates and determines the nature of philosophy.

Understanding makes determinations and maintains them. Reason is negative and dialectical because it dissolves into nothing the determinations of Understanding; Reason is positive because it is the source of the Universal in which the Particular is comprehended. Just as Understanding is commonly held to be something separate from Reason regarded generally, so dialectical Reason is held to be something separate from positive Reason. But in its real truth Reason is Mind—Mind which is higher than either Reason which understands, or Understanding which reasons. Mind is the negative, it is that which constitutes the quality alike of dialectical Reason and of Understanding; it negates the simple and thus posits that determinate distinction which is the work of Understanding, and just as truly it resolves this distinction, and is thus dialectical. Yet it does not abide in the negation which thus results, but is *therein just as much positive*,—thus it has thereby established the first Simple, but so that the Simple is also a Universal which is in itself concrete; under this universal a given Particular is not subsumed; but, in that determination, and in the solution thereof, the Particular has already been coincidently determined. This movement of Mind, which in its simplicity gives itself its determinateness and hence self-equality, and which thus is the immanent development of the Notion—this movement is the Absolute Method of knowledge, and at the same time the immanent soul of the Content of knowledge.—It is, I maintain, along this path of self-construction alone that Philosophy can become objective and demonstrated science.—It is after this fashion that I have tried to present consciousness in the *Phenomenology of Spirit*. Consciousness is Spirit as knowing which is concrete and engrossed in externality; but the *schema of movement* of this concrete knowing (like the development of all physical and intellectual life) depends entirely on the nature of the pure essentialities which make up the content of Logic. Consciousness, as manifested Spirit which as it develops frees itself from its immediacy and external concretions, becomes Pure Knowing, which takes as object of its knowing those pure essentialities as they are in and for themselves. They are pure thought, Spirit thinking its own essence. Their spontaneous movement is their spiritual life: by this movement philosophy constitutes itself; and philosophy is just the exhibition of this movement.

---

[1] *Phenomenology of Spirit*, Preface to the first edition.—The special exposition is the understanding of Method, and this has its place in Logic itself. [1831.]

I have thus indicated the relation to Logic of the science which I call *Phenomenology of Spirit*. With regard to the external arrangement, it was intended that the first part of the *System of Knowledge*[1] which contains the *Phenomenology*, should be followed by a second part which should contain Logic and the two concrete Philosophical Sciences, the Philosophy of Nature and the Philosophy of Spirit, thus completing the System of Knowledge. But the extensive elaboration demanded by *Logic* has caused me to allow this portion to be published separately; thus, in an enlarged scheme, Logic constitutes the first sequel to the *Phenomenology of Spirit*. I shall work out later the two concrete Philosophical Sciences already spoken of.—The First Book of this first volume of the Logic is *The Doctrine of Being*; the Second Book contains *The Doctrine of Essence*, which is the second division of the first volume; the second volume will contain *Subjective Logic*, or *The Doctrine of the Notion*.

NÜRNBERG,
22nd March 1812.

---

[1] (Bamberg and Würzburg, Göbhard, 1807.) This title will not be given to the second edition, which will come out next Easter.—Instead of the projected Second Part (mentioned in the sentences that follow above) containing all the rest of the philosophical sciences, I have since brought out the *Encyclopaedia of the Philosophical Sciences*, which last year was in its third edition. [1831.]

PREFACE TO THE SECOND EDITION

I HAVE embarked upon the new elaboration of the *Science of Logic* of which this is the first volume, with full consciousness both of the difficulties of the subject in itself and of its exposition, and also of the imperfection of the elaboration contained in the first edition; earnestly as I have striven, after many years' further occupation with the subject, to remedy this imperfection, I still feel that I have good reason to bespeak the indulgence of my readers. The title to such a claim for indulgence may well be based in the first instance upon the circumstance that for the content of the science hardly anything but merely external material was to be found in the earlier Metaphysic and Logic. Though both of these subjects have been universally and assiduously cultivated, the latter even up to our own day, the speculative aspect has met with but slight attention; on the contrary, we have, for the most part, the same things repeated again and again—sometimes thinned down to shallow triviality, while sometimes the old ballast is unloaded afresh and dragged about in ever greater bulk after such a fashion that the efforts expended—to a large extent merely mechanical—could bring no gain to the philosophic content. Hence it had come to pass that to present the realm of thought in its philosophical aspect—that is, in its own immanent activity, or (which comes to the same thing) in its necessary development—this had to be a new undertaking, and to be begun from the very beginning; but the traditional material—the well-known forms of thought—must be regarded as a highly important pattern—in fact a necessary condition, a presupposition to be thankfully acknowledged, even if only providing here and there a barren clue or, as it were, the lifeless bones of a skeleton, sometimes even flung together in disorder.

It is in human Language that the Forms of Thought are manifested and laid down in the first instance. In our day it cannot be too often recalled, that what distinguishes man from the beasts is *the faculty of Thought*. Language has penetrated into whatever becomes for man *something inner*—becomes, that is, an idea, something which he makes his very own;—and what man transforms to Language contains—concealed, or mixed up with other things, or worked out to clearness—a Category; so natural to man is Logic—indeed, Logic itself is just man's peculiar nature. But if Nature in general is opposed, as *physical*, to what is *mental*, then it must be said that Logic is rather that something Super-natural which enters into all the natural *behaviour* of man—Feeling, Intuition, Desire, Need, Impulse—and thereby alone transforms it all to something human—to ideas and purposes—though, perhaps, only formally human. It is a great advantage to a language when it has a wealth of logical expressions—that is, expressions characteristic and set apart—for the determinations of thought; of prepositions and articles many belong to those relationships which depend upon thinking; the Chinese language is said not to have developed so far, or only in a very small degree; these particles in fact perform an entirely subordinate office, the same as prefixes and suffixes, and in an only slightly more independent form. It is much more important that in a language the determinations of thought should be manifested in Substantives and Verbs and thus receive the stamp of objective form; the German language has here many advantages over other modern languages; indeed, many of its words have the further peculiarity that they have not only various, but even opposed, meanings, so that we must recognize here a speculative spirit in the language; it is a joy to thought to stumble upon such words, and to meet with the union of opposites (a result of Speculative Thought which to Human Understanding seems senseless) in the naïve shape of one word with opposite meanings registered in a dictionary. For this reason, in German, Philosophy for the most part requires no peculiar terminology. Of course some words from foreign languages (which indeed have already acquired by prescription the right of citizenship in the philosophic realm) have to be adopted in German, and an affected purism would be least in place where it is the thing and not the word that is of capital importance. The progress of culture generally, and of the sciences in particular, gradually brings to light higher relations of thought, or at any rate raises these relations to greater generality, and thereby attracts to them more attentive consideration. This is true even of those sciences which relate to what is empirical and sensuous, since they use in general the most familiar categories (for example, Whole and Parts, a Thing and its Properties, and the like). For instance, though in Physics the idea of Force had become supreme, in more modern times the most important part here has been played by the Category of Polarity—which indeed has been too much dragged in everywhere at random, and even into the theory of Light. In this determination of thought a distinction is drawn, while the things distinguished are inseparably bound up together. It is of infinite importance that in this way the abstract form (Identity) by which a thought-determinateness is endowed with independence (as, for example, Force) has been abandoned, and the form of the determination, of a distinction which remains all the while in identity because it is inseparable, is emphasized and becomes a current idea. Owing to the reality which appertains to natural objects, the observation of nature compels us to establish those natural Categories which we cannot ignore even when they may be thoroughly incoherent with others to which also validity is allowed, and does not permit here that passage from opposites to abstracts and universals, which more easily takes place in the case of ideational objects.

But whilst logical objects and the expression of them are thus something that is everywhere familiar in *cultivated thought*, still, as I have elsewhere observed, what is familiar is not on that account necessarily understood. It even rouses one's impatience to have to go on merely busying oneself about what is thus familiar—and what is more familiar than just those determinations of thought of which we make use at every turn, which proceed out of our mouths with every sentence that we speak? This foreword is intended to give the fundamental points in that course of the progress of Cognition which starts from what is thus *known and familiar*, and in the relation of Scientific thinking to Natural thinking; this, together with what is contained in the earlier *Introduction*, will suffice to furnish that general notion of the meaning of logical Cognition which one is accustomed to demand in the case of any Science, as a preliminary to the presentation of the Science itself.

In the first place, it is to be regarded as an immense advance that the Forms of Thought should be disengaged from the *Matter of Thought* in which they are imbedded in self-conscious Intuition and Ideation as well as in Desire and Will—or rather (since there is no human Desire nor Will without Ideas) in ideating Desire and Will; it is an immense advance that these Universals should be drawn forth and set up as objects of contemplation on their own account, as was done by Plato and then more especially by Aristotle; we have in this the beginning of knowledge. "It was only," says Aristotle, "after nearly everything that was necessary, and that pertained to the convenience and intercourse of life, had been obtained, that people began to trouble themselves about philosophic knowledge."[1] "In Egypt," he had previously remarked, "the mathematical sciences were early developed, because there the priestly caste at an early period was in such a position as to make leisure possible."[2] In truth the need to busy oneself with pure thought presupposes a long stretch of road already traversed by the mind of man. It is, one may say, the need of a need already satisfied as regards necessaries, the need of an attained absence of need, of abstraction from the matter of intuition, imagination, and so forth—from the concrete interests of Desire, Impulse, and Will, in which the determinations of thought are wrapped up and concealed. In the still spaces of Thought which has come to itself and is purely self-existent, those interests are hushed which move the lives of peoples and of individuals. "In so many directions," says Aristotle in the same connexion, "the nature of man is dependent; but this science, which is not sought for the sake of utility, this alone, in itself and for itself, is free, and seems therefore to be a possession not wholly human."[3] Philosophy generally still has in its thinking to deal with concrete objects —God, Nature, Mind—; but Logic is concerned with such thought wholly and solely on account of the thought itself, in complete abstraction from its objects. It is customary to assign to Logic a place among the studies of youth, because the young have not yet entered upon the interests of concrete life. Youth lives at leisure in respect of these interests; its business is to acquire the means and power of entering actively upon the objects of these interests, and even these objects are considered in a merely theoretic manner. In opposition to the view of Aristotle already quoted, logical science is reckoned as part of this equipment; occupation with Logic is a preliminary business, its place is said to be the school, which ought to precede the seriousness of life and action for solid ends. In Life, categories are *used*—they are degraded from the honour of being contemplated on their own account to *serve* in intellectual exercise upon living content by production and interchange of the ideas appropriate thereto. They serve, first, as *abbreviations*, in virtue of their generality; for what an endless multitude of particulars of external existence and of action do ideas comprise, for example, Battle, War, Nation, or Sea, Animal, and so forth;—how in the Idea of God, or of Love, and so on, in the simplicity of such ideating, we have an endless multitude of ideas, activities, conditions, and so on, epitomized! Secondly, Categories serve for the closer determination and discovery of objective relations, in which, however, content and purpose, the validity and truth of the thought which enters into this process, are made entirely dependent on the material presented, and no efficacy in determining the content is ascribed to the determinations of thought in themselves. Such use of the Categories, which has above been called Natural Logic, is unconscious, and when in scientific contemplation the mind assigns to the Categories the function of serving as means, then Thinking in general is turned into something subordinate to other mental functions. We do not say of our Feelings, Impulses and Interests that they serve us —rather, they are regarded as independent faculties and powers, the fact being that so to feel, thus to desire and will, to take interest in this or that—all this is just what we are. On the other hand again, we are likely to become conscious that we are at the disposal of our Feelings, Impulses, Passions, and Interests, let alone Habits, rather than that we are the owners of these or (still less) that they serve us as means, seeing how intimately they are incorporated in us. Such determinations of disposition and intellect soon show themselves as *Particulars* in contradistinction to that Universality of which we are conscious in ourselves and in which we find our Freedom; and indeed we think that we are entangled in these

---

[1] *Metaph.*, A 2. 982*b*.

[2] *Ibid.*, A 1. 981.

[3] *Ibid.*, A 2. 982*b*.

Particularities, and that they tyrannize over us. Consequently we are much less able to hold that the Forms of Thought serve us than that we serve them—that we are their masters and not much rather that they are our masters—those Forms of Thought which permeate all our ideas, whether those ideas are just theories or have a content of Sensation, of Impulse, or of Will. What surplus have *we* as against them, how shall we—how shall I—set myself up as more universal than they—they who are the Universal itself? When we give ourselves up to a sensation, a purpose, an interest, and then feel ourselves to be limited and unfree, the place into which we can withdraw therefrom and get back into freedom is the place where we are certain of ourselves, the region of pure abstraction, the region of Thought. So, when we mean to speak of things, we call the Nature or Essence of them their Concept—and this exists only for thought: but of the concepts of things we cannot say that we govern them, or that the thought-determinations of which they are the complexes serve us; on the contrary, our thought has to limit itself in accordance with them, and our arbitrary choice or Freedom ought not to try to frame them after its own fancy. In so far, then, as subjective Thinking is our own-est and innermost act, and the objective concept of things constitutes their own reality, we cannot get beyond that own act of ours, we cannot stand above it, and just as little can we get beyond the nature of things. We can, however, disregard the latter determination; it coincides in so far with the former, since it would furnish a relation of our thoughts to the thing, giving, however, only an empty result, the real thing being set up as a standard for our concepts, while that thing can for us be nothing else than our concepts of it. When the Critical Philosophy understands the relation of these three Terms so as to make *Thoughts* intermediary between *Us* and *Things* in such a sense that this intermediary rather excludes us from things than connects us with them, this view may be met by the simple observation that these very things which are supposed to stand beyond ourselves, and beyond the thoughts referring to them, at the opposite extreme, are themselves things of thought, and, as being quite undetermined, are just one such thing (the so-called Thing-in-itself), the product of empty abstraction.

Enough said, however, of that point of view at which the relation vanishes away, according to which determinations of thought are regarded merely as means and instrument; more important is the further point connected therewith, according to which it is customary to regard these determinations as external Forms.—That activity of Thinking which works in all our ideas, purposes, interests, and deeds is, as has been already remarked, un-self-consciously active (Natural Logic); what is present to consciousness is the content, the objects of our ideas, that in which we are *interested*; in this connexion the determinations of Thought are regarded as *Forms*, which are not the content itself but only *attached to the content.* But if it is true, as was asserted above and is generally admitted, that in an object the nature, the peculiar essence, the truly permanent and substantial among the multiplicity and contingency of its appearance and fleeting manifestation, consists in the concept of the thing, in the Universal immanent in it; as every human individual, though infinitely unique, is so only *because* it belongs to the class of man, every animal only *because* it belongs to the class of animal: if this be true, what remains of such an individual if this basis (i.e. the universal) be removed, however many other predicates it have, and although the universal be only one among them? The indispensable basis, the Concept, the Universal, which is Thought itself—in so far, that is, as in using the word *Thought* one can abstract from the idea—this cannot be regarded as a *merely* indifferent form which is attached to some content. But these thoughts of all natural and spiritual things, even the substantial content, are yet such as to possess manifold determinations and to contain the distinction between Soul and Body, between a concept and its respective reality; the deeper basis is the soul in itself, the pure concept, which is the very core of objects, their very life-pulse, as it is the core and pulse of subjective thinking itself. To bring into clear consciousness this *logical* character which gives soul to mind and stirs and works in it, this is our problem. Instinctive action is distinguished from intelligent and free action broadly by this, that the latter is accompanied by clear consciousness; when the content of that which stirs the mind is drawn out of its immediate unity with the Subject, and made an Object for it, then there begins

Freedom for the mind, which while caught in the workings of instinctive mental activity is broken up within the meshes of its Categories into an infinitely various material. In this web strong knots are formed now and then, which are foci of arrest and direction in mental life and consciousness: they owe their firmness and strength to the fact that, brought before consciousness, they are found to be independent concepts of the latter's essentiality. The most important point for the nature of mind is not merely the relation of that which it is *in itself*, but furthermore of that as which it *knows itself*, to that which it is in actuality; this self-knowledge, because it is essentially consciousness, is the fundamental determination of mind's actuality. These Categories function only instinctively and as impulses—they are at first introduced into consciousness piecemeal, and therefore are mutable and mutually confusing, and thus yield to mind only a piecemeal and insecure actuality. To purify these Categories and to raise the mind through them to Freedom and Truth, this it is which is the loftier task of Logic.

What we have laid down as the first step in Philosophy (a step the high value of which both on its own account and also as a condition of genuine cognition we have already recognized)—namely, that concepts in general and the moments of concepts (that is, the determinations of thought) should be treated at first as Forms which are distinct from Matter, and merely attached to it,—this procedure advertises itself at once as inadequate for the attainment of truth—truth, which is announced as the subject-matter and goal of Logic. For, taken thus as bare forms, as distinct from the content, they are taken to be standing in a determination which stamps them as finite, and makes them incapable of comprehending the truth, which is in itself infinite. If the true can, in whatever reference, be elsewhere associated with limitation and finitude, this is its aspect of negation, of untruth and unreality—in fact of its end—not its aspect of affirmation, which it is by virtue of being truth. Against the baldness of the merely formal categories, the instinct of common sense has at last felt itself so confirmed as contemptuously to abandon the knowledge of them to the domain of School Logic and School Metaphysics, with a want of appreciation of the intrinsic value that this clue possesses, and completely unaware that it (that is, common sense) is itself held captive when it adopts the instinctive activities of Natural Logic, and still more when it deliberately rejects both cognition and recognition of thought-determinations,—captive by a mode of thinking that is unpurged and therefore unfree. The simple basic or common determination of all these forms is Identity, which as the Law of Identity as A = A, and as the maxim of contradiction, is maintained in the Logic of this collection of forms. Common sense has so thoroughly lost its reverence for the school which is in possession of these laws of truth and still fosters them, that it derides the school on account of the laws, and would regard anyone as insufferable who, in accordance with these laws, made true statements such as, "The plant is—a plant," "Science is—Science," and so *ad infinitum*. And as to the formulas which constitute the rules of syllogizing (which, in fact, is one of the principal employments of Understanding)—mistaken as it would be not to recognize their place and validity in cognition and the fact that they are essential material for rational thought, yet the equally just view has been established that these laws are, quite as much, impartial instruments of error and sophistry, and—however Truth may be determined —that they are unserviceable for higher Truth—for instance, for religious Truth;—that, broadly, they are merely a matter of epistemological correctness and not of Truth itself.

The inadequacy of this way of regarding thought, which leaves Truth on one side, can only be supplemented when in the contemplation of Thought, Content is included as well as that which is habitually reckoned as belonging to external form. It very soon appears that what at first is to ordinary reflection, as Content, separated from Form, cannot in fact be formless, cannot be without internal determination; if it were so, it would be only emptiness, the abstraction of the Thing-in-itself. It appears that, on the contrary, Content has in itself Form, indeed it is only through Form that it has Soul and subsistence, and that it is Form itself which changes only into the show of a Content, as also into the show of a something external to this show. With this introduction of Content into logical consideration, the concrete concepts of things become the object of thought, instead of the mere things. And in this connexion we may recall that there *is* a multitude of Concepts, a multitude of concrete Things. The way, however, in which this multitude is limited and confined is (as has already been pointed out) partly this, that the Concept as Thought in general, as Universal, immeasurably abbreviates the endless Particularity of mere Things, which in their multitudinousness hover before the eye of indeterminate intuition and ideation; partly, however, *a* Concept is, to start with, *the* Concept in itself, and the Concept is One, and is the substantial basis; and secondly though it is a definite Concept, and this definiteness in it is what appears as Content, yet on the other hand the definiteness of the Concept is a Form-determination of that substantial One-ness, a phase of Form as Totality, of *the Concept itself*, which is the basis of determinate Concepts. This is not sensuously intuited nor ideated; it is only the object, product, and content of Thinking, and the Real Thing which exists in and for itself, the Logos, the reason of that which is, the Truth of what we call *a mere thing* (*Ding*); and it is Logos which should least of all be left outside logical science. We cannot therefore include it in the science, or leave it outside, at our discretion. When the thought-determinations which are only external forms are truly considered in themselves, only their finitude, and the untruth of attempts to make them exist for themselves, show up, and what shows up as their truth is the concept. Hence logical science, since it deals with the determinations of thought (which generally run through our mind instinctively and unreflectively, and—even though they enter into language—remain unidentified and unregarded)—logical science, I say, will be the re-construction of those thought-determinations which are thrown into relief by reflection, and by reflection are fixed as subjective forms, forms external to matter and content.

No unfolding of any subject-matter of Thought is in itself capable of such strictly immanent plasticity as is that of the development of Thinking in accordance with its own necessary Laws; no other carries with it this demand in such a degree of intensity; in this respect the science of Logic must surpass even Mathematics, for no subject-matter of thought has in itself this freedom and independence. By such a free and independent exposition—which after its fashion is present in the process of mathematical thought—it is demanded that there should not appear at any stage of the development, any thought-determination, any reflection, that does not immediately arise at this stage, and that has not passed into it from the stage preceding. However, such abstract perfection of exposition must, I confess, be generally given up; since the Science must begin with the absolutely simple, that is, with what is most general and most empty, the exposition would permit only quite simple expression of that simplicity, without the addition of a single word;—what would be admissible in accordance with the conditions of the case would be negative considerations, the objects of which were to ward off and to banish elements which might otherwise be introduced by imagination or unregulated thinking. Such invasions of the simple immanent process of development are, however, in themselves contingent, and consequently the effort to ward them off is itself tainted with contingency; besides which just because such occurrences lie outside the subject-matter it is vain to try to meet them all, and the systematic completeness demanded would itself be something imperfect. But the peculiar unrest and distraction of our modern consciousness force us to take into account reflections and fancies that lie more or less near at hand. And an explanation which lends itself to the subject calls for a corresponding docility of temper in those who would apprehend and comprehend; but youths and men thus docile, with such tranquil self-denial in the matter of reflections and fancies of *their own*, in which "original" thought is so impatient to manifest itself—listeners (such as Plato feigns) who only want to follow the argument—such could hardly be set up as interlocutors in a modern dialogue; still less could we reckon upon readers of such a temper of mind. Quite the reverse—I have been only too often and too fiercely attacked by opponents who are incapable of the simple reflection that their onslaughts and objections contain categories which are themselves assumptions and themselves need to be criticized before being employed. Want of knowledge in this matter goes incredibly far; it is guilty of the fundamental misunderstanding, the uninstructed and barbarous procedure of taking a category which is under consideration for *something else* and not for what it is. This ignorance is the less to be justified because this something else consists of other thought-determinations and concepts, and in a system of Logic these other categories themselves must likewise have found their place, and be themselves awaiting consideration in their place in the system. Where this is most surprising is in the great majority of the objections and attacks which are directed against the primary concepts or starting-points of Logic, Being and Nothing, and Becoming, as that which contains the two previous determinations (Being and Nothing) as Moments;—Becoming, which is itself without question a simple determination, as appears on the simplest analysis. Thoroughness seems to demand that the beginning—as the foundation upon which everything is to be built—should be examined before anything else, in fact that we should not proceed further until it has proved itself solid, and on the other hand, if it does not prove so, that everything that follows should be rejected. This thoroughness of procedure has at the same time the advantage that it guarantees a vast easing of the work of thought; it has before it, enclosed in this germ, the whole development of the science, and regards itself as having done the whole of its business when it has done the first part of it. This piece of the work is the easiest to dispatch, for it is the simplest, it deals with simplicity itself; it is the trifling labour that is here required that really recommends this "thoroughness" of procedure, which is so well satisfied with itself. This restriction to what is simple gives full scope to the free play of thought—thought which cannot go on being simple, but must bring in its reflections on the subject. Having good right to be busied at first *only* with the fundamental principle and, at that stage, not to allow itself to enter upon what is *beyond*, this thoroughness, even when employed on its own business, works in a contrary sense, and brings in what goes beyond the fundamental principle—it brings in, that is, other categories and other presuppositions and prejudices. Such presuppositions as that Infinity is different from Finitude, that Content is other than Form, that the Inner is other than the Outer, that Mediacy is not Immediacy—as though anybody did not know such things—are brought forward by way of information, and related and insisted upon rather than proved. A habit of such instruction is a childish procedure—we can call it nothing else; technically it is unjustifiable, because it presupposes and immediately assumes such things; and it is even more guilty of ignorance of the fact that it is the need and business of logical thought to investigate just this—whether a Finite without Infinity is something true, whether such abstract Infinity, and moreover a Content without Form and a Form without Content, similarly an Inner by itself that has no Outer, an Externality without Inwardness, and so forth—whether these can be *something true*, or *something actual.* But this education and discipline of thought, by which an adaptable attitude of thought is brought about, and the impatience of intrusive reflection overcome—all this can be procured only by going further, only by study, and by actualization of the whole course of development.

Anyone who works at building up anew an independent structure of philosophical science in modern times, may; when referring to the Platonic exposition, be reminded of the story of how Plato revised his *Republic* seven times over. The remembrance of this, the comparison in as far as such may seem to be implied here, should only urge one all the more to the desire, that for a work which, as pertaining to the modern world, has to deal with a deeper principle, a more difficult subject, and material of wider scope—for such a work leisure might have been vouchsafed, to go through it seventy times and seven. The author however, in face of the greatness of the task, has had to content himself with what it has been possible to accomplish under pressure of external necessity, inevitable distractions due to the greatness and many-sidedness of the interests of the time, and under a doubt whether the noisy clamour of everyday affairs, and the bewildering volubility of undisciplined fancy (which takes a pride in limiting its interests to such affairs)—whether these leave any room for sympathy with the passionless calm of purely speculative knowledge.

BERLIN,
7th November 1831.

INTRODUCTION

GENERAL CONCEPT OF LOGIC

THE need to begin with the subject itself, without preliminary observations, is felt nowhere more strongly than in the Science of Logic. In every other science, the Subject dealt with, and the Method of the Science, are distinguished from one another; and further the subject is not absolutely original, but depends upon other concepts, and is connected in all directions with other material. It is therefore granted to these other sciences to regard both their Principles (with the connexions of these) and also their Method, as starting from assumptions—to begin with applying forms of Definition and so on, which are presupposed as known and accepted, and to make use of familiar forms of reasoning for the establishment of their general concepts and fundamental determinations.

Logic on the other hand cannot take for granted any of these forms of reflection or rules and laws of thought, for these are a part of the very fabric of Logic, and must be demonstrated within the boundaries of the science itself. But not only the scheme of philosophic method, but also the very concept of philosophy in general belongs to the content of Logic and in fact constitutes its final result; what Logic is, cannot be set out beforehand—on the contrary this knowledge of what Logic is can only be reached as the end and consummation of the whole treatment of the subject. Moreover the subject of Logic (Thinking, or more precisely Conceptual Thinking) is really treated of within the boundaries of the science itself; the Concept of this Thinking is engendered in the course of development of the Science, and therefore cannot precede it. Therefore what is set forth in a preliminary way in this Introduction does not aim at establishing the concept of Logic at all, or at justifying beforehand its substance and method scientifically, but—by help of some reasoned and historical explanations and reflections—at bringing more clearly before the mind the point of view from which this science is to be regarded.

When Logic is taken as the science of Thinking in general, it is understood that this Thinking constitutes the *bare form* of cognition, that Logic abstracts from all *content*, and that the (so-called) other *constituent* of a cognition,—that is, its *Matter*,—must come from a different source; that thus Logic—as something of which this Matter is wholly and entirely independent—can provide only the formal conditions of true knowledge, and cannot, in and by itself, contain real truth, nor even be the path to real truth, because just that which is the essence of truth,—that is, its content—lies outside Logic.

But in the first place it is most inept to say that Logic abstracts from all *Content*, that it teaches only the rules of Thinking without going into what is thought or being able to consider its nature. For since Thinking and the Rules of Thinking are the subject of Logic, Logic has directly in them its own peculiar content;—has in them that second constituent of cognition—its Matter—about the structure of which it concerns itself.

But secondly, the ideas upon which the concept of Logic has hitherto rested have partly died out already, and, for the rest, it is time that they should disappear altogether, and that this science should be taken from a higher point of view, and should receive an entirely different structure.

The hitherto accepted concept of Logic rests upon the assumed separation of the *Content* of knowledge and the *Form* of knowledge (or *Truth* and *Certainty*)—a separation that is assumed once for all in ordinary consciousness. First, it is assumed that the material of knowledge is present in and for itself in the shape of a finished world apart from Thinking, that Thinking is in itself empty, and comes to that world from outside as Form to Matter, fills itself therewith, and only thus gets a content, and thereby becomes real knowing.

Next, these two constituents—for it is supposed that they have the reciprocal relation of constituents, and Cognition is constructed out of them in a mechanical or at best a chemical fashion—these constituents are placed in an order of merit in which the object is regarded as something in itself finished and complete, something which, as far as its reality is concerned, could entirely dispense with thought, while on the other hand, Thought is something incomplete which has to seek completion by means of some material, and indeed has to adapt itself to its material as if it were a form in itself pliable and undetermined. Truth is supposed to be the agreement of Thought with its object, and in order to bring about this agreement (for the agreement is not there by itself) thinking must accommodate and adapt itself to its object.

Thirdly, when the difference of Matter and Form, of Object and Thought, is not left thus nebulous and undetermined, but is taken more definitely, each is regarded as a sphere separated from the other. Thus Thought in its reception and formation of material is supposed not to go beyond itself--its reception of material and accommodation thereto is still regarded as a modification of self by which Thought is not transformed into its Other; moreover, self-conscious determination is held to belong to Thought alone; thus Thought in its relation to the Object of Thought does not go out of itself to the Object, while the Object, as a thing-in-itself, simply remains a something beyond Thought.

These views concerning the relation to one another of Subject and Object express the determinations which constitute the nature of our ordinary consciousness just as it appears; but these prejudices, translated into the sphere of Reason—as if the same relationship held there or had any truth by itself—are errors, the refutation of which throughout all departments of the spiritual and physical world is Philosophy itself; or rather, since these errors bar the way, they must be renounced at the very threshold of Philosophy.

The older Metaphysic had in this respect a loftier conception of Thought than that which has become current in more modern times. For the older Metaphysic laid down as fundamental that that which by thinking is known of and in things, that alone is what is really true in them; that what is really true is not things in their immediacy, but only things when they have been taken up into the Form of Thought, as conceptions. Thus this older Metaphysic stands for the view that thinking and the determinations of thinking are not something foreign to the objects of thought, but are rather of the very essence of those objects; in other words that *Things* and the *Thinking* of them are in harmony in and for themselves,—indeed language itself expresses an affinity between them,—that thought in its immanent determinations, and the true nature of things, are one and the same content.

But *reflective* Understanding assumed possession of Philosophy. We must learn precisely what is meant by this expression, which indeed is frequently used as a catch-word; by it is to be understood generally the abstracting and separating intelligence which clings tenaciously to the separations which it has made. Directed against Reason, this intelligence behaves as *crude Common Sense* and maintains the view that Truth rests upon sense-reality, that thoughts are *only* thoughts, meaning that it is sense-perception that first endows them with substance and reality, that Reason—in as far as it is merely Reason—can spin nothing but idle fancies. In this renunciation of Reason by itself, the concept of Truth is lost; it is restricted to the cognition of merely subjective Truth, of mere appearance, of something to which the nature of the thing itself does not correspond; *knowing* falls back into *opinion*.

But this turn which Cognition takes, and which has the air of being a loss and a retrogression, has something deeper behind it—something upon which the uplifting of Reason to the loftier spirit of the newer Philosophy chiefly depends. That is, the ground of this now everywhere prevalent idea is to be sought in a perception of the *necessary conflict* with each other of the determinations of Understanding. The reflection already mentioned is this, that the immediate concrete must be transcended, and must undergo determination and abstraction. But reflection must, just as much, transcend these its own separative determinations, and forthwith relate them to each other. Then at the standpoint of this relating, the conflict emerges. This relating activity of reflection belongs in itself to Reason; that transcending of these determinations which attains to a perception of their conflict, is the great negative step towards the true concept of Reason. But this perception, being merely partial, falls into the error of fancying that it is Reason which is in contradiction with itself, and does not recognize that the contradiction is just the lifting of Reason above the limitations of Understanding, and the dissolution of these. Instead of starting from this point to make the final step upwards, knowledge, recognizing the unsatisfactory nature of the determinations of Understanding, flies straight back to sensible existence, thinking to find therein stability and unity. But on the other hand, since this knowledge knows itself to be knowledge only of appearances, its insufficiency is confessed, yet at the same time it is supposed that things, though not rightly known in themselves, still are rightly known within the sphere of appearances; as though only the *kinds of objects* were different, and the one kind, namely Things in themselves, did not fall within knowledge, and the other kind, namely Appearances, did so fall. It is as though accurate perception were attributed to a man, with the proviso that he yet could not perceive Truth but only untruth. Absurd as this would be, a true knowledge which did not know the object of knowledge as it is in itself, would be equally absurd.

The *criticism of the Forms of Common Understanding* has had the result (mentioned above) that these Forms have no *applicability to things in themselves*. This can have no other meaning than that the Forms are in themselves something untrue. But if they are allowed to remain as valid for subjective Reason and for experience, then criticism has made no change in them, but leaves them in the same attitude towards the Subject of knowledge, as they formerly had towards the Object of knowledge. But if they do not suffice for the Thing in itself, then still less should common understanding, to which they are supposed to belong, put up with them and be content with them. If they cannot be determinations of the *Thing in itself*, they can still less be determinations of Understanding, to which we must allow at the very least the dignity of a Thing in itself. The determinations of Finite and Infinite are similarly in conflict—whether they are applied to the world, to time and space, or are determinations within the mind; just as black and white produce grey whether they are mixed on a canvas or on the palette; so if our World-representation is dissolved by having the determinations of Finite and Infinite transferred to it, still more must the Mind itself, which contains them both, be something self-contradictory and self-dissolving. It is not the constitution of the Matter or Object to which they are applied or in which they occur, that can make the difference, for the Object contains the contradiction only through these determinations and in accordance with them.

Thus this Criticism has only separated the forms of objective

Thinking from the Thing, and left them, as it found them, in the Subject of Thought. For in doing so, it has not regarded these Forms in and for themselves, according to their characteristic content, but has simply taken them up as a corollary from subjective Logic; so that it was not a question of the deduction of them in themselves, nor of a deduction of them as subjective-logical Forms, and still less a question of the dialectical consideration of them.

Transcendental Idealism, carried more consistently to its logical conclusion, has recognized the emptiness of that spectre of the *Thing-in-itself* which the critical philosophy left over—an abstract shadow, detached from all content—and had it in view to demolish it altogether. Also this philosophy made a beginning of letting Reason produce its own determinations out of itself. But the subjective attitude of this attempt did not admit of its being carried to completion. Henceforth this attitude—and with it that beginning, and the development of pure philosophy—was given up.

But that which has commonly been understood by Logic is considered without any reference to metaphysical import. In its present condition, this Science has indeed no Content of such a kind that it can be regarded by ordinary consciousness as reality and truth. But Logic is not on this account a mere formal science, destitute of significant truth. In any case, the province of truth is not to be looked for in that subject-matter which is lacking in Logic, and to the want of which the inadequacy of the Science is commonly attributed: the emptiness and worthlessness of the logical forms reside solely in the way in which they have been considered and treated. Whilst as fixed determinations they fall apart and cannot be held together in organic unity, they are mere dead forms, and have not dwelling in them the spirit which is their living concrete unity. Thus they are destitute of solid content and substantial filling. The content which we miss in the logical forms, is nothing other than a solid foundation and concreting of those abstract forms, and it is customary to seek this substantial essence for them, from outside. But it is just logical Reason which is that substantial or real, which holds together in itself all abstract determinations, and is their solid absolutely concrete unity. Thus we do not need to seek far afield for what is usually regarded as a filling or content; it is not the fault of the subject-matter of Logic if it is supposed to be without content or filling, but of the way in which Logic is conceived.

This reflection leads us nearer to the problem of the point of view from which Logic is to be regarded; how it is distinguished from the mode of treatment which this science has hitherto received, and to what extent it is the only true point of view upon which Logic is in the future to be permanently based.

In the *Phenomenology of Spirit* (Bamberg and Würzburg, 1807) I have set forth the movement of consciousness, from the first crude opposition between itself and the Object, up to absolute knowledge. This process goes through all the forms of the *relation of thought to its object*, and reaches the *Concept of Science* as its result. Thus this concept (apart from the fact that it arises within the boundaries of Logic) needs here no justification, having already received its justification in that place; the concept is incapable of any other justification than just this production by consciousness; for to consciousness, all its forms are resolved into this concept, as into the truth. A reasoned deduction or elucidation of the concept of science can at best render this service, that by it the concept is presented to the mind, and a historical knowledge of it is produced; but a definition of science, or—more precisely—of logic, has its evidence solely in the inevitableness (already referred to) of its origin. The definition with which any science makes an absolute beginning can contain nothing other than the precise and correct expression of that which is presented to one's mind *as the accepted and recognized* subject-matter and purpose of the science. That exactly this or that is thus presented is a historical asseveration, in respect of which one may indeed appeal to certain facts as commonly accepted; or rather the request can be made that certain facts may be granted as accepted. And still we find that one man here and another there will bring forward, here a case and there an instance, according to which something more and other is to be understood by various expressions, into the definition of which therefore a narrower or more general determination is to be admitted, and in accordance with which the science is to be arranged. It further depends upon argument *what* should be admitted or excluded, and within what limit and scope; and there stand open to argument the most manifold and varied opinions, among which only arbitrary choice can make a fixed and final decision. In this mode of procedure, of beginning a science with its definition, nothing is said of the need that the *inevitableness* of the *subject-matter*, and therefore of the Science itself, should be demonstrated.

The concept of pure Science, and the Deduction of it, are assumed in the present treatise so far as this, that the *Phenomenology of Spirit* is nothing other than the Deduction of this concept. Absolute Knowledge is the *Truth* of all modes of Consciousness, because according to the process of knowledge, it is only when absolute knowledge has been reached that the separation of the *Object of Knowledge* from *Subjective Certainty* is completely resolved, and Truth equated to this Certainty, and this Certainty equated to Truth.

So pure Science presupposes deliverance from the opposition of Consciousness. Pure Science includes *Thought in so far as it is just as much the Thing in itself as it is Thought*, or *the Thing in itself in so far as it is just as much pure Thought as it is the Thing in itself*. Truth, as *Science*, is pure Self-Consciousness unfolding itself, and it has the form of Self in that what exists in and for itself is the known concept, while the Concept as such is that which exists in and for itself.

This objective thinking is then the content of the pure science. Hence Logic is so little merely formal, so little destitute of the matter necessary for real and true knowledge, that on the contrary its Content is the only Absolutely True, or (if we wish still to employ the word *matter*) is the true genuine matter —a Matter, however, to which Form is not external, since this Matter is in fact Pure Thought, and thus Absolute Form itself. Logic is consequently to be understood as the System of Pure Reason, as the Realm of Pure Thought. *This realm is the Truth as it is, without husk in and for itself.* One may therefore express it thus: that this content *shows forth God as he is in his eternal essence before the creation of Nature and of a Finite Spirit.*

Anaxagoras is praised as the man who first gave voice to the idea that we ought to lay down, as the World-principle, Nous, that is Thought, and Thought as the World-essence.

He thus laid the foundation of an intellectualist view of the Universe, and of this view Logic must be the pure form. In it we are not concerned with thinking *about* something lying outside thought, as the basis of thought, nor with Forms which serve merely as *signs* of Truth; on the contrary, the necessary Forms and characteristic determinations of thought are the Content and the Supreme Truth itself.

In order that we may at least envisage this we must put aside the opinion that Truth is something tangible. Such tangibility has for example been imported even into the Platonic Ideas, which are in the thought of God, as though they were things existing, but existing in a world or region outside the world of Reality, a world other than that of those Ideas, and only having real Substantiality in virtue of this otherness. The Platonic Idea is nothing other than the Universal, or more precisely the Concept of an Object of Thought; it is only in its concept that anything has actuality; in so far as it is other than its concept, it ceases to be actual and is a non-entity; the aspect of tangibility and of sensuous externality to self belongs to that non-entical aspect.—From the other side, however, one can refer to the characteristic ideas of ordinary Logic; for it is assumed that, for instance, Definitions comprise not determinations which belong only to the cognizing Subject, but determinations which belong to the Object, and constitute its most essential and inmost nature. Again, when from given determinations we conclude to others, it is assumed that what is concluded is not something external to the Object and foreign to it, but that it belongs to the object,—that Being corresponds to Thought.—Speaking generally, it lies at the very basis of our use of the Forms of Concept, Judgment, Inference, Definition, Division, and so on, that they are Forms not merely of self-conscious Thinking but also of the objective understanding.—*To think* is an expression which attributes specially to Consciousness the determination which it contains. But in as far as it is allowed that *Understanding, and Reason, are of the World of Objects*, that Spirit and Nature have General Laws in accordance with which their life and their mutations are governed, in so far is it admitted that the determinations of Thought also have objective validity and existence.

The Critical Philosophy has indeed turned Metaphysics into Logic, but—as already mentioned—like the later idealism it shied at the Object, and gave to logical determinations an essentially subjective signification; thus both the Critical Philosophy and the later idealism remained saddled with the Object which they shunned, and for Kant a "Thing-in-itself," for Fichte an abiding "Resistance-principle," was left over as an unconquerable Other. But that freedom from the opposition of consciousness which Logic must be able to assume, lifts these thought-determinations above such a timid and incomplete point of view, and requires that those determinations should be considered not with any such limitation and reference, but as they are in and for themselves, as Logic, as Pure Reason.

Kant considers that Logic—that is, the aggregate of Definitions and Propositions which are called Logic in the ordinary sense—is fortunate in that it has fallen to its lot to attain so early to completion, before the other sciences; for Logic has not taken any step backwards since Aristotle,—but also it has taken no step forwards—the latter because to all appearance it was already finished and complete.—If Logic has undergone no change since Aristotle—and in fact when one looks at modern compendiums of Logic the changes consist to a large extent merely in omissions—what is rather to be inferred from this is, that Logic is all the more in need of a thorough overhaul; for when Spirit has worked on for two thousand years, it must have reached a better reflective consciousness of its own thought and its own unadulterated essence. A comparison of the forms to which Spirit has risen in the worlds of Practice and Religion, and of Science in every department of knowledge Positive and Speculative,—a comparison of these with the form which Logic—that is, Spirit's knowledge of its own pure essence—has attained, shows such a glaring discrepancy that it cannot fail to strike the most superficial observer that the latter is inadequate to the lofty development of the former, and unworthy of it.

As a matter of fact the need of a transformation of Logic has long been felt. It may be said that, both in Form and in Content, as exhibited in text-books, Logic has become contemptible. It is still trailed along rather with a feeling that one cannot do without Logic altogether, and from a surviving adherence to the tradition of its importance, than from any conviction that that familiar content, and occupation with those empty forms, can be valuable and useful.

The additions—psychological, educational, even physiological—which Logic received during a certain period were, later, almost universally recognized as disfigurements. In themselves, a great part of these psychological, educational, and physiological observations, laws, and rules, must appear very trivial and futile, whether they occur in Logic or anywhere else. Besides, such rules as for instance that one should think out and test what one reads in books or hears by word of mouth, that when one does not see well, one should use spectacles to help one's eyes—rules which in text-books on so-called Applied Logic are put forward with great seriousness and formality to help us to attain to truth—these must appear to all the world to be superfluous—except indeed to the writer or teacher who is at his wits' end to know how to piece out the inadequate lifeless content of his Logic.[1]

As to this content, we have given above the reason why it is so empty and lifeless. Its determinations are assumed to stand immovably rigid and are brought into a merely external relation with one another. Because in the operations of judgment and syllogism it is chiefly their quantitative element that is referred to and built upon, everything rests on an external difference, on mere comparison, and becomes a wholly analytic procedure, a matter of merely mechanical calculation. The deduction of the so-called rules and laws (especially of Syllogism) is not much better than a manipulation of rods of unequal length in order to sort and arrange them according to size—like the child's game of trying to fit into their right places the various pieces of a picture-puzzle. Not without reason, therefore, has this Thinking been identified with Reckoning, and Reckoning with this Thinking. In Arithmetic the numbers are taken as non-significant, as something that, except for equality or inequality—that is, except for quite external relations—has no significance,—that contains no Thought, either in itself or in its relations. When it is worked out in a mechanical way that three-fourths multiplied by two-thirds make a half, this operation involves about as much or as little thought as the calculation whether in any Figure of Syllogism this or that Mood is admissible.

---

[1] *Observation in first edition.* A work on this science which has recently appeared, *System of Logic by Fries*, returns to the anthropological foundations. The ideas and opinions on which it is based are so shallow in themselves and in their development that I am saved the trouble of having to take any notice of this insignificant performance.

In order that these dead bones of Logic may be re-vivified by Mind, and endowed with content and coherence, its Method must be that by means of which alone Logic is capable of becoming a Pure Science. In the present condition of Logic, hardly a suspicion of scientific Method is to be recognized. It has very nearly the structure of merely Empirical Science. For attaining their purpose, empirical sciences have hit upon a characteristic Method of defining and classifying their material as best they can. Pure Mathematics again has its own Method, which suits its abstract objects and the quantitative determinations with which alone it is concerned. I have in the Preface to the *Phenomenology of Spirit* said what is essential concerning this Method and especially concerning the subordinate nature of such Science as can find a place in Mathematics; but it will also be more closely considered within the bounds of Logic itself. Spinoza, Wolf, and others, have allowed themselves to be misled into applying this method in Philosophy, and identifying the external process of concept-less quantity with the conceptual process, which is self-contradictory. Hitherto Philosophy had not discovered its own method; it regarded with an envious eye the systematic structure of Mathematics and, as already remarked, borrowed this, or sought help in the method of Sciences which are only a medley of given material and empirical maxims and ideas—or took refuge in a crude rejection of all Method. But the exposition of that which alone is capable of being the true Method of philosophic Science belongs to Logic itself; since method is the consciousness of the form taken by the inner spontaneous movement of the content of Logic. In the *Phenomenology of Spirit* I have set out an example of this Method as applied to a more concrete object, namely to Consciousness.[1] We have here modes of consciousness each of which in realizing itself abolishes itself, has its own negation as its result,—and thus passes over into a higher mode. The one and only thing *for securing scientific*

---

[1] And later as applied to other concrete objects, and corresponding departments of Philosophy.

progress (and for quite *simple* insight into which, it is essential to strive)—is knowledge of the logical precept that Negation is just as much Affirmation as Negation, or that what is self-contradictory resolves itself not into nullity, into abstract Nothingness, but essentially only into the negation of its *particular* content, that such negation is not an all-embracing Negation, but is *the negation of a definite somewhat* which abolishes itself, and thus is a definite negation; and that thus the result contains in essence that from which it results—which is indeed a tautology, for otherwise it would be something immediate and not a result. Since what results, the negation, is a *definite* negation, it has a *content.* It is a new concept, but a higher, richer concept than that which preceded; for it has been enriched by the negation or opposite of that preceding concept, and thus contains it, but contains also more than it, and is the unity of it and its opposite.—On these lines the system of Concepts has broadly to be constructed, and to go on to completion in a resistless course, free from all foreign elements, admitting nothing from outside.

I could not of course imagine that the Method which in this System of Logic I have followed—or rather which this System follows of itself—is not capable of much improvement, of much elaboration in detail, but at the same time I know that it is the only true Method. This is already evident from the fact that the Method is no-ways different from its object and content;—for it is the content in itself, *the Dialectic which it has in itself*, that moves it on. It is clear that no expositions can be regarded as scientific which do not follow the course of this Method, and which are not conformable to its simple rhythm, for that is the course of the thing itself.

In accordance with this Method I would observe that the divisions and headings of the Books, Sections, and Chapters which are given in the work, as well as to some extent the explanations connected with them, were made for the purposes of a preliminary survey, and that in fact they have only a *historical* value. They do not belong to the content and body of the Science, but are compiled by external reflexion, which has already run through the whole of the scheme, and hence knows and indicates in advance the sequence of its phases, before these introduce themselves in the subject itself.

In the other Sciences too, such preliminary Definitions and Divisions are in themselves no other than such external specifications; but even within each science they are not raised above this status. Even in Logic, for example, we may be told that "Logic has two principal parts, (1) the Doctrine of Elements and (2) Methodology";—then under the first head we forthwith find, perhaps, the superscription: *Laws of Thought*;—and then: *Chapter I—Concepts. First Section*: *Of the Clearness of Concepts*—and so on.—These Determinations and Divisions, made without any deduction or justification, furnish the systematic framework and the whole bond of connexion of such sciences. Such a Logic regards it as its business to say that Concepts and Truths must be *derived* from Principles; but in what this Logic calls Method, derivation is the last thing that is thought of. The procedure consists, it may be, in grouping together what is similar, in putting what is simpler before what is compound, and other external considerations. But as for any inner necessary connexion, this goes no further than the list of Sections, and the transition consists merely in saying *Chapter II*;—or *We now come to Judgment*, and the like.

The headings and divisions which occur in this system too are designed in themselves to have no other significance than that of a Table of Contents. But in addition to this the *necessity of connexion* and the *immanent origination* of distinctions must show themselves in the discussion of the subject-matter, for they are part of the self-development of the concept.

That by means of which the Concept forges ahead is the above-mentioned Negative which it carries within itself; it is this that constitutes the genuine dialectical procedure. *Dialectic*—which has been regarded as an isolated part of Logic, and which as regards its purpose and standpoint has, one may aver, been entirely misunderstood—is thus put in quite a different position.—The Platonic Dialectic too, even in the *Parmenides* (and still more directly in other places), is sometimes intended merely to dispose of and to refute through themselves limited assertions, and sometimes again has nullity for its result. Dialectic is generally regarded as an external and negative procedure, that does not pertain to the subject-matter, that is based on a mere idle subjective craving to disturb and unsettle what is fixed and true, or that at best leads to nothing except the futility of the dialectically treated matter.

Kant set Dialectic higher, and this part of his work is among the greatest of his merits,—for he freed Dialectic from the semblance of arbitrariness attributed to it in ordinary thought, and set it forth as *a necessary procedure of Reason.* Since Dialectic was regarded merely as the art of producing deceptions and bringing about illusions, it was straightway assumed that it played a cheating game, and that its whole power depended solely on concealment of the fraud; that its results were reached surreptitiously, and were a mere subjective illusion. When Kant's dialectical expositions in the *Antinomies of Pure Reason* are looked at closely (as they will be more at large in the course of this work) it will be seen that they are not indeed deserving of any great praise; but the general idea upon which he builds and which he has vindicated, is the *Objectivity of Appearance* and the *Necessity of Contradiction* which belong to the very nature of thought-determinations; primarily indeed in so far as these determinations are applied by Reason to *Things in themselves*; but further, just what these determinations are *in Reason* and *in respect of that which is self-existent*,—just this it is which is their own nature. This result, *grasped on its positive side*, is nothing other than the inherent *Negativity* of these thought-determinations, their self-moving soul, the principle of all physical and spiritual life. But if people stop short at the abstract-negative aspect of the Dialectic, they reach only the familiar result that Reason is incapable of cognition of the Infinite;—a strange result, for—since the Infinite is the Reasonable—it amounts to saying that Reason is incapable of cognizing that which is Reasonable.

It is in this Dialectic (as here understood) and in the comprehension of the Unity of Opposites, or of the Positive in the Negative, that *Speculative knowledge* consists. This is the most important aspect of the Dialectic, but for thought that is as yet unpractised and unfree, it is the most difficult. If thought is still in the process of cutting itself loose from concrete sense-presentation and from syllogizing (*Räsonnieren*), it must first practise abstract thinking, and learn to hold fast concepts in their definiteness and to recognize by means of them. An exposition of Logic with this in view must, in its Method, follow the division above mentioned, and with regard to the more detailed content must hold to the determinations of the particular concepts without embarking upon the Dialectic. As far as external structure is concerned, this Logic would be similar to the usual presentation of the science, but as regards content would be distinct from it, and still would serve for practice in abstract thinking, though not in speculative thinking (a purpose which could not be in any degree fulfilled by the Logic which has become popular by means of psychological and anthropological trappings). It would present to the mind the picture of a methodically ordered whole, although the soul of the structure, the Method itself (which lives in Dialectic), would not be apparent in it.

As regards *education and the relation of the individual to Logic*, I observe in conclusion that this Science, like grammar, has two different aspects or values. It is one thing to him who approaches Logic and the Sciences in general for the first time, and another thing to him who comes back from the Sciences to Logic. He who begins to learn grammar, finds in its Forms and Laws dry abstractions, contingent rules, briefly an isolated multitude of determinations which only indicate the worth and significance of their face-value. At first, knowledge recognizes in them nothing whatever but barely themselves. On the other hand, if anyone has mastered a language, and has also a comparative knowledge of other languages, he and he only is capable of discerning the spirit and the culture of a people in the grammar of their language. Those same dry Rules and Forms have now for him a full and living value. Through grammar he can recognize the expression of mind in general—that is, Logic. Thus he who approaches Logic finds in the science at first an isolated system of abstractions that is self-contained and does not reach out to other knowledges and sciences. On the contrary, contrasted with the wealth of our world-presentations and the apparently real content of the other sciences, and compared with the promise of absolute Science to unfold the essential character of this wealth, the *inner nature* of Spirit and of the world, and to unveil *the Truth*, this science—in its abstract form, in the colourless cold simplicity of its purely formal determinations—looks, rather, as if the last thing to be expected from it were the fulfilment of such a promise, and as if it would stand empty in face of that wealth. On a first acquaintance, the significance of Logic is limited to itself; its content is regarded as only an isolated occupation with thought-determinations, *alongside of* which other scientific activities have their own material and their own intrinsic worth, upon which Logic may perhaps have some formal influence which it seems to exercise spontaneously, and for which logical structure and logical study can certainly be dispensed with, at need. The other Sciences have mostly rejected the regular Method, of a connected series of Definitions, Axioms, Theorems and their Proofs, and so forth; while so-called Natural Logic plays its part automatically in such series, and works of its own motion, without any special knowledge having Thought itself for its object. Above all the matter and content of these sciences keeps entirely independent of Logic, and altogether makes its appeal more to our senses, feeling, impressions, and practical interests.

Thus then Logic must certainly be learnt, at first, as something of which one has indeed perception and understanding, but which seems at the beginning to lack scope, profundity, and wider significance. It is only through a profounder acquaintance with other sciences that Logic discovers itself to subjective thought as not a mere abstract Universal, but as a Universal which comprises in itself the full wealth of Particulars;—just as a proverb, in the mouth of a youth who understands it quite accurately, yet fails of the significance and scope which it has in the mind of a man of years and experience, for whom it expresses the full force of its content. Thus the value of Logic only receives due appreciation when it is seen to result from knowledge of the particular sciences; so regarded, it presents itself to the mind as Universal Truth, not as a *particular* department of knowledge *alongside of* other departments and other realities, but as the very essence of all these other Contents.

Now though when one begins to study it, Logic is not present to the mind in all this recognized power, yet none the less the mind of the student conceives from it a power which will lead him into all truth. The System of Logic is the realm of shades, a world of simple essentialities freed from all concretion of sense. To study this Science, to dwell and labour in this shadow-realm, is a perfect training and discipline of consciousness. In this realm the mind carries on a business which is far removed from the intuitions and aims of sense, from emotions, from ideas which are a mere matter of opinion. Regarded on its negative side, the work consists in holding at bay the accidentals of syllogizing thought and the arbitrary preference and acceptance from among opposing arguments.

But above all, Thought wins thus self-reliance and independence. It becomes at home in the region of the abstract and in progression by means of concepts which have no substratum of sensation, it develops an unconscious power of taking up into the forms of Reason the multiplicity of all other knowledge and science, comprehending and holding fast what is essential therein, stripping off externalities and in this way extracting what is logical,—or, which is the same thing, filling with the content of all truth the abstract outline of Logic acquired by study, and giving it the value of a Universal, which no longer appears as a Particular side by side with other Particulars, but reaches out beyond all this, and is the essential nature thereof,—that is, the Absolute Truth.

General Classification of Logic

In what has been said of the concept of this Science, and the direction which its justification must take, it is implied that a general classification can at this point be only provisional, and hence can be indicated only so far as the author already knows the Science and is thus in a position to present here, historically and in a preliminary fashion, the principal forms in which the concept will manifest itself in the course of its development.

A provisional attempt can, however, be made to render generally intelligible what is required for such a classification, although in doing so one must employ a method that will only receive its full elucidation and justification within the precincts of the Science itself. First, then, it is to be remembered that we here presuppose that the classification must harmonize with the concept, or rather must be immanent in it. The concept is not indeterminate; it is determinate in itself; and the classification is the developed expression of this its determinateness. It is the fundamental and significant classification[1] of the concept; not of anything taken from without, but the fundamental classification, that is, the determination of the concept by itself. The quality of being right-angled, acute-angled, or equilateral, the determinations according to which triangles are classified, are not contained in the determinateness of the triangle itself; that is, they are not contained in what we are accustomed to call the concept of the triangle, any more than the commonly admitted concept of Animal in general, or of Mammal, Bird, and so forth contains the determinations by which animals are classified into mammals, birds, and so on, and these classes are subdivided into further genera. Such determinations are obtained otherwise, that is, from empirical contemplation; they come to these so-called concepts from without; but in the philosophical treatment of classification, it must be shown that the classification has its origin in the concept itself.

But in the Introduction the concept of Logic itself is stated to be the result of a Science which lies outside Logic, and thus this concept too is here presupposed. Logic was there found to determine itself as the science of pure thought, having pure knowledge as its principle, which is not abstract, but a concrete living unity; for in it the opposition in consciousness between a subjective entity existing for itself, and another similar objective entity, is known to be overcome, and existence is known as pure concept in itself, and the pure concept known as true existence. These are then the two *Moments* which are contained in Logic. But they are now known as existing inseparably, and not as in consciousness each existing for itself; it is only because they are known as distinct and yet not merely self-existent that their unity is not abstract, dead, and immobile, but concrete.

This unity constitutes an element of the logical principle, so that the development of the distinction which is immediately latent within it takes place only within this element. We have said that the classification is the fundamental classification of the concept, the positing of the determination immanent in it and thus of its distinction; hence this positing is not to be understood as resolving that concrete unity back into its determinations regarded as self-existent entities, which here would be mere retrogression to the former position, namely the opposition of consciousness. This opposition, on the contrary, has vanished, and that unity remains the element, beyond which the distinctions which occur in the classification and the development generally, do not pass. And thus determinations (such as the subjective and the objective, thought and being, concept and reality, whatever the respect in which they were determined)—determinations which, at an earlier point on the road to truth, were self-existing, are now, in their unity (which constitutes their truth), degraded to the rank of forms. In their distinction they therefore remain, in themselves, the whole concept, and this is placed under its own determinations in the classification.

---

[1] German, *Urteil*; *see* Glossary (Translators' note).

Thus the whole concept is to be considered, first as existent concept, secondly merely as concept; in the former case it is merely the concept in itself, the concept of reality or being; in the latter, it is the concept as such existing for itself (as it is found, to give a concrete example, in thinking man, and even in the sentient animal and in organic individuality in general, though there it is not conscious, still less known: concept *in itself* it is only in inorganic nature).—Logic is accordingly to be divided into the Logic of the Concept as Being, and of the Concept as Concept; or, to employ terms more habitual though least definite and therefore most ambiguous, into *Objective* and *Subjective* Logic.

The basic element, then, is the unity of the concept in itself, and the inseparable nature of its determinations; these, therefore, in so far as they are distinct, and the concept is posited in their distinctness, must at least be somehow related. There results a sphere of mediation—the concept as a system of *determinations of reflection*, that is, of Being in transition to the being-in-self of the concept; thus the concept is not yet posited as such for itself; immediate being, as something external, still cleaves to it. This is the Doctrine of Essence, which is intermediate between the Doctrine of Being and that of the Notion. In the general classification of this logical work it has been placed under Objective Logic, since, though Essence already is the Inward, the character of Subject has been expressly reserved for the Notion.

In recent times Kant[1] has opposed to what is commonly called Logic yet another, namely *Transcendental Logic*. What has here been called Objective Logic would partly correspond to what is Transcendental Logic with him. He distinguishes it from what he calls General Logic because (α) it considers concepts which refer *a priori* to objects and thus does not abstract from the entire content of objective cognition,—in other words it contains the rules of pure thinking about an object; and (β) because it further considers the origin of our cognition, so far as this cannot be ascribed to the objects.—It is on the second of these two aspects that the philosophic interest of Kant is exclusively directed. His chief aim is to claim the Categories for Self-consciousness, for the Subjective Ego. By virtue of this determination, his point of view remains within the boundaries of consciousness and its opposition, and there is a surplus, beyond the data of sensation and intuition, which is not posited and determined by thinking self-consciousness and is foreign and external to thought, namely the thing-in-itself; though it is easy to perceive that such an abstraction as the thing-in-itself is itself only a product of thought, namely, of purely abstracting thought. Other disciples of Kant have expressed themselves concerning the determination of the object through the ego in this sense, that the objectifying of the ego is to be considered as an original and necessary activity of consciousness, so that this original activity does not yet contain the idea of the ego itself; this latter being the consciousness, or even the objectifying, *of* such consciousness. On such a view, this objectifying activity, freed from the opposition of consciousness, is just that which can, generally, be called *Thought* as such.[1] But this activity should no longer be called consciousness: consciousness comprehends within it that opposition of ego and object which does not exist in this original activity. The name of consciousness gives a greater appearance of subjectivity to it than the expression "thought," which, however, is here to be taken in the absolute sense as Thought *infinite* and untainted by the finitude of consciousness: briefly, as Thought as such.

---

[1] It is to be remembered that I frequently take the Kantian philosophy into consideration in this work (superfluous though this may seem to some), because, however its detailed determinations and the individual parts of its development may be regarded in this work and elsewhere, it still remains the basis and beginning of modern German philosophy; whatever faults we may find with it, this must be set down undiminished to its credit. And further, objective Logic must frequently refer to it for this other reason, that it treats important definite aspects of Logic in greater detail, whereas later philosophical expositions have either neglected these, or else displayed a mere crude contempt for them; which has not, however, remained unavenged. The philosophies which are most widely diffused among us did not get beyond the Kantian results, that reason can cognize no valid content, and with regard to absolute truth must be referred to faith. Kant's *results* are made the immediate *beginning* of these philosophies, so that the preceding exposition, from which those results are derived, and which is philosophic cognition, is cut away beforehand. Thus the Kantian philosophy becomes a pillow for intellectual sloth, which soothes itself with the idea that everything has been already proved and done with. Those who look for knowledge and a definite content of thought, which are not to be found in this dry and sterile acquiescence, must turn to that preceding exposition.

Kant's philosophy then directing its interest on the so-called transcendental element of the determinations of thought, the treatment of these received no attention: it was not considered what they are in themselves apart from the abstract relation to the ego common to all, or what are their reciprocal determinations and relations. Hence this philosophy has in no way contributed to knowledge of their natures. The only interesting matter bearing upon the point occurs in the Critique of Ideas. But for the real progress of philosophy it was necessary that the interest of thought should be directed upon the formal side, the ego, consciousness as such, that is, the abstract relation of subjective knowledge to an object; and that the cognition of infinite form, that is, of the concept, should be introduced in this manner. But in order that this cognition may be reached, that finite determination, in which the form still is ego, or consciousness, has still to be cast off. The form, thus developed into purity by thought, will then have in itself the capacity of self-determination, that is, of giving itself content, and that a necessary content, in the shape of a system of determinations of thought.

Objective Logic then takes the place of the former metaphysic considered as the scientific reconstruction of the world, which was to be built of thoughts alone. If we refer to the last stage in the evolution of this science, we find, first and immediately, that it is Ontology whose place is taken by Objective Logic—that part of this metaphysic which is to investigate the nature of *Ens* in general;—Ens comprehending both Being and Essence, a distinction for which the German language has fortunately preserved different terms.—Secondly, Objective Logic also comprises the rest of metaphysics, in so far as the latter attempted to comprehend with the pure forms of thought certain substrata primarily taken from sensuous representation, such as Soul, World, God; and the determinations of thought constituted what was essential in the method of contemplation. Logic, however, considers these forms detached from such substrata, which are the subjects of sensuous representation; it considers their nature and value in themselves. The old metaphysic neglected this, and thus earned the just reproach of having used these forms uncritically, without a preliminary investigation as to whether and how far they were capable of being determinations of the thing-in-itself, to use the Kantian expression, or, to put it better, determinations of the Rational.—Objective Logic thus is their true critique, a critique which considers the forms of thought not under the abstract form of apriority as opposed to the *a posteriori*, but considers each according to its particular content.

---

[1] If the expression "objectifying activity of the ego" appears to suggest other productions of Spirit, e.g. imagination, it is to be remembered that we are speaking of a determining of an object in so far as the moments of its content do not belong to sensation and intuition. Such an object is a thought: to determine it means, partly, first to produce it, partly, in so far as it is already posited, to have further thoughts about it, further to develop it by thought.

Subjective Logic is the Logic of the Notion,—of Essence which has transcended its relation to any mere being, real or apparent, and in its determination is no longer external, but is the free, independent, and self-determining Subjective, or rather the Subject itself.—Since the subjective involves the misconception of the contingent and arbitrary, and, more generally, of determinations belonging to the form of consciousness, no special weight is to be attached to the distinction between the Subjective and the Objective, which will develop itself more clearly within the body of the Logic.

Thus Logic is divided broadly into Objective and Subjective Logic; more definitely, it has three parts, namely—

1. The Logic of Being,
2. The Logic of Essence, and
3. The Logic of the Notion.

BOOK ONE

THE DOCTRINE OF BEING

WITH WHAT MUST THE SCIENCE BEGIN?

It has only recently been felt that there is a difficulty in finding a beginning in philosophy, and the reason for this difficulty, as well as the possibility of solving it, has been much discussed. The beginning of philosophy must be either mediate or immediate, and it is easy to show that it can be neither the one nor the other: so that either method of beginning is refuted.

It is true that the principle of any philosophy also expresses a beginning, but this beginning is objective and not subjective; it is the beginning of all things. The principle is a content somehow determined,—Water, the One, Nous, Idea,—Substance, Monad, and so forth; or, where it relates to the nature of cognition and so is designed rather to be a criterion than an objective determination (like Thought, Intuition, Sensation, Ego, or Subjectivity itself), it is still the determination of the content to which interest is directed. On the other hand the beginning as such, considered as something subjective in the sense of some contingent way of introducing the exposition, remains neglected and indifferent; and so the need of the question, with what we are to begin, still seems unimportant compared with the need of a principle, which alone seems to contain the interest of the matter,—the interest as to what is the truth and the absolute basis of all things.

The modern embarrassment about a beginning arises from yet another need with which those are unacquainted who, as dogmatists, seek a demonstration of the principle, or who, as sceptics, seek a subjective criterion with which to meet dogmatic philosophy;—a need which, finally, is entirely denied by those who begin with explosive abruptness from their inner revelation, faith, intellectual intuition, and so forth, and desire to dispense with Method and Logic. If thought at first is abstract and concerns itself merely with the principle regarded as content, but in the progress of its evolution is forced to regard also the other side, the behaviour of cognition, then subjective activity is perceived as an essential moment of objective truth, and the need arises of uniting method with content and form with principle. The principle is to be the beginning, and the actual "prius" for thought is also to be first in the logical thought-process.

We have here only to consider how the logical beginning appears; the two aspects in which it can be taken, mediately as a result or immediately as a beginning proper, have already been named. The question which seems so important to contemporary thought, whether knowledge of the truth is immediate and simply begins, whether it is an act of faith, or a mediated knowledge, is not to be discussed here. In so far as such consideration can be undertaken preliminarily, this has been done in another place (in my *Encyclopaedia of the Philosophical Sciences*, 3rd Edn. *Vorbegriff*, § 61 ff.). We need only quote here that there is nothing in Heaven, Nature, Spirit, or anywhere else, which does not contain immediacy as well as mediacy, so that these two determinations are seen to be unseparated and inseparable, and the opposition between them null. As regards philosophical discussion, the determinations of mediacy and immediacy, and hence the discussion of their opposition and truth, occur in every logical proposition. With regard to thought, knowledge, and cognition, this opposition receives the more concrete shape of knowledge mediate or immediate; and the nature of cognition is considered within the Science of Logic itself, in so far as cognition, in its wider and concrete form, falls within the Science of Spirit and the Phenomenology of Spirit. But to ask for clearness about cognition before the beginning of the Science, is to demand that it shall be discussed outside its precincts; but outside the precincts of Science this cannot be done—at least not in a scientific manner, and such a manner alone is here in question.

The beginning is logical in that it is to be made within the sphere of Thought existing freely for itself, in other words, in pure Knowledge; its mediacy here arises from the fact that pure knowledge is Consciousness in its last and absolute truth. We have remarked in the introduction that the "Phenomenology of Spirit" is the Science of Consciousness, demonstrating that consciousness has for result the concept of Science, that is, pure knowledge. The science of manifested Spirit, which involves and demonstrates the necessity, and hence the proof, together with the mediation in general, of that standpoint which is pure knowledge, is thus presupposed by Logic. In this science of manifested Spirit we start from empirical and sensuous consciousness; this is immediate knowledge proper, and in the work mentioned its validity is discussed. Other kinds of consciousness, such as faith in divine truths, inner experience, knowledge through inner revelation, and so on, are shown on slight reflection to be very inappropriate as instances of immediate knowledge. In the treatise referred to, immediate consciousness is also the first and immediate element in the science, and therefore the presupposition: in the Logic, that is presupposition which appeared as the result of those reflections,—namely, the Idea as pure knowledge. Logic is pure science, that is, pure knowledge in the whole extent of its development. In its result, this Idea has determined itself to be certainty become truth; certainty which in one aspect is no longer over against the object, but has incorporated it with itself and knows it to be itself; and, from another aspect, has given up its conviction that it is opposed to and destructive of the object: it has renounced this subjectivity and is one with this renunciation.

Starting from this determination of pure knowledge, the beginning of the science of knowledge is to remain immanent; and in order to effect this, no more is requisite than that we must contemplate, or rather, putting aside all reflections and opinions otherwise held, we must just absorb, that which is presented.

Pure knowledge, taken as shrunk into this unity, has transcended all reference to an Other and to mediation; it is the undifferentiated, and as such ceases to be knowledge; nothing is there but simple immediacy.

Simple immediacy is itself an expression of reflection, and refers to its distinction from the mediated: properly expressed, this simple immediacy is therefore Pure Being. Just as pure knowledge is to mean nothing except purely abstract knowing as such, so Pure Being is to mean nothing except Being in general; Being and nothing else, without any further determination or filling.

Being is here the beginning represented as arising from mediation, a mediation which transcends itself; it being assumed that pure knowledge is the result of finite knowledge, of consciousness. If no assumption is to be made, and the beginning is to be taken immediately, it determines itself only in this way, that it is to be the beginning of Logic, of independent thought. Nothing is there except the decision (which might appear arbitrary) to consider Thought as such. The beginning must be an absolute, or, what here is equivalent, an abstract beginning: it must presuppose nothing, must be mediated by nothing, must have no foundation: itself is to be the foundation of the whole science. It must therefore just be something immediate, or rather the immediate itself. As it cannot have any determination relatively to Other, so also it cannot hold in itself any determination or content; for this would be differentiation and mutual relation of distincts, and thus mediation.—The beginning therefore is Pure Being.

To this simple exposition of what is proper to the subject of the perfectly simple, which is the logical beginning, the following further reflections may be added; their function however cannot be to elucidate or confirm this exposition, which is complete in itself, since they are rather the result of ideas and reflections which, though they may come in our way at the outset, must, like every other preliminary prejudice, be disposed of within the Science itself; so that really they should be made to await such disposal.

It has already been perceived that the absolutely true must be a result, and that, conversely, a result presupposes some primary truth; which, however, because it is primary, is not necessary, considered objectively, and, from the subjective side, is not known. Consequently the idea has arisen in recent times that the truth with which philosophy begins must be hypothetical or problematical, and that hence philosophy at first must be mere seeking. This view, frequently urged by Reinhold in the later period of his philosophy, must in justice be allowed to be based on a genuine interest regarding the speculative nature of the Beginning in philosophy. The analysis of this view is also an occasion for commencing a preliminary understanding of the meaning of logical progress; for the view in question immediately implies a reference to progress. Progress in philosophy, on this view, is in fact retrogression and justification, the result of which is to show that the Beginning was not taken arbitrarily, but is, indeed, partly the truth, partly the primary truth.

If it is considered that progress is a return to the foundation, to that origin and truth on which depends and indeed by which is produced that with which the beginning was made, then it must be admitted that this consideration is of essential importance; and it will be more clearly evident in the Logic itself.—Thus consciousness is led back on its road from immediacy, with which it begins, to absolute knowledge as its inmost truth; and the first term, which entered the stage as the immediate, arises, precisely, from this last term, the foundation.—Still further, we see that Absolute Spirit, which is found to be the concrete, last, and highest truth of all Being, at the end of its evolution freely passes beyond itself and lapses into the shape of an immediate Being; it resolves itself to the creation of a world which contains everything included in the evolution preceding that result; all of which, by reason of this inverted position, is changed, together with its beginning, into something dependent on the result, for the result is the principle. What is essential for the Science is not so much that a pure immediate is the beginning, but that itself in its totality forms a cycle returning upon itself, wherein the first is also last, and the last first.

Hence it equally results on the other hand that we must regard as result that to which the movement returns as into its foundation. From this point of view the first is equally the foundation, and the last derived: it is a result, in so far as we start from the first and reach the last (the foundation) by a series of correct conclusions. And further, the movement away from the beginning is to be considered merely as a further determination of it, so that the beginning remains the foundation of all that follows without disappearing from it. The movement does not consist in the derivation of an Other, or in a transition into something veritably Other;—in so far as such a transition occurs, it cancels itself again. Thus the beginning of philosophy, the basis which is present and preserves itself in all the developments which follow, remains a something immanent throughout its further determinations.

What is one-sided in the beginning, owing to its general determination as something abstract and immediate, is lost in this movement: it becomes mediated, and the line of scientific advance becomes a circle.—It also follows that the constituents of the Beginning, since at that point they are undeveloped and without content, are not truly understood at the Beginning; only the Science itself fully developed is an understanding of it, complete, significant, and based on truth.

Now precisely because the Result stands out as the absolute foundation, the advance of this knowledge is not something provisional, problematical, or hypothetical; it must be determined by the nature of the subject and the content. This beginning is not arbitrary nor temporarily accepted, nor is it something which, appearing arbitrary and assumed under correction, in the event turns out rightly to have been made the beginning. (The case is not that of the construction we are directed to make in order to prove a theorem in geometry, where it is only the proof which shows that we did right to draw just these lines, and then, in the proof itself, to begin with comparisons of just those lines or angles. For itself, the drawing such lines and making such comparisons does not render the proof self-evident.)

In this way the reason why in the pure Science we begin from Pure Being was above indicated immediately in the Science itself. This Pure Being is the unity into which pure knowledge returns, or, if pure knowledge as form is to be kept separate from its unity, then Pure Being is the content of pure knowledge. It is in this respect that pure Being, the absolutely immediate, is also absolutely mediated. But it is equally essential to take it one-sidedly as pure immediacy, just because it is here taken as the beginning. Were it not to be taken as this pure indeterminateness, then, in so far as it were determinate, it would be taken as mediated and as thus already carried a step further; for what is determinate contains an Other for a first element. It is therefore in the nature of the Beginning to be Being and nothing else. For entering into philosophy there is therefore no further need of preparations, nor of other considerations or connections.

We cannot extract any closer determination or positive content for the beginning from the fact that it is the beginning of philosophy. For here at the beginning, where there is as yet no philosophy, philosophy is an empty word, or an idea taken at random and not justified. Pure knowledge affords only this negative determination, that the beginning must be the abstract beginning. In so far as Pure Being is taken as the content of pure knowledge, the latter must draw back from its content and leave it to itself without further determining it.—Or again, if Pure Being is to be regarded as the unity into which knowledge has collapsed at the point where its union with the object is consummated, then knowledge has disappeared into this unity, leaving no distinction from it, and hence no determination for it.—Nor is there any other something, nor any content, which could be used to make a more closely determined beginning.

But even the determination of Being, which has been accepted so far as beginning, could be omitted, so that the only requirement would be to make a pure beginning. There would then be nothing but the beginning itself, and it would remain to be seen what that is.—This position might be used to pacify those who partly will not be satisfied because we begin with Being (from whatever considerations), still less with the resulting transition of Being into Nothing, partly know no better than that in any science a beginning is made by presupposing some idea;—such idea being next analysed, so that it is only the result of this analysis which affords the first definite concept of the science. Were we too to observe this procedure we should have no particular Object before us, because the beginning, as being the Beginning of Thought, must be perfectly abstract and general, pure form quite without content: we should have nothing but the idea of a bare beginning as such. It remains to be seen what we possess in this idea.

So far, there is Nothing: Something is to become. The Beginning is not pure Nothing, but a Nothing from which Something is to proceed; so that Being is already contained in the Beginning. The Beginning thus contains both, Being and Nothing; it is the unity of Being and Nothing, or is Not-being which is Being, and Being which also is Not-being.

Further, Being and Nothing are present in the Beginning as distinct from one another: for the Beginning points forward to something Other;—it is a Not-being related to Being as to an Other: that which *is-beginning*, as yet *is* not: it is advancing towards Being. The Beginning therefore contains Being as having this characteristic, that it flies from and transcends Not-being, as its opposite.

And further, that which is-beginning, already is, and equally, as yet, is not. The opposites Being and Not-being are therefore in immediate union in it: in other words, it is the undifferentiated unity of the two.

The analysis of the Beginning thus yields the concept of the unity of Being and Not-being, or (in a more reflected form) the unity of the state of being differentiated and of being undifferentiated, or the identity of identity and non-identity. This concept might be considered as the first or purest (that is, most abstract) definition of the Absolute; which in fact it would be were we concerned with the forms of definitions and the name of the Absolute. In this sense, this abstract concept would be the first definition of the Absolute, and all further determinations and developments would be richer and more closely determinate definitions of it. Being may be rejected as a beginning by some because of its transition to Nothing and the resulting unity of Being and Nothing: let these see whether their beginning, which starts with the idea of the Beginning, and the analysis of this (which, though doubtless correct, also leads to the unity of Being and Nothing), will turn out more satisfactory than the method by which Being is made the beginning.

But there is yet a further observation to be made on this method. This analysis presupposes as already known the idea of the Beginning: it has therefore copied the methods of other sciences. These presuppose their object, and take leave to assume that everyone has the same idea of it, and is likely to discover in it roughly the same determinations that they themselves indicate and extract from the object in various ways by analysis, comparison, and other forms of reasoning. But that which constitutes the absolute Beginning must likewise be something otherwise known; now if it is something concrete, and therefore contains a multiplicity of determinations within itself, then this relation, which it is in itself, is assumed as something known: it is asserted to be something immediate, and this it is not; for it is merely a relation of distincts, and therefore contains mediation. Further, in the concrete the contingency and arbitrariness proper to analysis and to varying determination begin to operate. The determinations which are extracted depend on what each individual finds given in his own immediate and contingent idea. The relation contained in a concrete, that is, a synthetic unity, is necessary only in so far as it is not a datum, but is produced by the inherent movement of the moments tending back into this unity;—a movement which is the opposite of the analytic method, which is an activity belonging to the subject and external to the object.

What has been said implies this further point, that that with which we must begin, cannot be something concrete, something containing a relation within itself. For such presupposes a mediation and a transition within itself from a first to an other, of which process the concrete, now reduced to simplicity, would be the result. But the beginning must not be a first *and* an other: in a thing which in itself is first *and* an other, progress has already advanced a step. That which constitutes the beginning (and that is, the very Beginning itself) must therefore be taken, in its simple immediacy without content, as something not admitting analysis, hence as pure vacuity, as Being.

If anyone, impatient of the consideration of the abstract Beginning, should demand that we begin, not with the Beginning, but directly with the matter itself, the answer is that the matter is just this empty Being: it is in the course of the Science that we are to discover what the matter is; the Science must not therefore presuppose this as known.

If any other form is taken for the beginning in preference to empty Being, then the beginning suffers from the flaws mentioned. Those who remain dissatisfied with this beginning are asked to set themselves the task of beginning differently in order to avoid these faults.

There is however one novel beginning in philosophy, which recently has become famous and cannot be passed over without mention, namely that which begins with the Ego. It arose partly from the reflection that all that follows must be derived from the first truth, partly from the need that the first truth should be something known and, even more, something immediately certain. Such a beginning generally is not a contingent idea which can take a different form in different subjects. For, the ego, this immediate consciousness of self, first manifests itself partly as something immediate, partly as something known in a far higher sense than any other idea; things otherwise known, though they belong to the ego, are a content distinct from it and therefore contingent, whereas the ego is simple certainty of itself. But the ego in general is also something concrete, or rather the most concrete of all things—consciousness of self as of a world infinitely complex. In order to make Ego the Beginning and basis of philosophy, this concrete element must be removed by an absolute act by which the ego is purged of itself and is presented to its own consciousness as abstract ego. But this pure ego is now no longer immediate, nor is it the known and ordinary ego of our consciousness, to which the Science was to be linked immediately and equally for all. Such an act would really be an exaltation to the standpoint of pure knowledge, where the distinction between subjective and objective has disappeared. But as this elevation is demanded immediately, it is a subjective postulate: in order to prove itself a legitimate demand the movement of the concrete ego from immediate consciousness to pure knowledge would have to be proved and demonstrated upon itself from inner necessity. Without this objective movement pure knowledge, even when defined as "intellectual intuition," appears as an arbitrary standpoint, or even as one of the empirical conditions of consciousness, with regard to which all depends on whether one individual finds it within himself or can produce it, and another not. But this pure ego must be pure essential knowledge, and pure knowledge is posited in the individual consciousness only through the absolute act of self-exaltation—it is not present immediately in consciousness; and it is just in this respect that the advantage is lost which is to arise from this beginning of philosophy, namely that it is something thoroughly well known which everyone finds immediately within himself as the starting-point of further reflection. Rather, in its abstract essential nature this pure ego is something quite unknown to ordinary consciousness, something which it does not find in itself. And here, on the contrary, there sets in the disadvantage of the illusion that we are speaking of something known, namely the ego of empirical self-consciousness, whereas in fact we are speaking of something remote from this consciousness. The determination of pure knowledge as ego involves a permanent recollection of the subjective ego, the barriers of which are to be forgotten, and preserves the idea that the propositions and relations yielded in the further development of the ego occur or are found in ordinary consciousness, since it is this of which they are asserted. This confusion, instead of immediate illumination, produces so much the more glaring complications and indeed a total loss of direction; among laymen especially it has led to the crudest misunderstandings.

Next, as regards the fact that the ego in general is determined as subjective, it is true that pure knowledge clears the ego of its limited meaning, according to which the object presents to it an insuperable opposition. But for this very reason it would at least be superfluous to retain this subjective attitude and the determination of pure essence as ego. Such a determination, however, not only involves this disturbing ambiguity, but, more closely regarded, it still remains a subjective ego. The actual development of the Science which starts from the ego shows that the object there persists in having and retaining its determination as *Other* relatively to the ego, so that this ego from which we start is not pure knowledge which has veritably overcome the opposition of consciousness, but is still imprisoned in the sphere of appearance.

This further essential observation must here be made : it is true that the ego in itself can be defined as pure knowledge or intellectual intuition, and asserted as a Beginning; but in philosophy what matters is not that which is there already in itself, or internally, but the presence *in thought* of the internal element and the determinateness in which the latter thus presents itself. Whatever element of intellectual intuition is present at the beginning of the Science, it cannot be anything but primary, immediate, and simple determination; or, if the object of such intuition is called the eternal or the divine or the absolute, the same applies to whatever of these elements is present in the beginning. Whatever name be applied of richer content than that expressed by mere Being, the only matter for consideration is how such an absolute enters into thinking knowledge and the expression of this. True, intellectual intuition is the forcible rejection of mediation and of demonstrative external reflection. But if it asserts more than simple immediacy, it asserts something concrete, something containing distinct determinations. But it has already been remarked that to express and represent such a thing is a mediating movement, which begins with one of the determinations and proceeds to the other, even although the latter returns to the former; and further, such a movement must not be arbitrary nor assertoric. Hence in such a representation a beginning is not made with the concrete, but with the simple immediate whence the movement starts. Further, if a concrete thing is taken as the beginning, there is lacking the proof which is demanded by the complex of determinations contained in the concrete.

The expression of the absolute, the eternal, or God (and God has the most undisputed right that the Beginning should be made with him), or the contemplation or thought of these, may contain more than pure Being: if that is so, such content has yet to manifest itself to thinking, (and not to presentational,) knowledge; for, however rich this content, the first determination which emerges into knowledge is something simple, for it is only the simple which does not contain something more than pure Beginning: the immediate alone is simple, for there only no transition has taken place from one to an other. If these richer forms of presentation, such as the Absolute, or God, express or contain anything beyond Being, then this is, in the Beginning, but an empty word and mere Being; so that this simple vacancy without further meaning is, absolutely, the Beginning of philosophy.

This consideration is so simple that the Beginning as such requires no preparation nor further introduction; and this preliminary discussion was not so much intended to deduce it as to remove all preliminaries.

GENERAL CLASSIFICATION OF BEING

Being, first, is determined as opposed to Other in general;

Secondly, as determining itself within itself;

Thirdly, this preliminary classification being rejected, it is abstract indeterminateness and immediacy, and in this it must be the Beginning.

According to the first determination Being distinguishes itself from Essence because in its further development it shows itself in its totality to be no more than one sphere of the concept, opposing to it by way of moment another sphere.

According to the second determination it is the sphere wherein fall the determinations and the whole movement of its reflection. Being here will posit itself in the following three determinations:

I. as Determinateness, as such: Quality;
II. as Determinateness transcended: Magnitude, Quantity;
III. as Quantity qualitatively determined: Measure.

At this point this classification is a preliminary enumeration, as was pointed out in the Introduction concerning these classifications generally; their determinations must arise from the movement of Being itself, and must thereby define and justify themselves. We have nothing here to remark on the divergence of this classification from the ordinary enumeration of categories, namely as Quantity, Quality, Relation, and Modality (which same, we may note in passing, are intended by Kant himself merely as titles for his categories, though in fact they are categories themselves, only of a more universal nature); the reason why we make this observation is that the whole exposition will show the divergence from the common order and meaning of the categories.

This remark only might perhaps be made, that elsewhere Quantity is enumerated before Quality, and this—as most other things—without reason. It has already been shown that the Beginning is made with Being as such, that is, qualitative Being. And if Quality be compared with Quantity it is evident that the former is by its nature prior. For Quantity is Quality which has already become negative: Magnitude is Determinateness no longer identical with Being, but already differentiated from it; it is Quality transcended and rendered indifferent. It comprehends the mutability of Being, while the matter itself, Being, of which it is a determination, suffers no mutation through it. The qualitative determinateness, on the other hand, is one with its Being; it neither falls short of nor exceeds it, but is its immediate limitedness. Quality therefore is immediate determinateness, and as such is prior and must constitute the Beginning.

Measure is a relation, though not Relation in general, but a definite relation—that of Quality to Quantity; the categories which Kant includes under Relation will be treated in quite another place. Measure may also, if you will, be regarded as a kind of Modality; but since with Kant modality is no longer meant to constitute a determination of the content, but is merely intended to concern the relation of the content to thought (that is, to the subjective element), this is an entirely heterogeneous relation which does not belong to this place.

The third determination of Being falls under the heading of Quality because, as abstract immediacy, it reduces itself to an individual determinateness as opposed to other such within its sphere.

SECTION ONE

DETERMINATENESS (QUALITY)

Being is indeterminate immediacy; it is free from all determinateness as contrasted with Essence, and similarly from all that it can hold within itself. This unreflected Being is Being as it is immediately only in itself.

Since it is indeterminate it lacks all quality; but the character of indeterminateness in itself belongs to it only in contradistinction from that of being determinate or qualitative. Determinate Being as such opposes itself to Being in general, and hence it is just the latter's indeterminateness which constitutes its quality. Hence it will be shown that primary Being is in itself determinate, and therefore,

Secondly, passes over into Determinate Being, and is Determinate Being; which however transcends itself as finite Being, and passes over into the infinite relation of Being to itself, namely,

Thirdly, Being for Self

CHAPTER I

BEING

A

BEING

Being, pure Being—without any further determination. In its indeterminate immediacy it is similar to itself alone, and also not dissimilar from any other; it has no differentiation either within itself or relatively to anything external; nor would it remain fixed in its purity, were there any determination or content which could be distinguished within it, or whereby it could be posited as distinct from an Other. It is pure indeterminateness and vacuity.—Nothing can be intuited in it, if there is any question here of intuition, or again it is merely this pure and empty intuition itself; equally there is in it no object for thought, or again it is just this empty thought. In fact, Being, indeterminate immediacy, is Nothing, neither more nor less.

B

NOTHING

Nothing, pure Nothing: it is simple equality with itself, complete emptiness, without determination or content: undifferentiatedness in itself.—In so far as mention can here be made of intuition or thought, it is considered a distinction whether we intuit, or think, something or nothing. In that case, to intuit, or think, nothing, has a meaning: so if intuition or thought of Being and of Nothing are distinguished, then Nothing *is* (or does exist) in our intuition or thought; or rather, it is this empty intuition and thought itself: the same empty intuition or thought, as pure Being.—Nothing, therefore, is the same determination (or rather lack of determination), and thus altogether the same thing, as pure Being.

C

BECOMING

1. Unity of Being and Nothing

Pure Being and pure Nothing are, then, the same; the truth is, not either Being or Nothing, but that Being—not passes—but has passed over into Nothing, and Nothing into Being. But equally the truth is not their lack of distinction, but that they are not the same, that they are absolutely distinct, and yet unseparated and inseparable, each disappearing immediately in its opposite. Their truth is therefore this movement, this immediate disappearance of the one into the other, in a word, Becoming; a movement wherein both are distinct, but in virtue of a distinction which has equally immediately dissolved itself.

Observation 1

Nothing is generally opposed to Something; but Something is an already determinate existent distinguished from another Something: such a Nothing opposed to Something, therefore, such a negation of Something, is a determinate Nothing. Here, however, Nothing must be taken in its indeterminate simplicity. —If it were held more correct to oppose Not-being, instead of Nothing, to Being, no objection could be made as far as the result is concerned, for Not-being contains the relation to Being: both Being and its negation are simultaneously asserted, and this assertion is Nothing as it exists in Becoming. Primarily, however, we are not concerned to have a formal opposition (that is, a relation) so much as abstract and immediate negation, Nothing purely for itself, unrelated repudiation, or what one might express if one so wished, merely by "Not."

The simple idea of pure Being was first enunciated by the Eleatics, as the Absolute and as sole truth; especially by Parmenides, whose surviving fragments, with the pure enthusiasm of thought first comprehending itself in its absolute abstraction, proclaim that "Being alone is, and Nothing is not at all."—It is well known that in oriental systems, and essentially in Buddhism, Nothing, or the Void, is the absolute principle.—Herakleitos was profound enough to emphasize in opposition to this simple and one-sided abstraction the higher total concept of Becoming, saying: "Being is no more than Nothing is," or "All things flow," which means, everything is Becoming.—Popular sayings, chiefly oriental, to the effect that everything which is has in its birth the germs of its decay, while death conversely is entrance to a new life, express at bottom the same union of Being and Nothing. But such expressions have a substratum upon which the transition takes place: Being and Nothing are held apart in Time; they are presented as alternating in Time, and they are not thought in their abstraction, and therefore not thought in such a manner as makes them in and for themselves the same.

Ex nihilo nihil fit is one of the maxims to which great importance was at one time ascribed in metaphysics. Either it is just the empty tautology, "Nothing = Nothing;" or, should Becoming be supposed to have real meaning in it, there is in fact no Becoming contained in it, for, since only Nothing comes out of Nothing, Nothing still remains Nothing here. Becoming implies that Nothing should not remain Nothing but pass over into its Other, into Being.—In rejecting the proposition that nothing comes out of nothing, later (chiefly Christian) metaphysics asserted the transition of Nothing into Being; however synthetically or merely imaginatively this proposition was taken, even the least complete union contains a point where Being and Nothing coincide and their distinction vanishes.—The true importance of the proposition that Nothing comes out of Nothing, that Nothing is just Nothing, resides in its opposition to Becoming generally and therefore to the creation of the world out of nothing. Those who assert the proposition that Nothing is just Nothing, and even grow heated in its defence, do not know that in so doing they are subscribing to the abstract Pantheism of the Eleatics and, in essentials, of Spinoza. That view in philosophy which takes for principle that Being is merely Being, and Nothing merely Nothing, deserves the name of system of identity: this abstract identity is the essence of Pantheism.

If the result which identifies Being and Nothing is startling in itself or seems a paradox, no further attention is here to be paid to this: rather should we wonder at this wonderment which shows itself such a tiro in philosophy, and forgets that in this Science there occur determinations quite different from those of ordinary consciousness and so-called common-sense,—which is not exactly sound understanding, but understanding educated up to abstractions and the faith, or rather superstition, of abstractions. It would not be difficult to demonstrate this unity of Being and Nothing in every example, in every fact and thought. What was said above about immediacy and mediation (which latter contains a relation and therefore negation), must also be said of Being and Nothing: that neither in heaven nor on earth is there anything not containing both Being and Nothing. Since here we are speaking of a Something, of an actual fact, these determinations admittedly are no longer found in that complete untruth in which they manifest themselves as Being and Nothing: they have already been further determined and are taken, for example, as positive and negative, the former being posited and reflected Being, the latter posited and reflected Nothing: now of positive and negative, the former has Being and the latter Nothing for abstract basis.—And so, in God himself, his quality (energy, creation, might, and the like) essentially contains the determination of the negative,—it is the production of an Other. But an empirical elucidation of this assertion by means of examples would here be entirely superfluous. This unity of Being and Nothing, as being the primary truth, is, once and for all, the basis and the element of all that follows: therefore, besides Becoming itself, all further logical determinations, such as Determinate Being, Quality, and in short all philosophic concepts, are examples of this unity.—As for what calls itself sound or common sense, so far as it rejects the inseparateness of Being and Nothing, it might be invited to find an instance where One can be found separate from Other—Something from limit (or barrier), or the infinite (or God), as we have just remarked, from energy. Only the empty thought-structures, Being and Nothing, are thus separate, and those are they which common sense prefers to the truth which everywhere faces us—the unity of both.

It cannot be our intention to meet at all points the confusions into which ordinary consciousness falls when faced by such a logical proposition, for they are inexhaustible. Only a few can be mentioned. One source among others of such confusion is that consciousness imports into such an abstract logical proposition representations of a concrete something, forgetting that we are not dealing with this, but with the pure abstractions of Being and Nothing, and that these alone are to be grasped.

Being and Not-being are the same: *ergo* it is the same whether I am or am not, whether this house is or is not, whether these hundred Thalers are part of my fortune or not.—This inference from or application of the proposition radically alters its meaning. In the proposition we merely have the pure abstractions of Being and Nothing; in the application these become a determinate Being and a determinate Nothing. But it has already been said that we are not here dealing with determinate Being. A determinate or finite Being is such as refers itself to an other; it is a content which stands in a relation of necessity with other content or with the whole world. In view of the mutually determinant connection of the whole, metaphysics could make the assertion (which is really a tautology) that if the least grain of dust were destroyed the whole universe must collapse. In the instances which are adduced against our proposition, something appears for which it is not indifferent whether it is or is not; and this not because of its being or not-being, but because of its content, which connects it with other matter. If a determinate content or being is presupposed, then such determinate being, because determinate, stands in various relations with other content; and for such being it is not indifferent whether a certain other content with which it stands in a relation, is, or is not: for it is only such a relation which essentially makes it what it is. The same occurs with imagination (where we oppose not-being in the more determinate sense of imagination to actuality); and in the context of imagination, the being and the absence of a content which is imagined as determined and in relation with other, are not indifferent.

This consideration also contains what is of capital importance in Kant's critique of the ontological proof of the existence of God; but it can be noted here only with respect to the distinction which occurs in this proof between (*a*) Being and Nothing in general and (*b*) *determinate* Being and Not-being.—The reader will know that in this so-called proof the concept of an essence is presupposed to which all realities pertain, and therefore that of existence, which was also taken as one of the realities. The Kantian critique in the main followed the line that Existence or Being (here taken as equivalent) is no attribute nor real predicate, that is, is not a concept of something which could be added to the concept of a thing.[1]—Kant by this means to say that Being is not a determination of content.—Therefore, he proceeds, what is potential contains no more than what is actual: one hundred actual Thalers contain not one doit more than one hundred potential;—that is, the former have no other determination of content than the latter. It is, in fact, indifferent for this content, regarded as isolated, whether it is or is not: it contains no distinction of being and not-being, indeed this distinction does not touch it at all; the hundred Thalers become no less when they are not, and no more when they are. A distinction must come from another source.—"On the other hand," so Kant reminds us, "my fortune is larger when the hundred Thalers are real than when they are a mere concept or merely potential. For, where it is actual, the object is not merely contained analytically in my concept, but is added synthetically to my concept (which is a determination of my condition), without the aforesaid hundred Thalers being in the least increased because they exist externally to my concept."

Two conditions, to retain Kant's terminology, which is not without a certain confused clumsiness, are here presupposed,—(1) what Kant calls concept, meaning representation; and (2) my fortune. For each of these conditions, for fortune as well as representation, such hundred Thalers are a determination of content, or, as Kant expresses himself, "they are added synthetically to a condition." I as owner or not-owner of one hundred Thalers, or I as representing, or else as not representing, them to myself: here there is, of course, a difference of content. And, more generally, the abstractions both of Being and of Nothing cease to be abstractions when they are given a determinate content; Being then becomes reality, the determinate being of one hundred Thalers, and Nothing becomes negation, the determinate not-being of them. This determination of content itself, the hundred Thalers, has remained the same in the one as in the other; and this is also the case when it is taken abstractly by itself. But when, further, Being is taken as a condition of my fortune, then the hundred Thalers enter into a certain relation with this condition, and, for this latter, such determinateness as the Thalers have is not indifferent; this being and not-being of the hundred Thalers is merely change; they are transferred into the sphere of Determinate Being. Therefore, if anyone urges against the unity of Being and Nothing that after all it is not indifferent whether a certain thing (such as the 100 Thalers) is or is not, it is because he is under the delusion that the distinction between possessing and not possessing one hundred Thalers has been passed on merely to Being and Not-being; this delusion depending, as has been shown, on the one-sided abstraction which omits the Determinate Being present in such examples and retains merely Being and Not-being, conversely changing abstract Being and Nothing, which it is its task to comprehend, into Determinate Being and Determinate Nothing. It is only Determinate Being itself which contains the true distinction between Being and Nothing, namely Something and an Other.—This real distinction is vaguely presented to the mind instead of abstract Being and pure Nothing and the merely intended distinction between these.

---

[1] Kant's *Critique of Pure Reason*, 2nd Ed., pp. 628 *sqq*.

Kant employs the expression that "existence adds something to the fabric of total experience"; "it gives us one further object of perception, though our concept of the object is not thereby increased."—From the preceding explanation it follows that this means that through existence Something, essentially because it is determinate existence, is connected with Other, which also includes a percipient.—Kant says that the concept of one hundred Thalers is not increased by reason of their being perceived. The concept here means the before-mentioned hundred Thalers presented as isolated. It is true that in this isolated manner they are an empirical content, but content cut off and without connection or determinateness relatively to Other; the form of identity with self deprives them of relation to Other and makes their perception or non-perception a matter of indifference. But this so-called concept of one hundred Thalers is a spurious concept; the form of simple relation to self does not properly belong to such limited and finite content, it is a form applied and lent to it by subjective understanding. One hundred Thalers are not related to themselves alone: they are mutable and perishable.

Thought or representation which has before it only a particular kind of Being, namely Determinate Being, must be referred back to the already mentioned beginning of Science which was made by Parmenides: it was he who purified and elevated his own processes of representation (and so those of posterity) to pure Thought, to Being as such, and so created the Element of the Science.—What is first in science has had to show itself first, too, historically. And we must consider the Eleatics' One or Being as the first step in the knowledge of Thought; water and similar material principles are indeed intended for the Universal, but are material and therefore not pure thought; numbers are Thought quite external to itself, and neither primary and simple nor resting in itself.

It is to be considered not only the very first demand of theory, but also of practice, that Thought should be ordered back from particular and finite Being to Being as such in its purely abstract universality. For if such an ado is made about the hundred Thalers, and it is said that the state of my fortune is altered by their possession or non-possession, and that the difference is still greater between the being and not-being of myself, or of other; then—neglecting the fact that there will probably be such fortunes as are indifferent to the possession of such hundred Thalers—we must recall that it is the duty of man to rise to that abstract universality of outlook for which it is indeed indifferent whether the hundred Thalers exist or not, whatever their quantitative relation to his fortune, just as much as he should be indifferent to his existence or non-existence—in finite life (for a condition, a determinate being, is here intended), and so forth. *Si fractus illabatur orbis, impavidum ferient ruinæ* was said by a Roman: a Christian should be even more perfectly indifferent.

Finally, we must notice the immediate connection between this elevation above the hundred Thalers (and finite things in general), and the ontological proof, together with Kant's critique of this. This critique has made itself universally plausible with its popular example: who does not know that a hundred actual Thalers differ from a hundred merely potential ones, and that they constitute a difference in my fortune? Because this difference is so salient with the hundred Thalers, therefore the concept (in the sense of the determinateness of content as bare potentiality) and being also are distinct: hence the concept of God is distinct from his existence, and I can no more "extract" the existence of God from his concept than I can get the actual existence of one hundred Thalers from their potential existence; now the ontological proof is supposed to consist in this extraction of the existence of God from his concept. But if it is correct that concept is not the same as being, it is truer still that God is not the same as one hundred Thalers or other finite things. It is the definition of finite things that with them concept and being, concept and reality, soul and body, are different and separable, and that therefore they are perishable and mortal: while it is just the abstract definition of God that with him concept and being are unseparated and inseparable. The true critique of the Categories and of Reason consists precisely in explaining this difference to cognition, and in preventing it from applying to God the determinations and relations of the finite.

Observation 2

Another reason should be mentioned which strengthens the repugnance to the proposition about Being and Nothing: this is, that the result of the consideration of Being and Nothing, when expressed in the proposition that "Being and Nothing are one and the same," is incomplete. The accent is chiefly placed on the "are one and the same," as in a judgment generally, where it is only the predicate which tells what the subject *is*. The sense therefore appears to be that the distinction is denied, although it immediately occurs in the sentence; for the proposition asserts the *two* determinations of Being and Nothing and contains them as distinct.—At the same time the meaning cannot be that they are to be abstracted from, and only the unity retained. Such a meaning would proclaim its own inadequacy, since that from which abstraction is to be made is present, and is named, in the sentence. Now in so far as the proposition that "Being and Nothing are the same" asserts the identity of these determinations, while yet containing both as distinct, it is self-contradictory and dissolves itself. If we look more closely we find that a proposition has here been asserted which, carefully considered, has a movement by which, through its proper nature, it disappears. But in so doing it does what must be held to constitute its true content, it undergoes Becoming.

The proposition thus contains the result, it in itself is the result. That to which attention must here be drawn, is the defect that this result is not itself expressed in the sentence; it is an external reflection which perceives it in the proposition. —At the outset we must make the general observation on this subject that a proposition, in the form of a judgment, is ill fitted to express speculative truths; acquaintance with this fact would be apt to remove many misunderstandings of speculative truths. Judgment is a relation of identity between subject and predicate, and it abstracts from the fact that the predicate is not the only determinateness of the subject, and also is wider than the subject. Now when the content is speculative the non-identical nature of the subject and predicate is an essential moment; but this is not expressed in the judgment. The paradoxical and bizarre light in which many aspects of modern philosophy appear to those unacquainted with speculative thought, is frequently due to the form of the simple judgment when it is used to express speculative results.

In order to express the speculative truth, the defect is remedied, in the first place, by adding the contradictory proposition, namely, "Being and Nothing are not the same;" this proposition, too, is expressed above. But now a further defect arises: these propositions lack connection and therefore show their content only in the form of an antinomy, whereas the content refers to one and the same thing, and the determinations expressed in the two propositions should be absolutely united; so that the union can then be called only a restlessness between incompatible terms, a movement. The most common injustice done to speculative content is to make it one-sided, that is, to emphasize one only of the propositions into which it can be resolved. When this happens, it cannot be denied that that proposition is asserted; but this assertion is as false as it is true, for once one proposition is taken from the speculative complex the other, too, should equally be considered and enunciated.—Special mention must here further be made of this unhappy word, as it may be called, of "unity;" unity even more than identity indicates subjective reflection; it is generally taken as the relation arising from external reflection, from comparison. In so far as this finds the same thing in two distinct objects there is unity, but in this shape, that the complete indifference towards this unity of the objects compared is presupposed, so that comparison and unity in no way concern the objects themselves, being a procedure and determining quite external to them. Unity therefore expresses a quite abstract sameness, and sounds violent and striking in proportion as the objects of which it is asserted obviously show themselves as distinct. In this respect therefore mere Unseparateness or Inseparability would be a good substitute for Unity; but these would not express the affirmative nature of the relation of the whole.

The whole and true result, therefore, which has here been found, is Becoming, which is more than the one-sided or abstract unity of Being and Nothing. On the contrary it consists of this Movement—that pure Being is immediate and simple; that it therefore equally is Nothing; that there is a distinction between them, but equally this cancels itself, and is not. In the result, therefore, the distinction between Being and Nothing is equally asserted, but is asserted as a distinction that is merely intended.

It is *intended* to take Being, rather, as that which is simply other than Nothing; nothing is clearer than their absolute distinctness, and nothing seems easier than to be able to describe it. But it is equally easy to prove the impossibility of this, and that the distinction cannot be expressed. Those who insist on remaining at this standpoint of the distinctness of Being and Nothing, should set themselves the task of indicating in what it consists. If Being and Nothing had any determinateness to distinguish one from the other, they would be determinate Being and determinate Nothing, as has been recalled, and not pure Being and pure Nothing, as they still are at this stage. The distinction between them is therefore completely void; each is the indeterminate itself, and is so in the same manner; the distinction therefore does not lie in themselves but in a third thing, namely our intention. This however is a form of the subjective which does not belong to this stage of the exposition. But the third element which gives subsistence to Being and Nothing must occur here too; and indeed it has occurred here too, and is Becoming. In Becoming the two exist as distinct; and only in so far as they are distinct is there Becoming. This third is something other than themselves;—they subsist solely in an Other, which also means that they do not subsist for themselves. Becoming is the subsistence of Being as much as of Not-being; in other words, their subsistence simply is their being in One: it is just this their subsistence which equally transcends their distinction.

The challenge to indicate the distinction between Being and Nothing includes a challenge to say what Being, and what Nothing, is. Those who struggle against recognizing, in either, any more than a transition to the other, and make various assertions about Being and Nothing, should indicate what they are speaking of; that is, they should give a definition of Being and Nothing, and demonstrate its correctness. Unless they satisfy this first demand of the old science of which they elsewhere admit and employ the logical rules, all such assertions about Being and Nothing are merely asseverations, scientifically invalid. If it has elsewhere been said that Existence (here regarded as equivalent to Being) is the complement to Possibility, then another determination, that of Possibility, is presupposed, and Being is asserted, not in its immediacy, but as dependent and conditioned. We shall reserve for such Being as is mediated the term of "Existence." Of course Being is sometimes imaged, for instance as pure light, as the clearness of unclouded vision, and Nothing as pure night; and their distinction is thus connected with this well-known sensuous difference. But in fact, if this pure seeing be imagined more exactly, it is easy to perceive that in absolute clearness as much and as little is seen as in absolute darkness, and that one kind of seeing, as much as the other, is pure seeing, that is, a seeing of nothing. Pure light and pure darkness are two voids which are the same. We can distinguish only in determinate light, that is, since light is determined by darkness, in clouded light,—and, equally, in determinate darkness, that is, since darkness is determined by light, in illuminated darkness: and this is just because clouded light and illuminated darkness contain distinction in themselves and therefore are Determinate Being.

Observation 3

The unity whose moments, Being and Nothing, exist as inseparable, is at the same time different from them, and therefore stands to them in the relation of a third: this, in its most characteristic form, is Becoming. Transition is the same thing as Becoming; only in the former the two, between which, as One and an Other, the transition takes place, are imagined rather as resting apart, transition taking place between them. Now, in whatever place or manner Being or Nothing is discussed, this third must be there too; for these two do not subsist for themselves, but are only in this third, in Becoming. This latter however has many empirical forms which abstraction shelves and neglects, in order to retain severally these its creations, Being and Nothing, and to prove them armed against transition. This simple attitude of abstraction is met, equally simply, by the reminder that in empirical Existence this abstraction itself is only Something, and has Determinate Being. Or the attempt is made to give fixity to this separation of inseparables by other Forms of Reflection. Such a determination contains its own opposite in itself, and this determination of reflection is refuted in itself, without any return or appeal to the nature of its object, by taking it as it is and showing it to contain its Other within itself. It would be wasted labour to spread a net for all the twistings and objections of reflection and its reasonings, in order to cut off and render impossible all the evasions and digressions which it uses to hide from itself its own internal contradictoriness. I therefore refrain from noticing much that calls itself objection or refutation of the assertion that neither Being nor Nothing is true and only Becoming is their truth. That temper of mind which alone can perceive the futility of such refutations, or rather alone can expel them, is only brought about by critical cognition of the forms of Understanding; but those who are readiest with such objections immediately assault with their reflections the very first propositions; not attaining nor having attained, through a further study of Logic, to any consciousness of the nature of these crude reflections.

We must now consider some of the phenomena which result from the isolation of Being and Nothing, when one is placed without the sphere of the other, and transition is thus denied.

Parmenides held fast to Being, and was the most consistent of all when at the same time he said of Nothing that it is not at all; Being alone is. Being thus isolated is the indeterminate; it is not related to Other; it therefore seems that no progress could be made from this Beginning (that is, from out of Being itself): progress could only be effected by connecting something foreign from without. The progress made by asserting that Being is the same as Nothing, therefore, appears like a second and independent Beginning,—a transition dependent on itself and merely added to Being from without. In short, Being would not be the absolute Beginning if it were in any way determined; for, if it were, it would be dependent on something else, would not be immediate, would not therefore be the Beginning. But if Being is indeterminate and therefore the true Beginning, it lacks whatever could transform it into an Other; it is End as well as Beginning. Nothing can assault it from without; but also, nothing can sally forth from within; Parmenides, equally with Spinoza, will not admit progress from Being, or the absolute Substance, to the negative or finite. We have remarked that, on this showing, Being admits of no relation and therefore no progress; if a progress is made nevertheless, it can only be effected in an external manner, and must be a second and new Beginning. Thus Fichte's most absolute and unconditional, fundamental assertion, $A = A$, is Thesis; his second is Antithesis: the latter is supposed to be partly conditioned, partly unconditioned, and thus self-contradictory. This progress is therefore one of external reflection, which, first, denies what, as being an Absolute, it made its Beginning (Antithesis being the negation of the first identity); and, secondly, forthwith expressly makes conditioned its second unconditioned element. But if there were any justification for the progress, that is, for transcending the first Beginning, then this first Beginning would itself have to possess the capacity of having an Other related to itself: it would have to be determinate But such neither Being nor Absolute Substance professes to be: quite the reverse. It is the immediate, that which is as yet utterly indeterminate.

The most eloquent descriptions—perhaps already forgotten—

of the impossibility of passing on from an abstract to something beyond, and to a union of the two, are given by Jacobi in support of his attack on the Kantian "*a priori* Synthesis" of self-consciousness, in his dissertation on the undertaking of the Critical Method to bring Reason within limits (Jac., *Works*, Vol. III). He puts the problem in this way, that the origin or creation of a synthesis is to be demonstrated in some pure sphere, whether in pure consciousness, pure space or pure time (p. 113). "Let space be one, let time be one, let consciousness be one;—say now, how does any one of these three Ones, purely and in itself, engender multiplicity? Each is one and no other; an indifference, a self-identity of space, time, and consciousness, where not even space, time, and consciousness can be asserted; for these nouns still slumber in the infinite naught of the indeterminate, whence each and every determinate has yet to issue. What introduces finitude into these three infinities? What renders Space and Time, *a priori*, pregnant with Number and Measure and changes them into a pure multiplicity; what causes pure spontaneity (Ego) to oscillate? Whence does the pure vowel get its consonant, or rather, how does its soundless and unbroken note break off and interrupt itself in order to get at least a kind of independent sound, an accent?"—It will be seen that Jacobi saw very clearly the unsubstantial nature of abstraction, whether of so-called absolute, that is, of abstract Space, or of abstract Time, or of abstract pure Consciousness or Ego; and he remains in this position in order to uphold the impossibility of a progress to Other (which is the condition of a Synthesis), and so to the Synthesis itself. The Synthesis here in question must not be taken as a connection of determinations already present externally;—the question is, partly, the creation for a first element of its second, and for the originally indeterminate, of something determinate; partly, it is the immanent synthesis or synthesis *a priori*,—the self-existent unity of distincts. Becoming is this immanent synthesis of Being and Nothing; but, since "synthesis" suggests external juxtaposition of things already there externally to one another, the word synthesis, and synthetic unity, rightly has not been employed.—Jacobi asks how the pure vowel of the ego gets its consonant, and what introduces determinateness into indeterminateness. The latter question is easily answered, and has been answered by Kant after his fashion; the former asks after the particular manner, the relations and so forth, and thus demands that a special category should be indicated; but here there can be no question of particular manner or of such categories of understanding. The question "how" is itself one of the bad habits of reflection, which asks how a thing can be conceived, but at the same time presupposes its own fixed categories, and thus knows itself armed against the answer to its question. With Jacobi too the question lacks the higher significance of a demand after the necessity of the synthesis, for, as was said, he stays firm in his intention to remain with the abstractions, so as to assert the impossibility of a synthesis. His description of the procedure for reaching the abstraction of Space is especially graphic (p. 147). "For a time I must seek utterly to forget whatever I saw, heard, touched or handled, myself expressly not excepted. Utterly and purely must I forget all motion and, just because it is hardest, be at greatest pains to achieve this forgetting. Whatsoever I have thought away I must also wholly and entirely keep away, nothing being retained save only the intuition, forcibly retained, of infinite unchangeable Space. Nor must I re-introduce myself into Space as something distinct from it and yet connected: I must not merely allow myself to be surrounded and penetrated by it: I must pass over into Space, become one with it, be changed into it; nothing must remain of me save this my intuition itself, so that this may be contemplated as a presentation truly self-subsistent and independent, one, and one only."

Such being the perfectly abstract purity of Continuity, that is, of indeterminateness and vacuity of presentation, this abstraction may indifferently be termed Space, or pure Intuition, pure Thought;—the Indian, who, externally motionless, and impassive also as regards sensation, fancy, imagination, desire, and so forth, contemplates the point of his nose for years on end, rehearsing his *Om Om Om*, or else nothing at all, has one name for all these concepts—Brahma. This torpid and vacuous consciousness, taken as consciousness, is Being.

Now in this void, so Jacobi continues, his experience is the reverse of what it should be, according to Kant's assurance: he does not find himself to be a multiplicity and manifold, but rather a One quite lacking either of these attributes; indeed, "I am impossibility itself, the annihilation of all manifold and multiplicity; from my pure, utterly simple and unalterable essence I cannot reconstruct, nor spirit into myself, any least thing; thus in this pure state the external and separate existence of entities, and every manifold and multiplicity based on these, are manifested as a pure impossibility" (p. 149).

This impossibility is simply the tautology of saying, "I hold fast to abstract unity and exclude all multiplicity and manifold; I persist in the undifferentiated and indeterminate, neglecting all that is differentiated and determinate." The Kantian *a priori* synthesis of self-consciousness, the active force, which this unity contains, whereby it splits itself up and sustains itself in this state of diremption, is, with Jacobi, watered down to the same abstraction. This "synthesis in itself," this "original judgment," with him is one-sidedly turned into "the Copula in itself;—an 'is,' 'is,' 'is,' without beginning or end, without what, who, or which; this repetition of repetition *ad infinitum* is the one business, function, and output of this purest of pure synthesis; which itself is just pure, absolute, and simple repetition" (p. 125). Or in fact, there being no break, that is, no negation or distinction, in it, it is not repetition, but merely undifferentiated simple Being.—But is this synthesis, if Jacobi omits precisely that which makes unity into synthetic unity?

It must be remarked, first, that when Jacobi thus entrenches himself in absolute or abstract Space, Time, and Consciousness, he has in this manner entered, and retains, a sphere *empirically* false; there is, given empirically, neither Space nor Time which is spatially or temporally unlimited, or whose continuity is not filled with the manifold limits of Determinate Being and Change, so that these limits and changes belong, unseparate and inseparably, to spatiality and temporality; similarly consciousness is filled with determinate sensation, presentation, desire, and so forth; it exists only in connection with some particular content.—The empirical transition in any case goes without saying; it is true that consciousness can make empty space, empty time, and even empty consciousness, or again empty Being, its object and content; but it does not remain there; it proceeds, or rather bursts on from such vacuity to a content which is better, that is, in some way more concrete;

which, however bad it may be in other ways, is, by this much at least, better and truer; and precisely such content is synthetic in itself, synthetic, taken in the more general sense. Thus Parmenides is faced with Appearance and Opinion, the opposite of Being and Truth; Spinoza with the Attributes, the Modes, Extension, Movement, Understanding, Will, and so forth. The synthesis contains and demonstrates the invalidity of these abstractions: in the synthesis they are united each with its Other and therefore exist, not as self-subsistent and absolute, but as entirely relative.

But we are not here concerned to demonstrate the empirical invalidity of empty space, and so forth. It is true that, by abstracting, consciousness can fill itself with those indeterminates; and the abstractions, thus fixed, are the ideas of pure space and time, pure consciousness, and pure being. The task is, to demonstrate as invalid the idea of pure space, and so on, that is, pure space itself, and so on, *in itself*: to demonstrate, in other words, that, being what it is, it is its own opposite, that its opposite has penetrated into it as being what it is, that it itself is the perfected transition from itself, namely, determinateness.

This, however, is immediately evident in these ideas. They are, as Jacobi abundantly describes them, results of abstraction, they are expressly determined as indeterminate; and the indeterminate, in its simplest form, is Being. But it is just this indeterminateness which constitutes its determinateness; for, the former being the opposite of the latter, indeterminateness, as being opposite, is itself the determinate, or negative, and, moreover, the pure and entirely abstract negative. It is this indeterminateness or abstract negation, thus inherent in Being, which is asserted by external as well as internal reflection when it equates Being with Nothing, declaring it to be an empty fabrication of thought: that is, Nothing.—Or we may say that, since Being is the indeterminate, it is not what it is (namely, affirmative determinateness): it is not Being, but Nothing.

In the pure reflection of the Beginning, as it is made in this Logic with Being as such, the transition is as yet hidden; since Being is posited as merely immediate, Nothing can break forth from it only immediately. But every subsequent determination (as will immediately be seen in Determinate Being) is more concrete; in Determinate Being that is already posited which contains and produces the contradiction, and hence the transition, of those abstractions. While Being is considered as this simple immediacy, the recollection that Being is the result or perfect abstraction and by this alone abstract negativity, of Nothing, is left outside the science, which, within itself, will, under the category of Essence, exhibit this one-sided immediacy as mediated; for Being there is *posited* as Existence and as that which mediates this Being, as Ground.

With this recollection the transition from Being to Nothing can even be imagined as something easy and trivial, or, as it is called, can be explained and made conceivable, thus: Being, which was made the Beginning of the Science, is of course Nothing, for we can abstract from everything, and this having been done, Nothing remains. But, so one might continue, in this way the Beginning is not affirmative, it is not Being but just Nothing, and Nothing then is the End, at least as much as immediate Being and even more so. It is simplest to let such reasoning be and look for the nature of the results of which it boasts. If Nothing were the result of this reasoning, and we should begin with Nothing (as does Chinese philosophy), this need not cause us to raise a finger, for before it could be raised this Nothing would have changed into Being (see above, *B*. Nothing). But if this abstraction from everything, since everything after all has the quality of *being*, is presupposed, it must be examined more exactly; the result of abstracting from all beings is, first, abstract Being, Being in general; just as in the cosmological proof of the existence of God we rise above the accidental being of the world, but, in rising, take with us this Being: Being is determined as infinite Being. But we can also abstract from this pure Being, Being can be added on to all that from which we have already abstracted: then Nothing remains. Now if we wish to forget the thinking of Nothing, that is, its metamorphosis into Being, or know nothing about it, we can continue further to do what we *can*: for we can, further, (heaven be thanked) abstract from Nothing (—the Creation of the World is an abstraction from Nothing), and then not Nothing remains, for it is just this from which we are abstracting; and so Being has been reached again.—This method of "can" affords an external play of abstraction where abstraction itself is a one-sided activity of the negative element.

First, then, to this method, Being is as indifferent as Nothing, and each arises as much as it vanishes; but it is, equally, indifferent whether we begin with the doing of Nothing, or with Nothing; for the former, that is, pure abstracting, is neither more nor less real than pure Nothing.

The dialectic which Plato applies to the One in the *Parmenides* must also be taken rather as a dialectic of external reflection. Being and the One are both Eleatic concepts, and are identical. But they are also to be distinguished, and this is the aspect which Plato seizes upon in this dialogue. He removes from the One its various determinations, *e.g.* whole and part, being in itself and being in an other, shape, time, and so forth, and reaches the result that Being does not pertain to the One, since Being pertains to nothing save in one of these modes (p. 141, e. Vol. III, ed. Steph.). Plato next deals with the proposition, "The One is;" and Plato himself should be consulted for the transition from this to the proposition that the One is not. It is effected by comparing the two determinations of the proposition which is assumed, namely, "The One is," which contains "One" and also "Being": now "The One is" contains more than the simple assertion of "One." The negative element of the proposition is demonstrated from the difference between these two. It is clear that this method presupposes something and is an external reflection.

The One is here connected with Being: similarly Being which it is desired to retain abstractly for itself, is best shown, without any reference to thought, in a connection which contains the contrary of that which is to be asserted. Being, taken in its immediacy, belongs to a Subject; it is asserted, has an empirical existence in general, and therefore is within the sphere of limitation and negation. Let understanding take what turns or phrases it will in its repugnance to the unity of Being and Nothing and in its appeal to that which is immediately at hand, it will find in this experience nothing but determinate Being, Being with some limitation or negation, which is just that unity which it rejects. The assertion of immediate Being is thus reduced to empirical Existence, nor can it reject the demonstration of this empirical Existence, since it desires to cling to the immediacy external to thought.

The case of Nothing is the same, but in the opposite manner, and this reflection is well known and has often enough been made about Nothing. Nothing, taken in its immediacy, appears as being; for in its nature it is identical with Being. Nothing is the object of thought, of imagination, of speech: it therefore is; it has its being in thought, imagination, speech, and so forth. But, further, this Being also differs from it; it is therefore said that, although Nothing is in thought or imagination, yet it cannot for that reason *be*, and being does not belong to it as such, since such being is just thought or imagination. It is undeniable that where this distinction is made Nothing is in a relation to Being; but this relation, although it contains the distinction, also contains a unity with Being. In what manner soever Nothing is asserted or demonstrated, it shows itself in connection or, if that is preferred, in contact, with some Being, and unsevered from some Being; that is, precisely, in a Determinate Being.

But, where the presence of Nothing in a Determinate Being is thus demonstrated, its distinction from Being is vaguely presented in the form that Being does not pertain to Nothing in itself, and that it has not in it the quality of being for itself, that it is not Being as such; and that Nothing is just absence of Being, and darkness absence of light, coldness absence of warmth, and so on. It is said that darkness has meaning only with reference to the eye, by virtue of an external comparison with the positive element (light); and that similarly coldness is something only in our sensation; whereas light, warmth, and Being, are, in themselves, the Objective, the Real and Active, having a quality and dignity quite different from that of those Negatives, or of Nothing. It is a common thing to find it cited as a most important reflection and significant recognition that darkness is mere absence of light and cold mere absence of warmth. Here, within the sphere of empirical objects, this one empirical remark may be made about this acute reflection, that darkness does indeed show itself active in light by determining it to colour, and thereby alone endows it with visibility; since, as was said above, as little is seen in pure light as in pure darkness. But visibility is activity within the eye, an activity in which this negative element has as great a share as light which counts for real and positive; similarly cold manifests itself adequately to water, to our sensation, and so on, and if we deny it so-called objective reality, our case against it is not strengthened thereby in the least. But this further censure might here be made, that here as well as above the negative of a determinate content is discussed, and the argument does not stop at Nothing itself, which, as regards empty abstraction, is neither superior nor inferior to Being.—But let cold, darkness, and other determinate negations be taken for themselves, and let it be seen what is thereby posited with regard to their general determination according to which they are here introduced. They are to be, not Nothing in general, but the not-being of light, warmth, and so on, of something determinate, of a content: they are, if one may say so, Nothings determinate and having content. But a determinateness, as will also be seen below, is itself a negation; hence they are negative nothings: but a negative nothing is something affirmative. This transformation of Nothing into something affirmative, by virtue of its determinateness (which above appeared as a Determinate Being in the subject, or whatever else it might be), seems the greatest of paradoxes to the consciousness which stands firm in the abstractions of Understanding. The recognition (in spite, or perhaps because, of its simplicity) that the negation of a negation is something positive, appears to be a triviality beneath the notice of Understanding in its pride, though no doubt correct enough: but this recognition not only is correct, but, because of the universality of such determinations, is of such infinite scope and universal application that it is well worthy of attention.

This further remark may be made about the determination of the transition of Being and Nothing into one another, that it is similarly to be taken without further determination of reflection. It is immediate and quite abstract, because of the abstract nature of the moments which pass into one another, since there is not yet posited in either moment the determinateness of that other which brings about the transition: although Being is essentially Nothing, yet Nothing is not yet posited in Being, and *vice versa*. It is not therefore here admissible to employ mediations further determinate, and to take Being and Nothing as in any way related; this transition is not yet a relation. It is therefore incorrect to say that Nothing is the ground of Being, or that Being is the ground of Nothing; that

Nothing is cause of Being, and so on; or that the transition to Nothing depends on this condition, that there *is* something, or the transition to Being on the condition that there should be Nothing. The kind of relation cannot be more closely determined without simultaneously determining the terms of the relation. In the connection between ground and consequence, and so on, the terms connected are no longer mere Being and Nothing, but, expressly, Being which is ground, and something which, though merely posited and not independent, yet is not abstract Nothing.

Observation 4

What has been said throws light on the quality of the dialectic directed against the beginning of the world, as also against its destruction,—the dialectic which was to establish the eternity of matter and refute Becoming, arising, and passing away in general.—The Kantian antinomy of the spatial and temporal finitude or infinity of the world will be more closely considered below, under the concept of Quantitative Infinity.—This simple, ordinary dialectic is based on the retention of the opposition between Being and Nothing. The impossibility of a beginning of the world, or of anything, is demonstrated in the following manner:—

Nothing can begin, either in so far as it is or in so far as it is not: for in so far as it is, it is not merely beginning; in so far as it is not, it is not even beginning.—If the world, or anything, is supposed to have begun, it must have begun in Nothing; but Nothing is not beginning, neither is there any beginning in it: for beginning comprehends a Being, and Nothing contains no Being. Nothing is only Nothing. In a ground, cause, and so on (if Nothing is determined in that way), there is contained an affirmation, a Being.—For the same reason something also cannot cease to be. For then Being would contain Nothing; but Being is only Being and not its own opposite.

It is evident that nothing is adduced here against Becoming, or against beginning and ceasing, against this unity of Being and Nothing, save a dogmatic denial,—validity being ascribed to Being and to Nothing, each distinct from the other.—At any rate this dialectic is more consistent than reflective imagination. This latter thinks it perfect truth that Being and Nothing can exist only as separate; but on the other hand it admits beginning and ceasing as equally valid determinations, and therein in fact assumes the inseparate nature of Being and Nothing.

If Being and Nothing are supposed to be absolutely separate, then—as we so often hear—beginning and Becoming are indeed something inconceivable; for an assumption is made which destroys beginning and Becoming; and yet these are again admitted: the contradiction, thus introduced and made incapable of solution, is then called the inconceivable.

What has been adduced is the same dialectic as is employed by Understanding against the concept of the infinitely small magnitudes of the higher analysis. This concept is dealt with at greater length below.—These magnitudes have been defined as such as are *in process of* disappearing,—neither *before* their disappearance, for then they are finite magnitudes, nor *after* it, for then they are nothing. Against this pure concept the objection has been made and has ever been repeated, that such magnitudes are either Something or else Nothing; that there is no mean condition (condition here is an improper and barbarous expression) between Being and Not-being.—Here, too, the absolute separation of Being and Nothing is assumed. But in fact the identity of Being and Nothing has been demonstrated, or, to speak the language of the objectors, it has been shown that there exists nothing that is not a mean condition between Being and Nothing. Mathematics owes its most brilliant successes to the adoption of this determination which Understanding contradicts.

This argument, which wrongly presupposes the absolute separateness of Being and Not-being and there remains stationary, should be called, not dialectic, but sophistry. For sophistry is an argument proceeding from a baseless supposition which is allowed without criticism or reflection; while we term dialectic that higher movement of Reason where terms appearing absolutely distinct pass into one another because they are what they are, where the assumption of their separateness cancels itself. It is the dialectic immanent nature itself of Being and of Nothing that they manifest their unity (which is Becoming) as their truth.

2. Moments of Becoming: Arising and Passing Away

Becoming is the unseparateness of Being and Nothing, not the unity which abstracts from Being and Nothing; rather, Becoming as the unity of Being and Nothing is this determinate unity in which there *is* Being as well as Nothing. But each, Being and Nothing, in so far as it is unseparated from its Other, is not. They *are*, therefore, in this unity: but only as disappearing and transcended. From the independence (which they were primarily imagined as possessing) they fall to the status of *moments*, which still are distinct, but at the same time are transcended.

The moments being thus taken in this their distinctness, each is in it as unity with its other. Becoming thus contains Being and Nothing as two such unities, each of which itself is unity of Being and Nothing: one of them is Being taken immediately and as relation to Nothing; the other Nothing taken immediately and as relation to Being. The determinations are of unequal value in these unities.

In this manner Becoming is in a twofold determination. In one of these, Nothing is immediate, that is, the determination begins with Nothing which relates itself to Being, or passes over into it; in the other, Being is immediate, that is, this determination begins with Being which passes over into Nothing: Arising and Passing Away.

Both are the same thing, namely Becoming; and even when taken as these different directions they penetrate and paralyse each other. One direction is Passing Away: Being passes over into Nothing; but equally Nothing is its own opposite, a transition to Being, that is, Arising. This Arising is the other direction: Nothing passes over into Being, but Being equally cancels itself (*hebt sich auf*) and is rather a transition to Nothing, a Passing Away.—They do not cancel mutually, nor one the other externally; each cancels itself in itself, and in itself is its own opposite.

3. Transcendence of Becoming

The equipoise of Arising and Passing Away is, first, Becoming itself. But this equally collapses into static unity. In it Being and Nothing exist only in so far as they disappear, but Becoming as such exists only by virtue of their distinctness. Their disappearance therefore is the disappearance of Becoming, or the disappearance of disappearance itself. Becoming is a baseless unrest which collapses into a static result.

This might also be expressed as follows:—Becoming is the disappearance of Being in Nothing and of Nothing in Being, and the disappearance of Being and Nothing in general; but also it depends upon the difference between these. It is therefore self-contradictory, because it unites contradictories within itself; but such a union destroys itself.

The result is, that disappearance has taken place; but this is not Nothing: that would be merely a relapse into one of the determinations which have already been transcended (*aufgehoben*), and not the result of Nothing and of Being too. It is the union of Being and Nothing, which has become a static simplicity. But this static simplicity is Being, which again, however, exists no longer for itself, but as a determination of the whole.

Becoming, then, taken as transition into the unity of Being and Nothing, which exists because it is, or has the form of, the one-sided immediate unity of these moments, is Determinate Being.

Observation

To transcend (*aufheben*), and that which is transcended (the ideal), are among the most important concepts of philosophy,—a fundamental determination which reappears everywhere without exception, the meaning of which must be taken definitely, and must especially be distinguished from Nothing.—What transcends itself does not thereby become Nothing. What is Nothing is immediate: what is transcended is mediated, and, though it is not, yet it has reached nonentity as a result approached from Being. It therefore retains the determinateness whence it started.

To transcend (*aufheben*) has this double meaning, that it signifies to keep or to preserve and also to make to cease, to finish. To preserve includes this negative element, that something is removed from its immediacy and therefore from a Determinate Being exposed to external influences, in order that it may be preserved.—Thus, what is transcended is also preserved; it has only lost its immediacy and is not on that account annihilated.—In the dictionary the two determinations of transcending may be cited as two meanings of this word. But it should appear as remarkable that a language should have come to use one and the same word for two opposite determinations. It is a joy for speculative thought to find words which in themselves have a speculative meaning; the German language has several such. The double meaning of the Latin *tollere* (which has achieved fame through the Ciceronian pun *tollendum esse Octavium*) does not go so far; the affirmative determination only goes so far as to mean elevation. A thing is transcended only in so far as it has come into unity with its opposite; in this narrow determination, as something reflected, it may fittingly be called moment. With the lever, weight and distance from a point are called its mechanical moments, on account of the sameness of their operation and in spite of the difference between something real (such as a weight) and something ideal (the mere spatial determination, the line): see *Encyclopaedia*, 3rd Ed., section 261, note.—Even more frequently the observation will force itself into notice that the technical language of philosophy employs Latin expressions for reflected determinations, either because the mother tongue has no expressions for them, or because, when it has such, as happens here, its own expressions call to mind rather what is immediate, and the foreign language what is reflected.

The more precise meaning and expression which Being and Nothing receive, now that they are moments, must result from the consideration of Determinate Being as the unity in which they are preserved. Being is Being, and Nothing Nothing, only in the distinctness of one from the other; but, truly considered and in their unity, they have disappeared as these determinations, and are now something different. Being and Nothing are the same: but just because they are the same they no longer are Being and Nothing, and have a different determination. In Becoming they were Arising and Passing Away: in Determinate Being, as in a differently determined unity, they are moments differently determined. This unity now remains their basis, from which they no more issue to the abstract meaning of Being and Nothing.

CHAPTER II

DETERMINATE BEING

THE determinateness of Determinate Being, as such, is existent determinateness, or Quality. By virtue of its quality Something is opposed to an Other: it is variable and finite, and determined not only as contrasted with but also simply as negative against, an Other. First, then, this its negation opposed to the finite Something is the Infinite: the abstract opposition in which these determinations appear resolves itself into an Infinity which contains no oppositions, into Being for Self.

The treatment of Determinate Being has therefore three divisions:

(A) Determinate Being as such.
(B) Something and Other: Finitude.
(C) Qualitative Infinity.

A

DETERMINATE BEING AS SUCH

In Determinate Being

(*a*) as such, its determinateness

(*b*) as Quality, must first be distinguished. This, however, must be taken as well in the one determination of Determinate Being as in the other; as Reality as well as Negation. But equally in these determinatenesses Determinate Being is reflected into itself; and posited as such it is

(*c*) Something, or that which Is Determinate.

(*a*) DETERMINATE BEING IN GENERAL

Determinate Being issues from Becoming: it is the simple oneness of Being and Nothing. From this simplicity it derives its form as of something immediate. Becoming, which mediated it, is left behind; it has transcended itself, and Determinate Being therefore appears as something primary and as something from which a beginning is being made. First, then, it is one-sidedly determined as Being; the other determination which it contains, that of Nothing, will also develop itself in it, in opposition to this other.

It is not mere Being, but Determinate Being (*Dasein*), the etymology of which implies being at a certain place, though the spatial signification does not belong here. According to the manner of its Becoming, Determinate Being in general is Being together with a Not-being, so that this latter is taken up into simple unity with Being. Not-being taken up into Being in this manner, that the concrete whole is in the form of Being, of immediacy, constitutes *determinateness* as such.

Since, in Becoming, Being has proved itself to be no more than a moment, the Whole also, in the form—that is, in the determinateness—of Being, is something transcended and negatively determined; but it is such for us when we reflect, it is not yet so posited in itself. But the determinateness as such of Determinate Being is the determinateness which is posited and also is implied in the expression "Determinate Being."—The two must always be carefully distinguished: only that which is posited in a concept is proper to the contemplation which develops it, to its content. The determinateness, however, which is not yet posited in the concept is part of our reflection, whether it regards the nature of the concept itself, or is external comparison: if attention is drawn to a determinateness of the latter kind it can only serve to elucidate, or point the way of, the process which will clearly manifest itself in the course of development. It is an external reflection which holds that the whole, the unity of Being and Nothing, is contained in the one-sided determinateness of Being; but this reflection will actually be posited in the negation, in Something and an Other, and so forth.—Our aim here has been to draw attention to this difference; but, if we were to give an account of everything which reflection would presume to remark, we should fall into the prolixity of anticipating what must in any case result in the investigation. Such reflections may facilitate a general view and hence an understanding of the process; but they have this disadvantage, that they may be mistaken for mere unjustified assertions, grounds and foundations for the further development of the subject. We must therefore take them for no more than they are meant to be, distinguishing them from those terms which are moments in the development of the process itself.

Determinate Being corresponds to Being in the previous sphere; but, since Being is indeterminate, no determinations can be evolved in it. But Determinate Being is, as such, concrete; so that immediately various determinations and distinct relations of its moments disclose themselves in it.

(*b*) QUALITY

In Determinate Being, Being and Nothing are immediately one: they are therefore coextensive. In so far as Determinate Being is at all, in so far it is Not-being, and therefore determinate. Being is not the universal and determinateness not the particular. Determinateness has not yet severed itself from Being; and, indeed, it never will do so, for the truth which is now the basis is the unity of Being and Not-being; and all further determinations are based on this as their foundation. But the relation here existing between determinateness and Being is the immediate unity of both, so that as yet no distinctness between them is posited.

Determinateness, taken thus isolated and by itself as existent determinateness, is Quality,—something quite simple and immediate. Determinateness in general is that more universal something which may equally be the quantitative or something further determined. This simplicity makes it impossible to say anything further about Quality as such.

But Determinate Being, which contains Nothing as well as Being, is itself the measure of the one-sidedness of Quality as merely immediate or existent determinateness. It must equally be posited in the determination of Nothing; and then this immediate or existent determinateness is posited as one which is differentiated and reflected: Nothing, the determinate part of a determinateness, is equally reflected, is a negation. Quality which is to count as something separately existing is Reality; taken as infected with any negation, it becomes Negation in general, which also is a Quality (though it counts as a deficiency) and further determines itself as limit or barrier.

Each is a Determinate Being; but with Reality as Quality (with emphasis on the fact that it *is*) the fact is hidden that it contains determinateness and therefore negation; Reality therefore is taken as something positive from which negation, limitation, and deficiency are supposed to be excluded. Negation taken as mere deficiency would be equivalent to Nothing: in fact it is a Determinate Being, a Quality, but one which has for determination a Not-being.

Observation

Reality may seem to be an ambiguous word, since it is employed for different and even opposite determinations. In the philosophic sense, for instance, a merely empirical reality is spoken of as a worthless existence. On the other hand, if a thought, a concept, or a theory is said to have no reality, the meaning is that it has no actuality; while in itself or as a concept the idea, for instance, of a Platonic Republic might very well be true. The idea here is not denied its value, and it is allowed to remain by the side of the reality. But as opposed to "mere" ideas and "mere" concepts, the real is taken as the only truth.—If it is a one-sided sense in which external existence is taken as the criterion of the truth of a content, it is equally one-sided if the idea, the essence or internal feeling is imagined as indifferent to external existence or is even regarded as excellent in proportion as it is removed from Reality.

In dealing with the expression "Reality" we must make mention of the former metaphysical concept of God, which used especially to be made the basis of the so-called ontological proof of the existence of God. God was defined as the Sum-total of all Realities; and of this sum-total it was said that it contained no contradiction, none of the realities cancelling the other; for a reality was to be taken merely as something complete and affirmative, containing no negation. Thus the realities were neither opposed nor contradictory to one another.

This concept of Reality implies that it survives when every negation is thought away. But this removes every determinateness. Reality is Quality, or Determinate Being; it therefore contains the moment of negation: this alone makes it that determinate something which it is. Taken as infinite (in the ordinary meaning of the term), or in its so-called "eminent" sense (and this is what is demanded), it spreads out into indeterminateness and loses its meaning. The goodness of God is not to be goodness in the ordinary, but in the eminent, sense; it is not to be distinct from justice, but is to be tempered by it (this is a mediatory expression belonging to Leibniz); and contrariwise justice is to be tempered by goodness; and now goodness has ceased to be goodness, and justice, justice. Power is to be tempered by wisdom; but then it is no longer power as such, for it is subordinate to wisdom; wisdom is to be extended into power, but then it ceases to be that wisdom which determines end and means. The true concept of the infinite and its absolute unity, which will be discovered later, must not be taken as a tempering or mutual limitation or mingling; for as such it is a superficial relation held fast in an indeterminate mist, and satisfactory only to non-conceptual imagination.—Reality, as it is taken in this definition of God, as a determinate Quality, is driven beyond its determinateness and ceases to be Reality: it becomes abstract Being. God as the purely Real in all Reals, or as the Sum-total of all Realities, is that same indeterminate and nebulous something as the empty Absolute where everything is one.

If on the other hand Reality is taken in its determinateness, then, since it essentially contains the moment of negativeness, the Sum-total of all Realities equally becomes the sum-total of all negations and of all contradictions; it becomes, for instance, absolute power which holds absorbed everything determinate, but, since it exists only in so far as it has opposed to it something not yet subjected to it, any thought which extends it to perfected and unbounded power leads it back to abstract Nothing. This Real in all Reals, or Being in all Determinate Being, which is to express the concept of God, is nothing else than abstract Being, and identical with Nothing.

"Determinateness is negation posited affirmatively," is the meaning of Spinoza's *omnis determinatio est negatio*,—a proposition of infinite importance; only, negation as such is formless abstraction. Speculative philosophy must not be accused of making negation, or Nothing, its end: Nothing is the end of philosophy as little as Reality is the truth.

From the proposition that determinateness is negation it necessarily follows that there is One Substance only: the unity of Spinoza's Substance. Spinoza was forced thus to unite Thought and Being (or Extension), the two determinations which were present to his mind; for, taken as determinate realities, they are negations, the infinite nature of which constitutes their unity; according to Spinoza's definition (which will be further dealt with below), the infinity of anything is its affirmation. He therefore conceived of them as Attributes, that is, as such as, having no separate persistence or Being-in-and-for-self, exist only as transcended, or as moments; or, rather, they are not even moments, since Substance is that which in itself is quite indeterminate, the Attributes (as also the Modes) being distinctions made by an external Understanding.—Nor can the substantiality of individuals persist against this proposition. The individual is related to itself because it sets a limit to everything else: from this very fact the limits become limits of itself, relations to an other, and its existence no longer depends upon itself. It is true that the individual is more than a something limited in every direction; but this "more" belongs to a different sphere of the concept; in the metaphysics of Being it is simply something determinate; and it is determinateness, essentially in its negative force, which opposes itself to the existence, as such and in and for itself, of such a something, such a finite thing, whirling it into the same negative movement of Understanding which causes everything to disappear in the abstract unity which is Substance.

Negation is immediately opposed to Reality; further on and in the proper sphere of reflected determinations it is opposed to the positive, which is Reality reflecting upon negation,—Reality in which the negative element, hidden in Reality as such, shows forth.

Quality is chiefly property in this respect, that it shows itself as an immanent determination in an external relation. By properties, for instance those of herbs, we mean determinations which not only are proper to a something, but the degree in which it preserves itself in relation to others in a peculiar manner by virtue of them, not allowing free course to alien activities posited in it, but on the contrary enforcing its own determinations in an Other, at the same time not keeping this external to itself. A more passive determinateness, such as figure or form, is not, on the other hand, called property nor even Quality, since it is imagined as variable and not identical with Being.

"Qualation" or "Inqualation"—an expression belonging to Jacob Böhme's philosophy, a philosophy which goes deep but into a murky depth—means the movement of a quality (that of being sour, astringent, fiery, and so forth) in itself, in so far as it posits and confirms itself in its negative nature (its *quale*) as opposed to an other and is its own restlessness, so that it is only by means of a struggle that it produces and maintains itself.

(*c*) SOMETHING

In Determinate Being its determinateness was distinguished in the shape of Quality: in Quality, so far as it exists, there exists distinction—that between Reality and Negation. But, however much these distinctions may have a place in Determinate Being, they are equally void and transcended. Reality itself contains negation, it is Being Determinate, and not indeterminate nor abstract. And, equally, Negation is Determinate Being: it is not a Nothing which is supposed to be abstract, but it is here posited as it is in itself, as being and as belonging to Determinate Being. Thus Quality is in no way separated from Determinate Being, which is just this—a qualitative Being.

But this transcendence of the distinction is more than a mere retraction and external re-omission of it, more than a mere return to the simple beginning, Determinate Being as such. The distinction cannot be omitted, for it *is*. Therefore, the truth which we have, is, Determinate Being in general, distinction in it, and the transcendence of this distinction; Determinate Being is not merely undifferentiated, as at the beginning; it is *once more* identical with itself through the transcendence of the distinction, and the simple nature of Determinate Being is mediated through this transcendence. The fact of this transcendence of distinction is the peculiar determinateness of Determinate Being; it is Being-in-itself, that which is-determinate, or Something.

Something is the first negation of negation as simple existent self-relation. Determinate Being, living, thinking, and so on, determine themselves essentially as that which exists, lives, thinks (ego), and so on. This determination is of the utmost importance, as it saves us from stopping at Determinate Being, living, or thinking, and so on, or at Godhead (instead of God), as universals. Imagination rightly holds Something to be something real. But Something is still a very superficial determination; just as Reality and Negation, so Determinate Being and its determinateness, though not completely void like Being and Nothing, yet are quite abstract determinations. It is just for this reason that they are the commonest expressions; reflection untrained in philosophy most uses them, pours its distinctions into them, and imagines that now it has something really well and firmly determined.—The negative of the negative, as Something, is only the beginning of the subject; and the Being-in-self is quite indeterminate. It next determines itself as being-for-self and so on, till finally as Notion it receives the concrete intensity of the subject. The negative unity with self is the basis of all these determinations; though here the first negation (negation in general) must carefully be distinguished from the second, the negation of negation, which is concrete and absolute negativity, while the first is only abstract negativity.

Something, so far as it is, is because it is the negation of negation; for this is the restoration of simple self-reference; but, for this very reason, Something is equally self-mediation. Already in the simple form of Something, and, more definitely, in Being-for-Self, Subject, and so on, self-mediation is present; even in Becoming there is a quite abstract mediation: in Something the self-mediation is posited in so far as it is determined as a simple identity.—Attention may be drawn to the presence of mediation in general, as against the principle of the alleged mere immediacy of knowledge, from which all mediation is supposed to be excluded: but there is no further need to draw attention particularly to the moment of mediation, for it is to be found everywhere and in every concept.

This self-mediation, which Something in itself is, taken merely as negation of negation, has no concrete determinations for its sides: it therefore collapses into that simple unity which is Being. Something is, and therefore also is a Determinate Being; and, further, it also in itself is a Becoming, having, however, something more than Being and Nothing for moments. One of these, Being, has now become Determinate Being, and, further, *a* Determinate Being. The second also is a Determinate Being, but one determined as a negative of Something,—as an Other. Something as Becoming is transition, having Somethings for its moments; it therefore is change,—a Becoming which has now become concrete.—At first, however, Something merely changes in its concept; it is not yet thus posited as mediating and mediated, but merely as preserving itself in its simplicity as self-related; and its negative is posited equally as something qualitative, as merely Other in general.

B

FINITUDE

(*a*) Primarily Something and Other are mutually indifferent; an Other, too, is an immediate existence, a Something; so that negation falls outside the two. Something is in itself, as opposed to its Being-for-Other. But determinateness also belongs to its in-itself, and is (*b*) its determination. This in turn passes over into Modification, which is identical with the determination and constitutes the immanent and simultaneously negated Being-for-Other, which is the limit of Something. (*c*) This limit is the immanent determination of Something itself, which accordingly is the Finite.

In the first section in which Determinate Being was considered it had the determination, as there taken, of *being*. Hence Quality and Something, the moments in its development, also have an affirmative determination. In this section, on the other hand, the negative determination immanent in Determinate Being develops itself. At that point it was merely negation in general, or First negation; here it is precisely determined as the "Being-in-itself" of Something, or the negation of negation.

(*a*) Something and an Other

1. First, Something and Other each are determinate Beings, or Somethings.

Secondly, each is also an Other. Which is mentioned first, and for this reason only has the name of Something, is immaterial. (In Latin, where they occur in one sentence, both are called *aliud*; "one another" is *alius alium*; when there is reciprocity the expression *alter alterum* is analogous.) If we call one determinate Being *A* and another *B*, then for the present *B* is determined as the Other. But *A* is just as much Other to *B*. Both are in the same way Others. The word "this" serves to fix the distinction and that Something which is to be taken affirmatively. But "this" clearly expresses that this distinction and emphasizing of one Something is a process of designation, subjective and external to the Something. The entire determinateness falls within this external designation, and even the expression "this" contains no distinction; each and every Something is "this" as much as it also is Other. We fancy that by "this" we are expressing something perfectly definite, overlooking that language, being a work of understanding, expresses universals only, except when it *names* individual objects; the individual name, however, is meaningless in this sense that it expresses no universal, for which reason it also appears as something merely posited and arbitrary; and indeed proper names may be assumed, bestowed, and also altered at will.

Otherness thus appears as a determination alien to Determinate Being determined in this manner: it is the other and is external to the one Determinate Being; it seems as though one Determinate Being were determined as other partly only by some external intelligence, partly only by virtue of some Other which is outside it, while, for itself, it is not other at all. Simultaneously, as we remarked, every one Determinate Being equally determines itself as an other Determinate Being, even for imagination, so that there does not remain one Determinate Being which is determined merely as such and is not also external to another, and therefore itself an Other.

Both are determined alike as Something and as Other; they are therefore the same, and as yet there is no distinction between them. But also this sameness of the determinations falls within external reflection, within comparison of the two; but according to the manner in which Other is provisionally posited it is indeed in itself in relation to Something, but it is also in itself external to this latter.

Thirdly, therefore, Other must be taken as isolated and in relation only to itself, as Other in the abstract, as τὸ ἕτερον of Plato, who takes it as one of the moments of totality, opposing it to the One and thus ascribing to Other a nature of its own. Thus Other, taken by itself as such, is not Other to Something but Other by itself, that is, its own Other.—Physical Nature is such an entity which is Other according to its determination: it is Other to Spirit; and thus preliminarily this determination is merely relative, and expresses not a quality of Nature but a relation external to it. But Spirit is true Something, and hence Nature by itself is only that which it is relatively to Spirit; therefore, if Nature is taken by itself, its quality is just that it is Other in itself, or that which is external to itself (in the determinations of space, time, and matter).

The Other for itself is the Other in itself, and hence that which is Other to itself, or Other to Other; that which in itself is the absolutely non-identical, the self-negating or changing. But also it remains identical with itself, for the only thing into which it could change is the Other which has no determination except this. But that which changes is determined in no different manner, but in the same, namely, to be Other; consequently the Other in the Other merely collapses into itself. It is thus posited as Something intro-reflected, added to which is a transcendence of otherness; it is a self-identical Something, and otherness, which also is one of its moments, is consequently something separate from it and not appertaining to it as Something.

2. Something preserves itself in its state of non-existence; it is essentially identical, and also essentially non-identical with this state. It therefore is in relation to its otherness, and is not simply its otherness. Otherness is simultaneously contained in and also separated from it: it is Being-for-Other.

Determinate Being as such is something immediate and unrelated: it is within the determination of Being. Since, however, it includes Not-being, it is a Being which is determinate and negated within itself; it next becomes Other; but, since it also preserves itself in its negation, it is merely Being-for-Other.

It preserves itself in its non-existence, and is Being; not Being in general but as relation to itself as opposed to its relation to Other, or as self-equality as opposed to inequality to itself. **Such a Being is Being-in-Self.**

Being-for-Other and Being-in-Self constitute the two moments of Something. Two pairs of determinations occur here—(1) Something and Other, (2) Being-for-Other and Being-in-Self. The first contains the unrelatedness which constitutes their determinateness: Something and Other fall apart. But their relation is their truth; therefore Being-for-Other and Being-in-Self are the above determinations posited as moments of one and the same thing, determinations which are relations and remain in their unity, the unity of Determinate Being. Thus, also, each of them contains the other moment which is distinct from it.

Being and Nothing in their unity (which is Determinate Being) are no more than Being and Nothing; it is only outside their unity that they are more: but in their restless unity, in Becoming, they are Arising and Passing Away.—In Something, Being is Being-in-Self. Being, or self-relation and self-identity, is now no longer immediate; it is self-relation only in so far as it is the Not-being of otherness, that is, intro-reflected Determinate Being.—Similarly, Not-being as moment of Something in this unity of Being and Not-being is not non-existence in general, but Other, and, more definitely, after the distinction of Being from it, it also is relation to its non-existence, or Being-for-Other.

Thus Being-in-Self is, first, negative relation to non-existence; otherness is external to and opposed to it; so far as Something is *in itself* it is removed beyond the sphere of otherness and of Being-for-Other. But, secondly, it also has Not-being in itself, since it is itself the Not-being of Being-for-Other.

But Being-for-Other is, first, the negation of the simple self-relation of Being, which in the first instance is supposed to be Determinate Being, and Something: in so far as Something is in or for an Other, it lacks its own Being. But, secondly, it is not non-existence equivalent to pure Nothing: it is a non-existence which points to Being-in-Self as its own intro-reflected Being; just as conversely Being-in-Self refers to Being-for-Other.

3. Both these moments are determinations of one and the same thing, namely, of Something. Something is in itself in so far as it has returned to itself from Being-for-Other. But, also, Something has a determination, or circumstance, in itself (the

accent here is on "in"[1]), or in *it*, in so far as this circumstance is "in" it externally, and is a Being-for-Other.

This leads to a further determination. Primarily, Being-in-Self and Being-for-Other are distinct; but, further, Something also has in itself that which it is in itself, and conversely it is in itself what it is as Being-for-Other; and this fact constitutes the identity of Being-in-Self and Being-for-Other, according to the determination that Something is equivalent to each of the moments, which therefore are comprehended in it as unseparated.—Formally, this identity already results in the sphere of Determinate Being, but more expressly in the consideration of Essence and further of the Relation of Internality and Externality, and most precisely in the contemplation of the Idea as unity of Notion and Actuality.—We imagine that we express something lofty in saying "in itself," as in saying "inner"; but that which Something is only in itself, this is only "in" this Something: "in itself" is a merely abstract and therefore external determination. The expressions "there is nothing in him," or "there's something in it," imply, though dimly, that that which is "in" a thing belongs to its Being-in-Self, to its true and inner worth.

We may remark that the meaning of the Thing-in-itself here becomes plain: it is a very simple abstraction, though for some time it was considered a most important and, as it were, high-class determination, just as the proposition, that we do not know what Things-in-themselves are, was a piece of wisdom held in wide esteem.—Things are called "in themselves" in so far as we abstract from all Being-for-Other, which means that they are thought of as quite without determination, as Nothings. In this sense it is indeed impossible to know what the Thing-in-itself is. For the question "what" demands that determinations should be indicated; and since it is postulated that the things of which these are to be predicated must be Things-in-themselves, that is, indeterminate, the question, in sheer thoughtlessness, is so put as to render an answer either impossible or self-contradictory.—The Thing-in-itself is like that Absolute of which we know only that in it all things are one. It is therefore easy to know what is in these Things-in-themselves: as such, they are mere abstractions, void of truth and content. It is Logic which demonstrates what a Thing-in-itself is in truth, and what truly is in itself; but here "in itself" is taken as something better than abstraction, namely, as that which something is by virtue of its concept: and the concept is concrete in itself; being a concept it may be conceived, and, being determinate and a concatenation of its determinations, it is in itself knowable.

---

[1] The German *an* denotes (in space) contiguousness and not inclusion; but it has seemed best to render it by "in" in order to make possible an intelligible translation of the sentences where it is used half punningly at the end of the next paragraph. (Translators' Note.)

Being-for-Other stands to Being-in-Self in the relation of contrasted moment; but Positedness also is contrasted with Being-in-Self. Though this expression, too, contains Being-for-Other, it also contains, definitely, the already accomplished refraction of that which is not in itself into its Being-in-Self, where it is positive. Generally Being-in-Self may be taken as an abstract manner of expressing the concept; Positing properly falls only within the sphere of Essence, or objective Reflection; Ground posits that which is grounded in it; and Cause even more produces an effect—a Determinate Being, of which the independence is immediately negated and which has in it the meaning that it has its base, or being, in an Other. In the sphere of Being, Determinate Being merely proceeds forth from Becoming; or, with Something an Other is posited and with the Finite the Infinite; but the Finite does not produce, does not itself posit, the Infinite. In the sphere of Being the self-determination of the concept is only *in itself*, and is called a transition; and the reflective determinations of Being, like Something and Other or Finite and Infinite, although they essentially point to one another, or exist as Being-for-Other, yet count as qualitative and as persisting for themselves; Other is, and Finite is counted, as being immediately and persisting for itself, no less than Infinite; the meaning of each appears complete even without its other. But positive and negative, and cause and effect, in however isolated a manner they are taken, are meaningless one without the other; they show in each other, and each shows in its other, and this is implied in themselves.—In the several circles of determination, and more especially in the progress of the demonstration or rather in the progress of the Notion towards its demonstration, it is a main point always to distinguish what is still *in itself* and what is *posited*; as with the determinations, whether they are in the concept, or posited and being-for-other. This distinction is proper only to dialectical development and is unknown to the metaphysical (which includes the Critical) philosophy. The definitions, postulates, distinctions, and conclusions of metaphysics intend to produce and assert only what is in the category of Being, and, more precisely, of Being-for-Self.

In the self-unity of Something, Being-for-Other is identical with its in-itself; thus Being-for-Other is in Something. Determinateness thus reflected into self is therefore once more one which simply is, and thus is a Quality—Determination.

(*b*) DETERMINATION, MODIFICATION, AND LIMIT

This "in-itself," into which as back upon itself Something is reflected from its Being-for-Other, is no longer abstract "in-itself" but mediated, since it is the negation of its Being-for-Other; this, therefore, is its moment. It is not merely the immediate self-identity of Something, but that identity through which Something is in it (the in-itself) that which it is in itself: it has Being-for-Other in itself because "in-itself" is the transcendence of this latter and is in itself having passed out from it; but also because it is abstract and therefore essentially infected with negation and Being-for-Other. Here there is not only Quality and Reality, or existent determinateness, but determinateness which is in itself: the course of development consists in positing it as this intro-reflected determinateness.

1. Quality, which constitutes the in-itself in the simple Something, essentially in unity with its other moment, being-in-it, may be called its Determination, in so far as this word is distinguished in its exact meaning from determinateness. Determination is affirmative determinateness: it is Being-in-Self to which existent Something remains conformable, opposing any entanglement with Other, which would determine it; it thus remains equal to itself, and gives weight to this equality in its Being-for-Other. It fulfils its Determination in so far as the further determinateness which springs up in many shapes owing to its relation to Other, becomes adequate to its Being-

in-Self, becomes its fulfilment. Determination contains this, that what Something is in itself, it also has in itself.

The Determination of man is thinking reason: thinking in general is his simple determinateness, which distinguishes him from the beast: he is Thought in himself, in so far as this is also distinct from his Being-for-Other which immediately connects him with Other—namely his natural and sensuous quality. But also man has thought in himself: man is thought, it is as thinking that he is, and this is his existence and actuality; and, further, since it is in his Determinate Being and his Determinate Being is in thought, it must be taken as concrete, as having content and filling; it is thinking reason, and thus the Determination of man. But, in itself, even this determination is no more than an Ought; that is, with the filling which is incorporated with its in-itself, it is in the form of in-itself in general in opposition to such Determinate Being as is not incorporated with it, which latter may also be taken as an externally opposite and immediate sensuousness and nature.

2. The fulfilling of Being-in-Self with determinateness is also distinct from such determinateness as is only Being-for-Other and remains outside the determination. For, in the field of the qualitative, distinctions, when they exist as transcended, retain, relatively to one another, immediate and qualitative Being. That which Something has in itself divides up in this manner, and in this direction is external Determinate Being of the Something, which also is its Determinate Being, but does not belong to its Being-in-Self.—Thus determinateness is Modification.

Modified in this or that manner, Something is conceived as being externally influenced and related. This external relation on which Modification depends, and the fact that determination takes place through the medium of an Other, appears as contingent. But it is the quality of Something to be subject to this external influence and to have a Modification.

In so far as Something changes, the change falls within its Modification: it is that element within the Something which becomes an Other. The Something preserves itself in the change, which affects only this unstable surface of its otherness and not its Determination.

Thus Determination and Modification are distinct: according to its Determination, Something remains indifferent to its Modification; while that which Something has in itself is the middle term of this syllogism and connects the two. But it appeared that to belong to Something analysed itself into these two extremes. Determinateness as such is the simple mean; and both Determination and Modification belong to its identity. But, for itself, Determination passes over into Modification, and Modification into Determination. This much is implied in what has been said already; more precisely, the connexion is as follows: In so far as Something has in itself what it is in itself, it is infected with Being-for-Other; and hence Determination as such remains liable to a relation to Other. Determinateness is also a moment; but simultaneously it contains this qualitative distinction, that it differs from Being-in-Self and is the negative of Something, an other Determinate Being. Determinateness which thus comprehends Other, when united with Being-in-Self, introduces otherness into Being-in-Self (or Determination); the latter thus is reduced to Modification.—Conversely, Being-for-Other posited as Modification—that is isolated and by itself—is, in itself, the same as is Other as such, that is, that which is Other to itself, or of itself; taken thus it is self-related Determinate Being, or Being-in-Self with a determinateness; that is, Determination.—But the two must also be kept distinct: hence Modification, whose very basis appears to be in something external, or Other, also depends upon Determination, and the alien determining is also determined by the determination proper to and immanent in the Something. But, further, Modification belongs to that which the Something is in itself: Something changes with its Modification.

This mutation of Something no longer is its first change according to its Being-for-Other: the former was change in itself only, and proper to the inner concept; now the change is that which is posited of Something.—Something has itself been further determined, and the negation has been posited as immanent in it, or as its developed Being-in-Self.

At first the transition into each other of Determination and Modification is the transcendence of the distinction between them; here Determinate Being or Something in general is posited. It is the result of the distinction which comprehends within itself qualitative otherness: there thus are two Somethings, which are not merely Other to each other (in which case this negation would be only abstract and dependent on comparison), but the negation is immanent in these Somethings. As existent they are indifferent to each other, but this their affirmation is no longer immediate, and each is related to itself through the transcendence of otherness, which, in the determination, is reflected into Being-in-Self.

Something is in this relation to Other from its own nature and because otherness is posited in it as its own moment: its Being-in-Self comprehends negation, through which alone it now has its affirmative existence. From this, however, Other is distinct qualitatively, too, and thus it is posited as outside Something. The negation of its Other is only the quality of Something, for it is just as this cancellation of its Other that it is Something. It is only through this that Other opposes itself to a Determinate Being: it is only in an external manner that Other is opposed to the first Something; or else, if they are in fact connected absolutely (or according to their concept), the connection is merely this, that Determinate Being has passed over into otherness, or Something into Other, so that Something is Other just as much as Other is. Now in so far as Being-in-Self is the Not-being of otherness, which is contained in it, but also distinguished from it by virtue of its quality of being, Something itself is negation, namely this negation, that an Other ceases when Something begins: it is posited as remaining negative towards it, and as thus preserving itself;—and this Other, the Being-in-Self of Something taken as negation of negation, is its Being as such; and also this cancellation as simple negation is a quality it has, the quality of negating the other Something external to it. It is one determinateness of these, and it is identical with the Being-in-Self of Somethings (as negation of negation); and it also (since these negations now stand in the relation of other Somethings to each other) makes them spontaneously to coalesce while simultaneously severing them, one negating the other. This is Limit.

3. Being-for-Other is indeterminate and affirmative community of Something with its Other: in Limit, Not-being-for-Other becomes prominent, which is qualitative negation of

Other; the latter thereby is kept away from intro-reflected Something. It remains to see the development of this concept, which soon, however, reveals itself as a confusion and a contradiction. The latter is immediately found in this fact, that Limit, as the intro-reflected negation of Something, contains ideally the moments of Something and Other, while these as distinct moments are posited in the sphere of Determinate Being as real and as qualitatively distinct.

α. Something then is immediate and self-related Determinate Being, and has a Limit, first, as in relation to Other: it is the not-being of Other, not of Something itself; in the Limit Something delimits its Other.—But Other is, itself, and in general, a Something: therefore, the Limit which Something has relatively to Other is Limit also of Other relatively to Something, it is that Limit by means of which Other keeps distinct the first Something as its own Other; in other words, it is a not-being of this Something. It is thus not only not-being of Other but of both Somethings equally, and thus of Something in general.

But essentially also it is the not-being of Other, and thus Something also *exists* by virtue of its Limit. True, in so far as Something exists as limiting, it itself is degraded to the rank of something limited; but its Limit (which is the fact that Other ends where it begins) also is no more than the Being of Something: through it the latter is what it is and has its quality in it.—This relation is the external aspect of the fact that Limit is simple (or first) negation, while Other is negation of negation or the Being-in-Self of Something.

Something as immediate Determinate Being is, therefore, Limit with reference to another Something; but this Limit is also its own, and if it is Something, it is so only through the mediation of this which is equally its not-being. Limit is the mediation through which Something and Other is and also is not.

β. Now, in so far as Something simultaneously is and is not in its Limit, and these moments are an immediate and qualitative distinction, the Determinate Being and not-being of Something fall apart. Something has its Determinate Being without (or also, as we imagine it, within) its Limit; and so too Other, because it is Something, is outside it. Limit is the mean between the two, in which they cease. They have their Determinate Being beyond one another and beyond their Limit: Limit as the not-being of either is the Other of both.

According to this distinction of Something from its Limit, the line appears as line only outside its limit, the point; the plane as plane outside of the line; and body as body only outside its limit, the plane.—It is from this aspect that Limit first strikes imagination (the externalization of the concept), and is also chiefly applied to spatial objects.

γ. But further, Something as it is outside the Limit, or unlimited Something, is just Determinate Being in general. Here it is not distinct from its Other: it is only Determinate Being and has the same determination as its Other; each is only Something in general and also Other: both are the same. But this their at first immediate Determinate Being is now posited with determinateness as Limit, in which both are what they are distinctly from one another. But also Limit is their common distinctness, their unity and distinctness, like Determinate Being. This double identity (Determinate Being and Limit) of both, implies that Something has its Determinate Being only in Limit, and also (since immediate Determinate Being and Limit are negative of one another) that Something, which only exists within its Limit, severs itself from itself and points beyond itself to its not-being and predicates this as its Being, thus passing over into it. To apply this to the former example, one determination is that Something is what it is only within its Limit;—thus the point is not limit of the line only in the sense that the line only ceases in it and, as Determinate Being, exists outside it; the line is not limit of the plane only in the sense that the plane only ceases in the line; and so with the plane as limit of the body: rather the line begins in the point; it is its absolute beginning; even in so far as it is imagined as unlimited in either direction, or produced to infinity, as it is put,—even then the point is its element, and the line the element of the plane, and the plane of the body. These limits are the principle of that which they limit; just a one (*e.g.* as hundredth) is limit, but also is element of the whole hundred.

The other determination is the unrest of Something in its Limit; it is immanent in the Limit to be a contradiction which send; Something on beyond itself. Thus point is this self-dialectic which becomes line, line that which becomes plane, and plane that which becomes total space. There is a second definition of line, plane, and cubic space, namely that the line is created through the movement of a point, the plane through the movement of a line, and so forth. This movement of the point, the line, and so forth, is, however, taken as something merely contingent or imaginary. This assumption is really, however, retracted when it is said that the determinations from which line and so forth are to be evolved are their elements and principles; and these also are nothing else than their Limits: this creation, therefore, is not looked on as merely contingent or imaginary. It is part of the concept of the Limit which is immanent to Something that point, line, and plane in themselves, as self-contradictory, are self-repellent beginnings; so that the point (for instance) passes from itself into the line because its concept is what it is; it moves in itself, and thus creates line. The application, however, belongs to the consideration of Space; to give an indication of it here, point is purely abstract Limit, but it is so within a Determinate Being: and as yet the latter is taken in a quite indeterminate manner, being so-called absolute (that is, abstract) Space, or entirely continuous being-separate. And Limit is not abstract negation but is in a given determinate Existence and is spatial determinateness; and hence the point is spatial, the contradiction of abstract negation and of continuity, and so incipient and perfected transition into line, and so forth; and indeed "there is" no such thing as point, or line, or plane.

Something, posited with its immanent Limit as self-contradiction through which it is driven and forced beyond itself, is the Finite.

(*c*) FINITUDE

Existence is determinate; Something has a quality, and in it is limited as well as merely determined: its quality is its limit, and, affected with this, it at first remains affirmative and quiescent Determinate Being. But when this negation is developed to show that the opposition between its Determinate Being and negation as its immanent limit, is itself the Being-in-Self of Something, and that thus this latter is in itself just Becoming, then these facts constitute its Finitude.

When we say of things that they are finite, we mean thereby not only that they have a determinateness, that quality is here reality and self-existent determination, and that they are merely limited; for then they still have Determinate Being outside their limit;—but rather that Not-being constitutes their nature and their Being. Finite things are; but their relation to themselves is this, that, being negative, they are self-related, and in this self-relation send themselves on beyond themselves and their being. They are, but the truth of this being is their end. The finite does not only change, like Something in general, but it perishes; and its perishing is not merely contingent, so that it could be without perishing. It is rather the very being of finite things, that they contain the seeds of perishing as their own Being-in-Self, and the hour of their birth is the hour of their death.

(*a*) The Immediacy of Finitude

The thought of the Finitude of things brings with it this sorrow, because it is quintessential qualitative negation, and because in the simplicity of this kind of determination no affirmative Being is left them as distinct from their determination to destruction. Because of this qualitative simplicity of negation, which has gone back to the abstract opposition between Nothing and Passing Away, on the one hand, and Being, on the other, Finitude is the most stubborn category of understanding. Negation in general, Modification, and Limit do not war with their Other, that is, Determinate Being; and even abstract Nothing is abandoned as an abstraction; but Finitude is negation based on itself and is therefore in abrupt opposition to its affirmative. It is true that the finite can be put in motion: it is itself the fact that it is determined as doomed to end, but only to end;—or, rather, it is the refusal to move affirmatively to its affirmative, the Infinite, and to allow itself to be connected with it; it is therefore posited inseparably from its Nothing, and every reconciliation with its Other, the affirmative, is thus precluded. The determination of finite things is none other than their end. Understanding persists in this sorrow of Finitude by making not-being the determination of things, and also making it at once enduring and absolute. Their perishability could perish only in their Other, the affirmative, but then their Finitude would leave them; but this Finitude is their unchanging quality,—quality, that is to say, which does not pass over into its Other, or, in other words, into its affirmative. It is thus eternal.

This consideration is of great importance; but certainly no philosophy, nor opinion, nor Understanding in general, will allow itself to be saddled with the point of view that the finite is absolute; rather, the opposite is expressly present in the assertion of the finite. The finite is that which is bounded and perishable: it is *only* finite and not imperishable: this is implied immediately in its determination and expression. But the question is whether speculation persists in the Being of Finitude, and perishability remains, or perishability and perishing perish? And it is precisely in that view of the finite which makes perishing the end of all finite things, that this does not happen. It is expressly asserted that between finite and infinite no peace and no union are possible, that finite is absolutely opposed to infinite. Being, absolute Being, is ascribed to the infinite: the finite is retained in opposition to it as its negative; and, incompatible with the infinite, it remains absolute upon its own side. From the affirmative, or infinite, it would receive affirmation, and thus itself would perish; but a union with it is just that which is declared to be impossible. If it is not to remain opposed to the infinite, but to perish, then, as was said above, it is perishing which is its end, and not the affirmative, which would only be a perishing of perishing. But, if the finite should not perish in the affirmative, and should its end be taken as being Nothing, then we should have returned to that first and abstract Nothing, which has perished long ago.

With this Nothing, however, which is to be merely Nothing, and yet is granted an existence in thought, imagination, or speech, the same contradiction occurs as was just indicated in the Finite, with this difference, that it merely *occurs* in the first and abstract Nothing, while in Finitude it is expressed. There it appears as subjective, here the perennial opposition of finite and infinite is upheld,—the finite being that which in itself is null, and exists as such. This must be made evident; and the development of the finite shows that it is here that, because it is this contradiction, it collapses into itself, while at the same time it actually resolves the contradiction by showing, not that it is merely perishable and perishes, but that perishing, or Nothing, is not the last word, but perishes itself.

(β) BARRIER AND OUGHT

Abstractly, indeed, this contradiction is at once present in the fact that Something is finite, or that the Finite is. But Something or Being is no longer posited as abstract, it is intro-reflected and developed as Being-in-Self having a determination and modification, or, more closely, a limit, which, as that which is immanent in the Something and constitutes the quality of its Being-in-Self, is Finitude. We must now see what moments are contained in this concept of the finite Something.

It was seen that determination and modification were different aspects for external reflection; but already the former contained otherness as belonging to the nature of Something by itself; the externality of otherness is on the one hand contained in the proper internality of Something, but, on the other, as externality it remains distinct from it; it is still externality as such, but applied to a Something. But since, further, otherness is itself determined as limit, or negation of negation, the otherness immanent to Something is posited as the relation of the two sides; and the self-identity of Something to which both determination and modification belong is its introverted relation, or that relation which negates its immanent limit relating the self-existent determination to it. Thus, self-identical Being-in-Self relates itself to itself as its own not-being, but in the shape of negation of negation, or as negating that which simultaneously retains Determinate Being in itself, since it is the quality of its Being-in-Self. The proper limit of Something, thus posited of it as something negative and also essential, is no longer Limit as such, but Barrier. But Barrier is not only that which is posited as negated: the negation acts both ways, since that which it posits as negated is Limit; for this latter is that which is common to Something and Other, and also is the determinateness of the Being-in-Self of the determination as such. This Being-in-Self is then the negative relation to its limit (which also is distinct from it), or to itself taken as Barrier: that is, Ought.

In order that the limit applying to Something in general should also be Barrier, Something must pass over into itself beyond the limit; it must, referring to itself, relate itself to it as to something which is not. The Determinate Being of Something lies undisturbed and indifferent, so to speak *beside* its limit. But Something passes beyond its limit only in so far as it accomplishes its cancellation and is negative Being-in-Self relatively to it. And since in determination Limit itself exists as Barrier, Something thus passes beyond itself.

Ought thus contains a double determination, first as self-existent determination relatively to negation, secondly as a not-being, which, as Barrier, is distinguished from it, but also itself is self-existent determination.

Thus the finite has determined itself as the relation of its determination to its limit: in this relation the former is Ought and the latter Barrier. Thus both are moments of the finite; and both themselves are thus finite, Ought as well as Barrier. But it is only Barrier which is *posited* as finite: Ought is restricted only in itself (and thus for us). It is restricted by the relation to the limit already immanent in it, but this restriction is wrapped up in Being-in-Self, for, according to its Determinate Being, that is, its determinateness in relation to Barrier, it is posited as Being-in-Self.

What ought to be is, and also is not; for if it were, then it could not also be the case that it ought to be. Thus, essentially, Ought has a Barrier. This Barrier is nothing alien: that which merely ought to be is the determination which now is posited as it is in fact, namely, as being also just a determinateness.

Thus the Being-in-Self of Something in its determination reduces itself to Ought by reason of the fact that that which constitutes its Being-in-Self exists, in exactly the same respect, as Not-being. This takes place in the following manner: in Self-existence (or the negation of negation) this Being-in-Self, as being one of the two negations (as that which negates), is one with the other, which also exists as a qualitatively other limit, whereby this unity exists as relation to it. The Barrier of the finite is nothing external; its own determination is also its barrier: and this latter is Ought as well; it is that which is common to both, or rather that wherein both are identical.

But the finite further, as Ought, passes beyond its Barrier; the very determinateness which is its negation is also transcended, and thus is its Being-in-Self: its limit is also not its limit.

Something, as Ought, thus rises superior to its Barrier; but, conversely, it has a Barrier only as Ought. Both are inseparable. Something has a Barrier in so far as it contains negation in its determination, and the determination is also the accomplished transcendence of the Barrier.

Observation

Ought has latterly played a great part in philosophy, chiefly in relation to morality and, in a more general metaphysical manner, as the last and most absolute concept of the identity of Being-in-Self, or self-reference, and determinateness or limit.

"Thou canst because thou oughtest"—this expression, by which much is intended, is implied in the very concept of Ought. For Ought is the accomplished transgression of the barrier; in it, limit is transcended, and thus Ought's Being-in-Self is identical self-relation and is hence the abstraction of "Can."—But the converse is equally correct: "Thou canst not just because thou oughtest." For Barrier, as Barrier, is equally implied in Ought; and the first-mentioned formalism of possibility finds in it a reality and qualitative otherness opposed to itself: the relation of the two to each other is one of contradictoriness, and thus of "Cannot," or rather of impossibility.

At Ought the transgression beyond finitude, Infinity, begins. Ought is that which, leaving behind impossibility, manifests itself in its development as progress into the infinite.

Coming to details we may criticize two prejudices with regard to the form of Barrier and Ought. First, the barrier of Thought, of Reason, and so on, is often held in high respect, and it is asserted that no progress can be made beyond the barrier. When this assertion is made it is not seen that by the very fact that something has been determined as barrier, it has already been surpassed. For any determinateness or limit is determined as barrier only as opposed to its Other in general, that is, something unrestricted: for the Other of a barrier is just the beyond. A stone or metal is not beyond its barrier just because, for it, it is not barrier. But if, in such general propositions of understanding thought as deny the possibility of transgressing the barrier, thought refuses to apply itself to examining the, contents of the concept, then it must be referred to actuality. where such propositions turn out the least actual things of all Just because thought "ought" to be above actuality, dwelling apart from it in higher regions, so that itself is determined as an Ought, it firstly does not pass on to the concept, and, secondly, finds itself in an attitude equally invalid relatively to actuality as to the concept.—The stone thinks not nor has it even any sensation; for it, therefore, its restrictedness is no barrier, that is, it is no negation of sensation, imagination, or thought for the stone, since it does not possess these faculties. But even the stone, as Something, is distinguished into its determination (or Being-in-Self), and its Determinate Being, and in this respect even it passes beyond its barrier; the concept which it is in itself implies the identity with its Other. If it is a base, and is capable of being acted on by an acid, it may be oxidized, neutralized, and so forth. In oxidization, neutralization, and so forth, its barrier, which is, to exist only as base, cancels itself: it passes beyond, and similarly acid cancels its barrier, which is, to be acid; and in the case of acid (as in that of caustic base) so much is it true that it "ought" to pass beyond its barrier, that force must be employed to keep them apart as (waterless, *i.e.* purely non-neutral) acid and caustic base.

But where an existent contains the concept, not as abstract Being-in-Self, but as a totality which is for itself, as impulse, life, sensation, imagination, and so on, then it has achieved through its own power the passage of the barrier and an existence beyond it. The plant passes over the barrier of existing as a seed, and over the barrier of existing as blossom, fruit, or leaf; the seed becomes the unfolded plant, the blossom fades, and so forth. In the barrier of hunger, of thirst, and so on, the sentient element is the impulse to pass beyond this barrier, and it also completes this egress. It feels pain, and it is the privilege of sentient nature to feel pain: it is a negation within its self, and it is determined as a barrier in its sensibility just because whatever is sentient has the sensation of its *self*, which is the totality that exists beyond this determinateness. If it were not beyond it, then it would not feel it as its negation, and would feel no pain.—And then we are told that Reason, and Thought, cannot pass beyond its barrier,—Reason, which is the universal and therefore in itself is beyond abstract, that is, beyond all, particularity; which is nothing if not egress beyond the barrier.—It is true that not every passage and existence beyond the barrier is a veritable emancipation from it, or a veritable affirmation. Ought itself, and abstraction in general, is such an incomplete egress. But it is enough to point to the purely abstract universal to refute the equally abstract asseveration that no progress beyond the barrier is possible; or to point to the infinite in general to refute the asseveration that no progress is possible beyond the finite.

We may here mention a notion of Leibniz, which appears ingenious; namely, that, if a magnet had consciousness, it would consider its direction to the north as a determination of its will, or a law applied to its freedom. Much rather, if it had consciousness, and therefore will and freedom, it could think: and then space for it would be a universal containing all directions, so that the one direction towards the north would be a barrier to its freedom, as much as it would be a barrier to the freedom of a man to be kept at one point, but not for a plant.

On the other hand Ought is an egress beyond the barrier, but one which itself is only finite. In the sphere of finitude therefore it has its place and its validity, where it maintains Being-in-Self against that which is restricted, asserting it as Rule and as essential against the void. Duty is Ought turned against individual will, against selfish desire and capricious interest: and this is put before will as an Ought in so far as, in its mobility, it can isolate itself from the veritable. There are those who esteem the Ought of morality thus highly, and see an attempt to destroy morality in the refusal to accept the Ought as something ultimate and veritable; there are reasoners whose understanding indulges in the unceasing satisfaction of being able to adduce, against everything which is, an Ought, and thus a superior knowledge, and therefore also decline to see themselves robbed of their Ought: these do not see that, as regards the finitude of their sphere, the Ought is fully recognized.—But in actual fact Reason and Law are at no such sorry pass as that they merely "ought" to be; it is only the abstraction of Being-in-Self which stops dead at that point; —nor yet is Ought in itself perpetual, nor finitude (which would be the same) absolute. The philosophy of Kant and Fichte calls Ought the highest point of the resolution of the contradictions of Reason: in truth, this is only the standpoint where they persist in finitude, and (which is the same thing) in contradiction.

(γ) TRANSITION OF FINITE INTO INFINITE

In itself Ought contains Barrier; and Barrier, Ought. Their relation to each other is Finitude itself, which, in its Being-in-Self, contains them both. These moments of its determination are qualitatively opposite, Barrier being determined as the negation of Ought, and similarly Ought as the negation of Barrier. The Finite thus is self-contradictory; it cancels itself and passes away. But this result, namely, the negative in general, is (α) its proper determination; for it is the negative of the negative. Thus the finite in perishing has not perished; so far it has only become another finite, which, however, in turn perishes in the sense of passing over into another finite, and so on, perhaps *ad infinitum*. But (β) if this result is more closely considered, the finite in perishing, in this negation of its self, has reached its Being-in-Self, and therefore has here collapsed into itself. Each of its moments contains just this result: Ought passes beyond the Barrier, that is, beyond itself: but that which is beyond it (its Other) is precisely the Barrier. But Barrier immediately points on beyond itself to its Other, which is Ought; this, however, is that same antagonism of Being-in-Self and Determinate Being, which Barrier is itself; it is the same thing. Thus it passes beyond itself only to collapse again into itself. This self-identity, or negation of negation, is affirmative Being, or the Other of the Finite, which is supposed to have the first negation as its determinateness;—this Other is the Infinite.

C

INFINITY

In its simple concept the Infinite may be regarded, first, as a fresh definition of the Absolute; as indeterminate self-relation it is posited as Being and Becoming. The forms of Determinate Being do not form part of the series of determinations which can be regarded as definitions of the Absolute, since each form of that sphere is posited immediately for itself merely as a determinateness, or as finite in general. But the infinite is considered as absolute in itself, since it is expressly determined as negation of the finite, and thus in the infinite an express attitude is taken to the quality of being subject to barrier (of which Being and Becoming, though in themselves they neither have nor show this quality, might yet be in some way capable); and in fact this quality is negated in the infinite.

But this in itself is not sufficient to remove the infinite beyond finitude and the quality of being subject to barrier; the main point is, to distinguish the true concept of infinity from bad infinity, the infinite of Reason from that of Understanding; the latter is the infinite made finite, and it will appear that, just when it is to be kept unspotted and removed from the finite, the infinite is made finite.

The Infinite

(*a*) is, in simple determination, the affirmative as negation of the Finite;

(*b*) thereby, however, is in Reciprocal Determination with the Finite, and is abstract or one-sided Infinite;

(*c*) the self-transcendence of this Infinite and of the Finite, taken as one process, is the veritable Infinite.

(*a*) The Infinite in General

The Infinite is the negation of negation, the affirmation or Being which has re-issued from its quality of being subject to barrier. The infinite is, and is in a more intensive sense than the first immediate Being; it is veritable Being, elevation above the barrier. The name of the Infinite kindles in the mind and the spirit a light; for here it not only rests abstractly by itself, but rather rises to itself, to the light of its thought, its universality, and its freedom.

It has appeared, first, with regard to the concept of the infinite, that Determinate Being in its Being-in-Self determines itself as finite and passes beyond its barrier. It is the nature of the finite to pass beyond itself, to negate its negation and to become infinite. The infinite thus does not stand above the finite as something complete in itself, the finite still permanently remaining without or beneath it. As yet *we* pass from the finite into the infinite only as a subjective Reason. Thus when we say that the infinite is a concept of Reason, and that we rise by Reason above temporality, we do this without infringing upon finitude, which this exaltation (since it remains external to it) does not concern. But also, since it is the finite which is raised into the infinite, it is no external power which impels it to do so; rather it is its nature to relate itself to itself as barrier, (both as barrier as such and as Ought,) and to pass beyond it; or, rather, to have negated it as relation to itself and to be in the state of having passed beyond it. Infinity in general is not created by the cancellation of finitude in general: the finite is just that which itself becomes infinity through its own nature. Infinity is its affirmative determination, or that which it veritably is in itself.

Thus the finite has disappeared in the infinite, and what is, is nothing but the infinite.

(*b*) RECIPROCAL DETERMINATION OF FINITE AND INFINITE

The Infinite is, and in this immediacy it also is the negation of an Other—the Finite. Since thus it is, and also is the not-being of an Other, it has fallen back into the category of Something with a limit (in the sense of anything determinate in general); more exactly, this is so because it is posited as intro-reflected Determinate Being which results from the cancellation of determinateness in general and thus is distinct from its determinateness.—In consequence of this determinateness, the Finite stands over against the Infinite as real Determinate Being; they thus stand in qualitative relation as entities remaining external to each other. The immediate Being of the Infinite resuscitates the Being of its negation, the Finite, which for the moment had appeared to vanish into the Infinite.

Infinite and Finite are not however in these two categories of relation only; the relation of the two sides is more closely determined than that they should be mere Others to each other. For the finite is barrier posited as barrier; it is Determinate Being posited with the determination to pass over into its Being-in-Self, to become infinite. Infinity is the Nothing of the Finite, it is its Being-in-Self and Ought, but is so as Ought fulfilled and intro-reflected, as Being self-related only and wholly affirmative. There is this to satisfy the mind in Infinity, that all determinateness, change, barrier, and therefore even Ought itself, have disappeared, and that, these cancelled, the Nothing of the Finite is now posited. Being-in-Self is determined as this negation of the Finite; as negation of negation it thus is affirmative in itself. But this affirmation, since it is qualitatively immediate self-relation, is Being; and thus the Infinite has been brought back to the category where it has the Finite over against it as an Other; its negative nature is posited as being, and therefore as first and immediate negation. Thus the Infinite is infected with the opposition to the Finite, which, since it is Other, remains determinate and real Being, although in its Being-in-Self—the Infinite—it is also posited as cancelled: this is the Non-finite,—a Being in the determinateness of negation. In opposition to the Finite, the sphere of existent determinatenesses, of realities, there stands the Infinite, the indeterminate void, the beyond of the Finite, whose Being-in-Self does not depend on its Determinate Being, which is a determinate.

When the infinite is posited in this qualitative relation of opposition to the finite as to its Other, it must be called the bad infinite or the infinite of Understanding, which takes it for the highest and absolute truth. The contradictions which assail it from every side when it undertakes to apply and explain these its categories, should suffice to make it aware that, while it thinks to have reached a point of satisfaction in this reconciliation of truth, it in fact remains in a state of contradiction unreconciled, unresolved, and absolute.

This contradiction is immediately found in the fact that the finite remains opposed to the infinite as a Determinate Being; thus there are two determinatenesses: there are two worlds, the finite and the infinite, which being related the infinite is but the limit of the finite, and thus is merely an infinite itself determinate and finite.

This contradiction develops its content into more express forms.—The finite is real Determinate Being, which remains so still even after the transition has been made to its not-being, the infinite;—and this, as has been shown, is determined relatively to the finite merely as first, or immediate, negation; and the latter, opposed to this negation, and as negated, has merely the meaning of an Other, and therefore still is Something. And thus when Understanding, rising from this finite world, climbs to its highest height, the infinite, the finite world still remains as something on this side of it; the infinite is placed merely above the finite and separate from it, by which process precisely the finite is separated from the infinite. Each is set in a different place: the finite is Determinate Being "on this side"; the infinite, although it is the in-itself of the finite, is yet placed on the far side, in a dim and inaccessible distance, outside of which the finite is supposed to be and remain.

Thus separated, however, they are also, and equally essentially, related to each other by the very negation which separates them. This negation which relates these intro-reflected Somethings is the mutual limit of One opposed to an Other; the manner in which this happens is this, that each has the negation not merely in its other, as opposed to the other; rather, the negation is their Being-in-Self, each has the limit in itself for itself in its separation from the other. But the limit here is the first negation: thus both are limited and finite in themselves. Also, however, each, as being affirmatively self-relating, is the negation of its limit; it immediately casts off the limit as its not-being, and thus qualitatively separated from it posits it as another Being outside itself: the finite positing its not-being as this infinite, and the infinite as the finite. It is gladly admitted that the transition from finite to infinite is necessary (that is, in accordance with the determination of the finite), and that the former is elevated to Being-in-Self; for the finite is determined as a Determinate Being which, though it persists, yet is determined as inherently working its own dissolution and nullity, while the infinite, though its determination infects it with negation and limit, also is that which is in itself, so that this abstraction of the self-relating affirmation constitutes its determination, and finite Determinate Being accordingly is not implicit in it. But it has been shown that the infinite itself reaches the result of affirmative Being only through negation (namely, as negation of negation), and that this its affirmation, taken as merely simple, qualitative Being, reduces the negation it contains to mere simple and immediate negation, and thus to determinateness and limit; and this, as contradictory to its Being-in-Self, is excluded from it, and is thus posited as not belonging to it but rather opposed to its Being-in-Self,—in short, as finite. Each in itself and from its determination thus being the positing of the other, the two are inseparable. But this unity lies hidden in their qualitative otherness, it is internal and only basic.

This determines the manner in which this unity manifests itself: posited in the category of Determinate Being, it is a metamorphosis or transition of finite into infinite, and conversely; the infinite being the background against which the finite merely shows forth, and similarly the infinite showing forth against the finite, or Other against Other. In short, each has its immediate origin in the other, their relation being merely external.

In detail the process of transition takes the following form. We pass beyond the finite to the infinite. This egress appears as an external activity. What is it that arises in this void which is beyond the finite? What is its positive element? Finite and infinite being inseparable (or, because the infinite, standing aloof on its own side, itself is subject to barrier), the limit arises: the infinite has disappeared, and its Other, the finite, has taken its place. But this entrance of the finite appears as an event external to the infinite; and the new limit as something which has not arisen out of the infinite itself, but likewise is accidentally discovered. This constitutes a relapse into the former, and vainly transcended, determination. The new limit itself, however, exists only to be cancelled or transcended. Thus, once more the void or Nothing has arisen, in which we once more find the above-mentioned determinateness, or new limit, *and so on to infinity*.

This is the Reciprocal Determination of Finite and Infinite: the finite is finite only with reference to Ought, or the infinite, and the infinite is infinite only with reference to the finite. They are inseparable and, at the same time, absolutely, in the relation of Other to each other: each has its Other in itself; thus each is the unity of itself and its Other, and in its determinateness is a Determinate Being which is not that which it, and its Other, is.

This Reciprocal Determination, negating both itself and its negation, appears as that *progress to infinity* which, in so many forms and applications, is taken as something ultimate beyond which no progress is possible: having reached this "and so on to infinity," the train of thought generally is at an end.—This progress takes place everywhere where relative determinations are forced into opposition, so that, while they are in indissoluble unity, each yet has independent existence as opposed to the other ascribed to it. Hence this progress is contradiction which is not resolved, but asserted to be still at hand.

We have here an abstract progress which remains incomplete because there is no progress beyond this progress itself. The infinite is indeed at hand, and we progress beyond it; for a new limit is posited: but this very act constitutes a return to the finite. In itself, this bad infinity is the same as perpetual Ought, it is negation of the finite, but cannot truly rid itself of finitude. The latter reappears within it as its Other, since the infinite exists only in relation to its Other, the finite. The progress to infinity is therefore only a recurring monotony, one and the same wearisome alternation of this finite and infinite.

The infinity of the infinite progress remains infected with the finite as such; it is limited by the finite, is finite itself. Thus it would indeed be posited as the unity of finite and infinite. This unity, however, is not kept in mind, though it is it alone which calls up the infinite in the finite and the finite in the infinite,—a mainspring, so to speak, of the infinite progress. This progress is the external aspect of this unity which is the *ne plus ultra* of imagination,—this perpetual repetition of one and the same alternation, the empty restlessness of a progress beyond the limit to infinity, which, finding a new limit in this infinite, can find a hold in the limit no more than in infinity. This infinite has the fixed determination of a *beyond*, which cannot be reached because it *ought* not to be reached, because the determinateness of the beyond, the existent negation, is never surrendered. According to this determination the finite remains opposed to it as a *hither*; and this likewise cannot rise into the infinite because it is determined as an Other, that is, as a Determinate Being, perpetual, and still reproducing itself in its beyond in the shape of something different from this beyond.

(*c*) Affirmative Infinity

We have demonstrated the alternating and reciprocal determination of finite and infinite: the truth of these terms here is already established, and all that is necessary is to seize this established truth. This alternation constitutes the external realization of the concept: in it the contents of the concept are posited, though in an external and non-coherent manner. For it is only necessary to compare the different moments for that unity to manifest itself, which in turn yields the concept;—this unity of finite and infinite (as has often been remarked, but must especially be remembered here) being the distorted expression for the unity as it in truth is; however, the exposition of the concept now before us must also contain the removal of this distorted determination.

Taken according to its first and merely immediate determination, the infinite is just a going beyond the finite; its determination makes it the negation of the finite; the finite, therefore, taken only as that which must be gone beyond, is in itself that self-negation which is infinity. Each, therefore, implies the determinateness of the other of the pair, which, according to the implication of the infinite progress, should exclude each other and follow alternately; neither can be posited or comprehended without the other, neither infinite without finite nor finite without infinite. When we put in words what the infinite is (namely, the negation of the finite), the finite itself is the word we pronounce; we cannot do without it in order to determine the infinite. We need only know what we are saying, in order to find the determination of the finite in the infinite. As regards the finite, it is immediately admitted on its side that it is null; but its nullity, precisely, is infinity, from which it is equally inseparable.—It might appear that in this view they are taken according to their relation to their Other. If, accordingly, they are taken as unrelated, so that they are connected solely by the "and," they are opposed to each other as independent, because each has its being entirely in itself. It remains to be seen what their nature is in this view. Thus placed, the infinite is only one of the two; but, as *only* one of the two, it is itself finite, it is not the whole but only one side; it has its limit in that which is opposed to it; it is thus the finite infinite. There are given only *two finites*. It is just the fact of the infinite's separation from the finite, and consequent one-sidedness, that constitutes its finitude, that is, its unity with the finite.—On the other hand the finite, when separated from the infinite, is that self-relation which excludes its relativity, dependence, and transitoriness; it is that independence and self-affirmation which the infinite is supposed to be.

These two ways of regarding the matter appear at first to have a different determinateness for their point of departure, inasmuch as the former must consider them purely as the relation of infinite and finite, of each to its Other, and the latter in their complete severance from each other. Both, however, give one and the same result: infinite and finite regarded in their relation to each other (a relation which was to be external to them, but really is essential, for without it neither is what it is) contain each its own Other in its own proper determination, just as much as each taken by itself and considered in itself possesses its Other in it as its own moment.

We thus have that ill-famed unity of Finite and Infinite;—that unity which is itself infinity, and comprises both itself and finitude;—infinity, therefore, in a sense different from that in which the finite is separated from and opposed to it. Since now they must still be distinguished, each in itself, as has been shown, is the unity of both; thus two such unities result. The common element, the unity of each determinateness with the other, first posits these (when taken as unity) as negated, since each is supposed to be that which it is when the two are held distinct: in their unity they thus lose their qualitative nature.—This reflection is of importance to refute the idea which, in the unity of finite and infinite, refuses to abandon their retention in the quality, which is supposed to be theirs, of being external to one another;—an idea which therefore sees in their unity only the contradiction and not its solution through the negation of the qualitative determinateness of the two. Thus the unity of finite and infinite, simple and general at first, is adulterated.

But, further, since they must also be taken as distinct, the unity of the infinite, which each of these two moments is itself, is determined in each of them in a different manner. That which is determined as infinite yet contains finitude, which is distinct from it: in this unity, the former is the "in-itself," the latter, only determinateness and limit; but it is a limit which is its absolute Other, its opposite; and its determination, which is Being-in-Self as such, is spoilt by the addition of a quality of this kind: it is thus an infinity made finite. Similarly, since the finite as such is only not-Being-in-Self, but in the unity also contains its opposite, it appreciates above its proper value, infinitely as one may say: it is posited as the finite made infinite.

Understanding, as before it adulterated the simple unity of infinite and finite, now likewise adulterates their twofold unity. Here again this is brought about when in one of the two unities the infinite is taken not as negated but as Being-in-Self, so that determinateness and barrier must not be applied to it; for else (it is thought) Being-in-Self is degraded and spoilt. Conversely, the finite, too, remains in its aspect of that which, though null, is not negated, so that in its connexion with the infinite it is held to be elevated to a status not its own, and therefore to be made infinite, in opposition to its determination which still persists and has not disappeared.

When understanding thus adulterates finite and infinite, holding fast their relation to one another as qualitative distinctness, and asserting that their determination is one of separateness (in fact, of absolute separateness), it can do so only by forgetting that which, for understanding, is the concept of these moments. For according to the concept, the unity of finite and infinite is not an external juxtaposition of these terms, nor an improper connexion contrary to their determination, and binding together entities separate and opposed and mutually independent and hence incompatible; on the contrary, each in itself is this unity, and is so only in transcending itself, neither excelling the other in Being-in-Self and affirmative Determinate Being. It has been demonstrated above that finitude exists only as a passing beyond itself; it thus contains infinity, which is its Other. And, similarly, infinity exists only as a passing beyond finitude; it thus essentially contains its Other, and so is in itself its own Other. The infinite does not transcend the finite as a power existing external to the latter; rather, it is the infinity of the finite to transcend itself.

This transcendence is not therefore alteration or otherness in general; it is not the transcendence of Something. The finite transcends itself in the infinite as negation of finitude; but the latter itself has for some time past been Determinate Being determined as a Not-being. Thus, what is transcended in the negation is itself negation. So, on its side infinity is determined as the negative of finitude and therefore of determinateness in general; as a vacant beyond; its self-transcendence in finitude is a return from this empty flight: it is a negation of that beyond, which in itself is a negative.

In both, therefore, we have the same negation of negation. But this is, in itself, a self-relation; it is affirmation, but affirmation in the sense of a return to itself, that is, through mediation, which is negation of negation. Mainly we must fix our attention upon these determinations; secondly, on the fact that they are also posited in the infinite progress, and on the manner in which they are so posited, namely, as not yet in their ultimate truth.

Here, then, first, both are negated—both finite and infinite; both are overpassed in the same manner: secondly, each after the other is posited as distinct and positive in itself. We thus select these two determinations for comparison; in this comparison (which is external) we compare how we have separated two modes of contemplation—on the one hand finite and infinite in their relation, on the other each taken for itself. The infinite progress, however, asserts more than this: in it is also posited the connexion of terms which also are distinct, though at present distinct only as transition and alternation; we need only a simple reflection in order to see what is in fact contained therein.

The negation of finite and infinite which is posited in the infinite progress may, first, be taken as simple, so that the terms are separate members of a series. Beginning with the finite we pass beyond the limit, and the finite is negated. We therefore have its beyond, which is the infinite; in it, however, limit again arises, and thus we have the progress beyond the infinite. Partly, however, this double transcendence is posited as a mere external contingency, an alternation of the moments; partly it is not yet posited as one unity; each transition is a fresh start or new act, so that they fall apart.—Further, however, the infinite progress also contains the relation of these terms. First, there is the finite; next we pass beyond it, and this negative or beyond of the finite is the infinite; thirdly, we pass once more beyond this negation, and a fresh limit—which once more is a finite—arises.—This is the complete and self-terminating movement, where the end reached is that from which a start was made; that from which we began has now arisen, that is, the finite has been reconstructed; it has thus collapsed into itself, has found only itself again in its beyond.

The same happens with regard to the infinite. In the infinite, or the beyond of the limit, a new limit arises, and its fate is the same: being finite it must be negated. Thus that which we now have is that same infinite which lately disappeared in the new limit; consequently through its transcendence,through the new limit, the infinite has not been pushed further; it has neither been removed from the finite, since this is the mere transition into the infinite, nor from itself, for it has returned upon itself.

Thus, both infinite and finite are this movement which through negation returns to itself: they are in themselves only as mediation, and the affirmative of both contains the negation of both and is the negation of negation.—Taken thus they are *result*, and not, therefore, that which they are in the determination of their beginning;—the finite is not a Determinate Being on its side, and the infinite a Determinate Being or Being-in-Self beyond this Determinate Being, that is, beyond that which was determined as finite. Understanding recoils from the unity of finite and infinite chiefly for this reason only, that it presupposes the barrier and the finite, as well as Being-in-Self, as perpetual; it thus overlooks the negation of both which in fact is present in the infinite progress, together with the fact that they occur in it only as moments of a whole and that they manifest themselves only through their opposite, but equally and essentially through the transcendence of their opposite.

The return upon itself has been considered both as return of the finite and of the infinite to itself; the result shows a flaw which is connected with the distortion just criticized; first, finitude, and then infinity, is taken as point of departure, and it is only thus that *two* results arise. But it is perfectly indifferent which is taken as beginning; and this disposes in itself of the distinction which caused the *double* result. This is likewise implied in the line of the infinite progress which is unlimited in either sense; each moment, there, occurs equally in alternation, and it is quite contingent what point is fixed on and which of the two is taken as the beginning.—They are here distinct, but also each is but the moment of the other. And as both finite and infinite are themselves moments of the progress, they jointly are the finite: since jointly they are negated in the progress and in the result, their result, which is the negation of the finitude of both, is justly called the infinite. Thus the double meaning proper to both is their distinction. The finite has this double meaning, that first it is finite only relatively to the infinite which is opposed to it, while secondly it is simultaneously the finite and also the infinite opposed to it. And the infinite too has the double meaning that it is *one* of these two moments (it then is the bad infinite), and *also* is that infinite in which those two, itself and its Other, are only moments. As we actually find the infinite, its nature is to be the process in which it first degrades itself to the rank of one only of its determinations, having the finite opposed to it and thus itself becoming a finite, and next transcends this distinction from itself, rises to self-affirmation, and through this mediation is true infinite.

This determination of the true infinite cannot be cast into the formula (already criticized) of the unity of finite and infinite; the unity is abstract and motionless self-equality, and the moments too are as unmoved existents. But the infinite, like its two moments, essentially exists as Becoming, a Becoming which is now further determined in its moments. Its determinations are at first abstract Being and Nothing; as Change, it has for moments determinate existents—Something and Other; as Infinite, it has Finite and Infinite, which are themselves in process of becoming.

This infinite is the accomplished return upon itself. As such it is self-relation or Being; but not abstract or indeterminate Being, for it is posited as negating negation; and thus it is also Determinate Being, for it contains negation as such, and, therefore, determinateness. It exists, and exists as a Determinate Being, present and before us. It is only the bad infinite which is the beyond, because it is the negation, and nothing more, of the finite posited as real; it is thus abstract and first negation; it is determined as merely negative, and is without the affirmation implicit in Determinate Being; and if held fast as mere negative it is even supposed to be non-existent and beyond reach. But to be thus beyond reach is not its glory but its shame; which, ultimately, is based on the fact that the finite as such is held fast as existent. That which is untrue is beyond reach; and it is evident that such an infinite is the untrue.—The image of the "progress to infinity" is the straight line, the infinite still remaining at its two limits and there only where the line is not; now the line is Determinate Being, which passes on to this its contradictory, that is, into the indeterminate. But as true infinity, turned back upon itself, it has for image the circle, the line which has reached itself, closed and wholly present and having neither beginning nor end.

True Infinity thus taken, in general, as Determinate Being opposed affirmatively to abstract negation, is Reality in a higher meaning than is that infinity which before was determined as *simple*; it has here received concrete content. It is not the finite which is the real, but the infinite; and thus Reality is further determined as Essence, Notion, Idea, and so forth. It is, however, superfluous to repeat these earlier and more abstract categories, such as "Reality," when the more concrete has been reached, and to employ them for determinations more concrete than these are in themselves. A repetition, such as is made when we say that Essence or the Idea is the Real, has its reason in the fact that, to uncultivated thought, the most abstract categories, such as Being, Determinate Being, Reality, and Finitude, are the most familiar.

Here there is a more definite reason for recalling the category of reality, for the negation to which it stands in the relation of affirmative is here the negation of negation: it is thus itself opposed to this reality, which is finite Determinate Being.—Negation is thus determined as ideality; that which partakes of the ideal nature (*das Ideelle*)[1] is the finite as it is found in true infinity,—as a determination or content, which though distinct does not exist independently, but only as moment. Ideality has this more concrete meaning, which is not fully expressed by negation of finite Determinate Being.—But with relation to reality and ideality the opposition of finite and infinite is taken in this manner, that the finite is taken as real and the infinite as of ideal nature; and such, indeed, and only such, the Notion is later on taken to be; whereas Determinate Being in general is taken as real. In this manner it is indeed of little avail if we reserve for the concrete determination of negation just indicated the term of "ideal in nature": in this opposition a return is once more made to the one-sidedness of the abstract negative which is proper to the bad infinite, and we remain fixed in the affirmative Determinate Being of the finite.

Transition

Ideality may be called the Quality of Infinity; but, as it is essentially the process of Becoming, it is a Transition, like that of Becoming in Determinate Being, and it must now be indicated. As transcendence of finitude (finitude as such) and also of the infinity which is merely opposed to it and merely negative, this return to self is self-relation or Being. But since this Being contains negation it is Determinate Being, and since this negation essentially is negation of negation, or self-related negation, it is that kind of Determinate Being which is called Being-for-Self.

---

[1] This has a narrower meaning than the Ideal (*das Ideale*) (the Beautiful and whatever tends thither); this does not yet belong to that plane: hence the expression in question "of ideal nature," is used. With *reality* this distinction is hardly made in ordinary parlance, and "real" (*das Reale*) and "of real nature" (*das Reelle*) are used almost interchangeably: the precise nuance of these expressions relatively to each other is of no interest. [Author's note.]

Observation 1

The infinite,—in the ordinary sense of bad infinity,—and the progress to infinity, are, like Ought, the expression of a contradiction, which pretends to be the solution and the ultimate. This infinite represents the first exaltation of sensuous imagination above the finite into Thought, the content of which, however, is Nothing, or that which is expressly posited as not-being; it is a flight from barrier, which, however, neither collects itself nor knows how to lead back the negative to the positive. This imperfect reflection has completely before it the two determinations of the true infinite—the opposition of finite and infinite, and the unity of these; but it fails to reconcile these two thoughts; either inevitably evokes the other, but in this reflection they merely alternate. An exhibition of this alternation, an infinite progress, everywhere takes place where a halt is made at the contradiction between the unity, and the opposition, of two determinations. The finite is self-transcendence and includes its own negation, which is infinity,—this is the unity of both; we pass beyond the finite to the infinite as its beyond,—and this is the separation of them. But beyond the infinite there is another finite: the beyond or finite contains finitude;—once more we have the unity of both; but this finite is also a negative of the infinite, and this is the separation of the two: and so forth.—Thus in the causal relation cause and effect are inseparable; a cause which should have no effect is no cause, and also an effect having no cause is no effect. This relation therefore yields the infinite progress of cause and effect; Something is determined as cause, but being finite—and finite it is just because it is separated from the effect—it also has a cause, that is, it is itself effect; thus that which is determined as cause is also determined as effect: this is the unity of cause and effect. And now that which is determined as effect again has a cause, that is, cause must be *separated* from its effect, must be posited as a different Something;—but the new cause being itself effect, we have unity of cause and effect. But it also has an Other for its cause; this is the separation of the two determinations; and so on to infinity.

A more peculiar form can thus be given to the progress. It is asserted that finite and infinite are one unity: this false assertion must be rectified by its contradictory,—they are absolutely different and opposite: this assertion must again be rectified in the sense that they are inseparable, one determination being contained in the other through the assertion of their unity; and so to infinity.—It is an easy demand, made in order that the nature of the infinite may be understood, that it should be clearly grasped that it is the nature of the infinite progress, or developed infinite of Understanding, to be an alternation of the two determinations, of the unity and separation of the two moments; and that further it should be grasped that this unity and separation are themselves inseparable.

The solution of the contradiction does not lie in the recognition of the equal validity and the equal invalidity of the two assertions (—for this is only another form of the abiding contradiction), but in the *ideality* of both, in which, in their distinction, and as mutual negations, they are only moments. That other monotonous alternation is in fact the negation as well of their unity as of their separation. But what was indicated above is equally definitely contained in it, namely, that the finite falls beyond itself into the infinite, but also finds itself regenerated beyond it; that thus it merely collapses into itself, as does the infinite; so that the same negation of negation in the result brings about affirmation, which result thus proves itself to be its truth and original nature. In this Being, which is the ideality of the distinct elements, the contradiction has not vanished abstractly, but it is resolved and reconciled, and the thoughts are not only complete but combined. The nature of speculative thought shows its characteristic method in this example just worked out: it consists solely in seizing the opposed moments in their unity. Here each in fact shows that it contains its opposite and coincides with it: and thus the affirmative truth is this unity which moves itself within itself, the compacting of the two thoughts, their infinity,—self-relation, not immediate, but infinite.

Those who are no tiros in thought have often placed the essence of philosophy in the problem of answering the question how the infinite passes beyond itself and achieves finitude.—It is supposed that this cannot be made conceivable. The infinite, the concept of which we have reached, will further determine itself in the course of this demonstration; it will exemplify in every variety of form what was demanded, namely, how it, if that expression is desired, achieves finitude. Here we are examining this question only in its immediacy, and with respect to the meaning, above considered, which infinity is wont to have.

On the answer to this question nothing less than the existence or non-existence of philosophy is supposed to depend; and, while it is pretended that this is still in doubt, the question itself is thought to be a sort of puzzle, an invincible talisman rendering us safe and secure against the answer, and therefore against philosophy and the arrival at philosophy.—In other subjects, too, the art of putting a question demands some education; still more so in philosophical subjects, if a better answer be required than this, that the question is idle.

In such questions it is claimed as reasonable that the wording is irrelevant, and that it shall be intelligible from one or other mode of expression, what the point is. Expressions of sensuous imagination such as "passing beyond" and the like, which are used in the question, raise the suspicion that it is derived from the sphere of ordinary imagination, and that in the reply also figures current in common life, and the form of a sensuous analogy, are expected.

If, instead of the infinite, Being in general is taken, the determining of Being, the existence of negation or finitude in it, seems more easily comprehensible. Being is itself the indeterminate; but it is not immediately expressed in it that it is the opposite of the determinate. The infinite on the contrary contains this expressly; it is the not-finite. Thus the unity of finite and infinite immediately seems precluded; and therefore incomplete reflection opposes itself most stubbornly to this unity.

But it has been shown, and is immediately evident without further investigation of the determination of finite and infinite, that infinite, in the sense in which it is taken by this reflection (namely, as opposed to the finite), has in it its Other just because it is opposed to it; that, therefore, it is limited and itself finite, and is the bad infinite. Therefore, if it is asked how the infinite becomes finite, the answer is that there is no infinite which first is infinite and then must become finite or pass on to finitude, but that for itself it is already finite as much as infinite. The question assumes that the infinite is on the one side, by itself; and that the finite, having passed out of it, or from some other source, into the state of separation from it, is, though finite, yet veritably real. In this assumption one should rather say that that which is unintelligible is the separation: neither finite nor infinite of such a kind is true; and what is without truth is unintelligible. But we must just as much say that they are intelligible; if, considering them even as they appear to sensuous representation, we see that one contains the determination of the other, then to have this simple perception of their inseparability is to understand them: this inseparability is their concept.—But when it implies the independence of finite and infinite the question sets up an invalid content and implies an invalid relation of the latter. Therefore, instead of answering the question, its wrong presuppositions (that is, the question itself) must be negated. The question about the validity of finite and infinite changes the standpoint, and this change will cause to recoil upon the first question that embarrassment which it was intended to produce. This question of ours is foreign to the reflection which gave rise to the first question; for that reflection does not contain that speculative interest which, before it relates determinations, proceeds for its own sake to see whether they are valid as they are assumed. But once we have perceived the invalidity of this abstract infinite, and of the finite which is equally supposed to remain fixed by its side, this may be said about the emergence of the finite from the infinite, that the infinite passes out into finitude just because, taken as abstract unity, it has in it neither validity nor permanence; and, conversely, the finite passes into the infinite for the same reason, because it is void. Or, rather, this should be said, that the infinite has ever passed out to finitude; that, absolutely, it does not exist, by itself and without having its Other in itself, any more than does Pure Being.

This question about the transition of infinite into finite may also contain this presupposition, that in itself the infinite includes the finite, being thus in itself the unity of itself and its Other; so that the difficulty relates chiefly to the *separation* as repugnant to the supposed unity of the two. Here the opposition which is retained has only another form; *unity* and *distinctness* are separated and isolated from each other. But, if this unity is taken not as the abstract and indeterminate unity, but as the determinate unity of finite and infinite (as it is in the former presupposition), then it already contains the distinction of the two moments;—a distinction which thus, further, does not set them free to achieve separate independence, but keeps them, as of ideal nature, in their unity. This unity and this distinctness of finite and infinite are the same inseparability as finitude and infinity.

Observation 2

The proposition that the finite is of ideal nature constitutes Idealism. In philosophy idealism consists of nothing else than the recognition that the finite has no veritable being. Essentially every philosophy is an idealism, or at least has idealism for its principle, and the question then is only how far it is actually carried through. This is as true of philosophy as of religion; for religion equally with philosophy refuses to recognize in finitude a veritable being, or something ultimate and absolute, or non-posited, uncreated, and eternal. The opposition of idealistic and realistic philosophy is therefore without meaning. A philosophy which should ascribe to finite Determinate Being, as such, veritable, ultimate, and absolute being, would not deserve the name of philosophy: the principles of old and new philosophies, such as water, matter, or atoms, are Thoughts, universals, and of ideal nature, not Things such as we find them immediately, that is, in their sensuous isolation. Such was not even that water which Thales feigned was the principle of things; for though it is empirical water, it is also the "in-itself" or essence of all other things; and these are not independent or based upon themselves, but posited in something else, namely, water; that is, of ideal nature. Above, the principle or universal was said to be of ideal nature; and the Notion, the Idea, the Spirit, still more deserve this name; whereas the individual things of sense-perception, as being of ideal nature in principle, exist in the Notion, and still more in the Spirit, as transcended. Herein attention must provisionally be drawn to the same double aspect as showed itself with the infinite, namely, that first that which is of ideal nature is the concrete and veritable; but, secondly, its moments are just as much that which is of ideal nature, that which is transcended in it; while, in fact, there is but the one concrete whole, from which the moments are inseparable.

By that which is "of ideal nature," the form of imagination is meant primarily; and this name is given to whatever is *in* my imagination in general, or *in* the concept, *in* the idea, *in* the fancy, and so forth; so that it comes to be counted equivalent only to fancies,—imaginations which are not only distinct from the real, but are supposed in their essence to be not real. The true idealist, indeed, is the Spirit. Its content, even when it only feels and imagines, much more when it thinks or conceives, is not so-called real Determinate Being; in the simplicity of the ego such external Being is merely transcended, it is *for me*, or *ideally* within me. This subjective idealism, whether as the unconscious idealism of consciousness in general or as consciously expressed and laid down as a principle, applies only to the *form* of imagination, according to which a content is my own; in the systematic idealism of subjectivity this form is asserted to be the one true form, excluding the form of objectivity or reality, or of the external existence of this content. This idealism is formal, for it does not consider the *content* of imagination or of thought, which can thus remain entirely in its finitude both in imagination and in thought. In such an idealism nothing is lost; for the reality of such finite content, Determinate Being filled with finitude, is preserved; and also, in so far as we abstract from this, this content in itself is supposed to be irrelevant. Nor is anything gained, precisely because nothing is lost, and the ego, imagination, or Spirit remains filled with the same content of finitude. It is true that the opposition between the forms of subjectivity and objectivity is one of the finitudes; but the content as taken up into sensation, intuition, or the more abstract element of imagination or of thought, contains finitudes in abundance; and though this mode of finitude, the form of subjective and objective, is excluded, it is but one, and the others have not been removed, still less fallen away spontaneously.

CHAPTER III

BEING FOR SELF

In Being for Self qualitative Being finds its consummation; it is infinite Being. The Being of the Beginning is indeterminate. Determinate Being is Being transcended, but transcended only immediately: it thus contains only first negation, itself immediate; Being indeed is preserved, and in Being Determinate both have entered into a simple unity; but for that very reason they are not yet in themselves equal to one another, and their unity is not yet posited. For this reason Determinate Being is the sphere of difference, of dualism, the field of finitude. The determinateness here is determinateness as such, a fact relative and not absolute. In Being for Self the difference between Being and determinateness, or negation, is posited and levelled; quality, otherness, limit, and reality, being-in-self, ought, and so forth, are incomplete adaptations of negation to Being, which still have for base the difference between the two. But in finitude negation has passed over into infinity, into the posited negation of negation; it is thus simple self-relation, and in itself is the identification with Being:—absolute determinateness.

Being for Self is such, first, *immediately*; it is One.

Secondly, One passes over into Plurality of Ones—Repulsion; which otherness of One transcends itself in its ideality—Attraction.

Thirdly, there is the mutual determination of Repulsion and Attraction in which they collapse into equipoise; here Quality, which reached its climax in Being for Self, passes over into Quantity.

A

BEING FOR SELF AS SUCH

We have reached the general concept of Being-for-Self. The task now is to demonstrate that the image which we connect with the term of "Being-for-Self" does correspond to this concept, if we are to be justified in employing it for this concept.

And so, indeed, it seems; we say that something is for itself in so far as it cancels its otherness, its relatedness to and community with Other, rejecting and abstracting from them. In it, Other only exists as having been transcended, or as its moment; and Being-for-Self consists in this, that it has passed beyond the Barrier and its Otherness in such a manner that, thus negating them, it is infinite return upon itself.—Consciousness as such already contains the determination of Being-for-Self; for, the object of its sensation, intuition, and so forth, is imagined, and it thus comprehends its content, which, therefore, is of ideal nature; in its intuition, and, generally, in its entanglement with its own negative or Other, it still remains at home. Being-for-Self is the polemical or negative attitude against the limiting other; by virtue of this negation it is an intro-reflectedness; although, parallel to this return of consciousness upon itself and the ideality of the object, the reality likewise of the object is preserved, since simultaneously it is known also to be an external Determinate Being. In this its phenomenal aspect consciousness is a dualism; first it has knowledge of another and external object; secondly it is for itself, it contains the object in an ideal manner; its being is not only with such an Other, but, being with such an Other, it is also at home with itself. On the other hand, self-consciousness is Being-for-Self accomplished and posited; the aspect of relation to an Other, an external object, has been removed. Self-consciousness is thus the nearest example of the presence of infinity;—an infinity abstract indeed, but also of a concrete determination very different from that of Being-for-Self in general, the infinity of which still has a wholly qualitative determinateness.

(*a*) Being-Determinate and Being-for-Self

It has already been pointed out that Being-for-Self is infinity which has collapsed into simple Being; it is Determinate Being in so far as the negative nature of infinity, which is negation of negation, exists only as negation in general or simple qualitative determinateness, in the form, now posited, of the immediacy of Being. In this kind of determinateness Being appears as Determinate Being, but further is distinct from that Being-for-Self which is only Being-for-Self in so far as its determinateness is of this infinite kind; but, also, Determinate Being is a moment of Being-for-Self, since it is true that the latter also contains Being infected with negation. Thus determinateness, which, in Determinate Being as such, is an Other and a Being-for-Other, is reflected into the infinite unity of Being-for-Self; and here the moment of Determinate Being is present as Being-for-One.

(*b*) Being-for-One

This moment expresses the manner in which the finite exists in its unity with the infinite, or as ideal in nature. In Being-for-Self negation is not applied as a determinateness or limit, not, therefore, as relation to another and distinct Determinate Being. This moment has been called Being-for-One, but so far there is nothing for which it could be;—there is no One, of which it could be a moment. And, indeed, nothing of this kind has yet been fixed in Being-for-Self; that for which Something would be (and, in fact, there is no Something), that which should be its general counterpart, is itself a moment and only Being-for-One; it is not yet One. Thus the two sides which may be suggested in Being-for-One are not distinguished; there is only one Being-for-Other, and just because it is one, it too is a Being-for-One; it is but one ideality of that, for which or in which a determination should exist as moment, and of that, which should be moment in it. Thus Being-for-One and Being-for-Self are not true determinatenesses relatively to one another. But let the distinction be granted for a moment; then, if we can assume a Being-for-Self, it is Being-for-Self, as cancellation of otherness, itself, which relates itself to itself as to the cancelled Other; it is thus for One, and in its Other relates itself to itself. That which is of ideal nature must needs be for One, but not for an Other: that One for which it is, is merely itself.—Ego, or Spirit in general, or God, are of ideal nature because they are infinite; but, as being for themselves, they are not distinct, ideally, from that which is for One. For then they would be merely immediate, or rather, they would be determinate beings, or being-for-other; for that which for them is would then be an Other and not themselves, if they had not the moment of being for One. God is thus for himself in so far as he is himself that which is for him.

Thus, Being-for-Self and Being-for-One are not really distinct meanings of ideality, but rather moments of it essential and inseparable.

Observation

The moment here considered is clearly emphasized in its intro-reflection by the expression of the German language, strange at first sight, in enquiring after quality by asking, "what for a thing is this?"[1] In its origin the expression is idealistic; it does not ask what this thing *A* is for another, *B*, or what this man for another man;—but, "what for a thing, what for a man, is this?" Thus, this Being-for-One has forthwith returned into this thing, or man, itself, and that which is, is identical with that for which it is,—an identity which must also be considered as ideality.

Ideality first applies to the already cancelled determinations as distinct from that wherein they are cancelled; this latter may be called the real. But thus once more that which is of ideal nature is one of the moments, and the real the other; but ideality consists in this, that both determinations in the same manner are only for one and count only as one, this one ideality thus being undifferentiated reality. In this sense self-consciousness, Spirit, God, are of this ideal nature as pure infinite self-relation;—ego is for ego, and both are the same, ego is named twice, but each of the two is for one, and of ideal nature; Spirit is only for Spirit, God for God, and it is only this unity which is God, God as Spirit.—But self-consciousness, as consciousness, enters into the distinction between itself and an Other, or between its ideality (where it exists as imagining) and its reality (inasmuch as its imagination has a definite content which has this further aspect of being known as untranscended negative, or as Determinate Being). But if Thought, Spirit, or God is called *only* of ideal nature, that point of view is assumed from which finite Determinate Being is counted as real, and that which is of ideal nature, or Being-for-One, has only a one-sided meaning.

---

[1] The equivalents of the German words (*was für ein Ding*) are used; the meaning is, "what kind of thing is this?" (Translators' note.)

In a previous Observation (p. 168), it was indicated what is the principle of idealism; and it was said that in any philosophy the next point of importance was, how far this principle was applied. As to the manner of application, a further remark may be made with reference to the category at which we have arrived. First, the possible independent existence of finite Determinate Being, alongside Being-for-Self, governs this application; and next, the possible positing of the moment of "for-one" (a self-relation of that which is of ideal nature, as such) in the infinite itself. The Eleatic Being, or the Substance of Spinoza, for instance, is only the abstract negation of all determinateness; but no ideality is posited in this negation. And with Spinoza (as will be mentioned below) infinity is only the absolute affirmation of a thing, and thus immobile one-ness; Substance thus does not even reach the determination of Being-for-Self, and much less that of Subject and Spirit. The noble idealism of Malebranche is, in itself, more explicit. It contains the following fundamental thought:—God contains all the eternal truths, the ideas and perfections of all things, so that they are his and his alone; therefore we see them in him only; God causes in us our sensations of objects by means of an act which has no sensuous element: we imagine that we obtain of the object not only the idea (which represents its essence), but also the sensation of its determinate existence (*De la Recherche de la Vérité*, *Eclairc. sur la nature des idées*, *etc.*). Thus the determinate existence, like the eternal truths and ideas (or essentialities) of things, is, in God, of ideal nature; it is not an actual Determinate Being; they are objects to us, and yet they are only "for-one." This moment of explicit and concrete idealism, lacking in Spinoza, is present here, absolute ideality being determined as knowledge. This idealism is pure and deep, but these relations partly contain much that is as yet indeterminate for thought, and partly their content is immediately wholly concrete (Sin and Salvation, and so on, enter immediately into play); the logical determination of infinity, which ought to be its foundation, is not carried through for itself; and thus, although this lofty and complete idealism is the work of a pure speculative spirit, it is not the work of a pure speculative and therefore alone truly fundamental thinking.

The idealism of Leibniz lies rather within the limits of the abstract concept.—Leibniz's imagining subject, the monad, is essentially of ideal nature. Imagination is a Being-for-Self, in which the determinatenesses are not limits and therefore not a Determinate Being, but only moments. To imagine is certainly a more concrete determination, but still it has here no further meaning than ideality; for with Leibniz even that which lacks consciousness in general has imagination and perception. Thus otherness is transcended in this system; Spirit and Body, or, the monads in general, are not Others for one another; they neither limit nor act on one another; thus all the relations based upon a Determinate Being are absent. The differentiation, therefore, is of ideal nature, and internal, and the monad here remains self-related; changes develop within it; they are not relations of it to others. Where the real determination sees a determinately existing relation of the monads to one another, there is in fact an independent and merely simultaneous Becoming, included in the Being-for-Self of each individual.—That there is a plurality of Monads, and that therefore each is also determined as an Other, does not concern them, for this is the external reflection of a third party; in themselves they are not related as Others, and Being-for-Self remains unadulterated by any parallel Determinate Being.—However, this also implies the incompleteness of this system. It is only in themselves, or in God (as the monad of monads), or in the system, that the monads thus exist as imagining. Otherness also exists, in imagination itself or in whatever other name we give to that third element which considers them as Others and as Many. The multiplicity of their Determinate Being has only been barred out, and that only for the moment, and it is only by abstraction that the monads have been posited as existing as not-others. If a third element posits their otherness, it also cancels it; but the whole movement which gives them their ideal nature is external to them. Should it be objected that the whole movement of thought only falls within a monad imagining, then it may be replied that the content of such thought in itself is external to itself. From the oneness of absolute ideality (the monad of the monad) an immediate and non-conceptual transition (—by means of the image of creation) is made to the category of abstract, or unrelated, multiplicity of Determinate Being; and an equally abstract return is made from it to the former oneness. Ideality and imagination in general remain formal, and formal too is imagination raised to the power of consciousness. We mentioned above Leibniz's notion about the needle of the compass which, if it had consciousness, would regard its northward direction as a determination of its freedom: there consciousness was taken as a one-sided form, supposed to be indifferent to its content and determination: here, similarly, ideality in the monads is a form which remains external to multiplicity. Ideality is supposed to be immanent in them, imagination is supposed to be their nature: but, first, their conduct is their Harmony, which is not within the sphere of their Determinate Being, and hence is pre-established; next, this their Determinate Being is not taken as Being-for-Other, nor, further, as Ideality; it is determined merely as abstract multiplicity; the ideality of multiplicity and its further determination to Harmony never become immanent and proper to this multiplicity.

Other forms of idealism, like that for instance of Kant and Fichte, do not get beyond Ought and the infinite progress; they remain in the dualism of Determinate Being and Being-for-Self. The Thing-in-itself, or the infinite Resistance-principle, does indeed enter immediately into the Ego in these systems, becoming "for-it" alone; but it starts from a free otherness, which persists as negative Being-in-Self. The Ego therefore may be determined as being of ideal nature, as infinite self-relation and being-for-self; but Being-for-One is not completed to that point where its beyond, or the trend towards the beyond, disappears.

(*c*) ONE

Being-for-Self is the simple unity of its self and of its moment, Being-for-One. Here there is only one determination, the self-relation of the transcendence. The moments of Being-for-Self have collapsed into indistinguishability; this is immediacy, or Being, but an immediacy based on that negation which has been posited as its determination. Being-for-Self thus is that which is for itself; and since in this immediacy its inner significance vanishes, it is its own completely abstract limit—One.

Attention may be drawn before beginning to the difficulty inherent in the following presentation of the development of One, and to the ground of this difficulty. The moments which constitute the concept of One as Being-for-Self here fall apart. They are (1) negation in general; (2) two negations; (3) negations, therefore, of two terms which are the same; (4) these are utterly opposite; (5) self-relation, identity as such; (6) negative relation which yet is self-relation. These moments here fall apart because the form of immediacy, or Being, enters into Being-for-Self as that which is for itself; by virtue of this immediacy each moment is posited as an independent determination which has Being; and yet also they are inseparable. Of each determination, therefore, its contradictory must be asserted: it is this contradiction which, together with the abstract nature of the moments, constitutes their difficulty.

B

THE ONE AND THE MANY

One is the simple relation of Being-for-Self to itself, where its moments have collapsed into themselves; here, therefore, it has the form of immediacy, and its moments now have determinate existence.

One, as self-relation of the *negative*, is determination,—and as *self*-relation, it is infinite self-determination. The resulting immediacy, however, brings it about that these distinctions are no longer posited merely as moments of one and the same self-determination, but also as being. The ideality of Being-for-Self as totality thus, first, passes into reality, and into the most fixed and abstract of all, into One. Being-for-Self, in One, is the posited unity of Being and of Determinate Being, as absolute union of relation to Other and Self-relation. Next, however, the determinateness of Being opposes itself to the determination of infinite negation (or self-determination); so that what One is in itself it now merely has in itself; the negative thus is an Other, distinct from it. That which shows itself present, as distinguished from it, is its own self-determination; and its unity with itself as thus distinct from itself is degraded to the rank of relation; and, as negative unity, it is negation of itself as an Other, an exclusion of the One as something Other to the One.

(*a*) THE ONE IN ITSELF

In itself, the One has existence in general; this its Being is not determinate or determinateness as relation to Other, nor is it Modification: what it is, is the fact that this circle of categories has been negated. One thus cannot become Other; it is immutable.

It is indeterminate, but not as Being is; its indeterminateness is that determinateness which is self-relation: the fact of being absolutely determined, posited Being-in-Self. Since in its concept it is self-related negation, it contains distinction,—it passes away from itself towards Other, which tendency, however, is immediately reversed; for this moment of self-determination brings it about that there is no Other to be a point of direction; it thus returns to itself.

In this simple immediacy the mediation of Determinate Being and of ideality, and thus all differentiation and multiplicity, has disappeared. There is nothing in it; and this nothing, which is the abstraction of self-relation, is here distinct from Being-in-Self; it is *posited*, since Being-in-Self no longer is that simplicity proper to the category of Something, but has a determination, namely that of being mediation and hence concrete. Being abstract, it is identical with One, but it is distinct from its determination. Nothing, thus posited as within the One, is Nothing as the Void.—This Void is thus the quality of One in its immediacy.

(*b*) THE ONE AND THE VOID

The One is the Void as the abstract self-relation of negation. But the Void, as Nothing, is absolutely different from the simple immediacy, the Being of One which is also an affirmation; and since they are in one relation (namely, that of One), their difference is also *posited*. The Nothing as Void differs from the existent, however, in that it is *outside* the existent One.

Thus Being-for-Self is determined as One *and* Void, and has once more acquired Determinate Being.—The One and the Void have for common and simple basis their negative self-relation. The moments of Being-for-Self leave this unity and become external to one another: since the determination of Being is introduced through the simple unity of the moments, it thus reduces itself to one side only (to the rank of Determinate Being), and thus its other determination, negation in general, likewise opposes itself in the shape of Determinate Being of Nothing, that is, as Void.

Observation

In this form of Determinate Being, the One is that grade of the category which occurs with the ancients as the atomistic principle, which places the essence of things in the atom and the void (*τὸ ἄτομον* or *τὰ ἄτομα καὶ τὸ κενόν*). Abstraction having reached to this shape has gained a greater determinateness than the Being of Parmenides or the Becoming of Herakleitos. Lofty as is its stand when, making the simple determinateness of the One and the Void into the principle of all things, it reduces the infinite variety of the world to this simple antithesis, daring to understand it through this alone, it is equally easy for imagining reflection to imagine, here atoms, and there beside them, the void. No wonder therefore that the atomistic principle has always survived; the equally trivial and external relation of combination, which must be added if we would reach the appearance of a concrete and a multiplicity, is as popular as the atoms themselves and the void. The One and the Void is Being-for-Self, the highest qualitative Being-in-Self, sunk back to complete externality; and the immediacy or Being of One, since it is the negation of all otherness, is so posited as to exclude all determination or variation; it is absolutely brittle, and thus all determination, multiplicity, or connection remains a wholly external relation.

They who first thought it did not however leave the atomistic principle in this externality; besides its abstraction it had a speculative determination when the Void was understood to be the source of motion. This is a relation of atom and void quite different from the mere parallelism and mutual indifference of these determinations. And when it was said that the void was the source of motion it had not this trivial meaning, that Something can have motion only into a Void and not into a space already filled, for then no room would be left open for it; in which case the Void would be only the presupposition or condition and not the ground of motion, motion itself being already assumed and its ground (which is the essential thing) forgotten. Rather, the view that Void is the ground of motion, contains this profounder thought, that the negative in general contains the ground of Becoming, the unrest of self-movement: and in this sense the negative must be taken as the true negativity of the infinite.—The Void is the ground of movement only as the negative relation of the One to its negative, to the One (that is, to itself); this, however, is posited as having determinate existence.

Apart from this the further determinations of the ancients about form and position of the atoms and about the direction of their movement, are sufficiently arbitrary and external; and, what is more, they are directly contradictory to the fundamental determination of the atom. The atoms are the principle of utter externality and utter absence of concept; this principle, in the shape of molecules and particles, causes physics to suffer as much as a theory of the State which begins from the separate wills of individuals.

(*c*) MANY ONES

Repulsion

The One and the Void constitute Being-for-Self in its closest Determinate Being. Each of these moments has negation for its determination, and is also posited as a Determinate Being. According to this, One and Void are the relation of negation to negation as of one Other to its Other; the One is negation in the determination of Being, the Void in Not-being. Essentially, however, the One is self-relation only as relating *negation*; that is, it is itself that which the Void is intended to be outside it. But each is also posited as an affirmative Determinate Being, one as Being-for-Self as such, the other as indeterminate Existence in general; their mutual relation is that of other Determinate Being. The Being-for-Self of One is, however, essentially the ideality of Being Determinate and of Other; it is related, not as to an Other, but to itself. But Being-for-Self is pegged down as One, as something which is for itself and is immediately present, and thus its negative relation to itself is also relation to a Being: and since the relation is negative, that to which it relates itself remains determined as a Being Determinate and Other; being essentially a self-relation, this Other is not indeterminate negation, as Void, but likewise is One. One thus is Becoming which differentiates itself into many Ones.

But, really, this is not properly a Becoming, for Becoming is a transition of Being into Nothing; while One only becomes One. One as the other term of the relation contains the negative as relation, and thus is itself affected with it. Instead of Becoming, there is then, first, the proper and immanent relation of One; next, since it is negative, and also the One has being, One repulses itself from itself. The negative relation of One to itself is Repulsion.

This Repulsion, taken thus as a positing of many Ones, is effected only by the egress of One beyond itself; but they which are beyond, are also Ones. This is Repulsion according to the concept, or, as it is in itself. The second Repulsion is distinct from it, it is that which is present to the imagination of external reflection: not the creation of Ones, but the mutual exclusion of Ones which are presupposed and already present. It remains to be seen how that first Repulsion, which is in itself, determines itself to the second, which is external.

It must first be settled what determinations the many Ones have as such. The multiple Becoming, or production of Many, vanishes immediately as the fact of being posited; the products are Ones, but not for Other; they are infinitely self-related. The One repels only itself from itself: it does not become, but is already; what is imagined as the object of repulsion is also a One, a Being; each is equally the subject and object of a Repulsion which makes no difference.

Thus, relatively to one another, the Ones are *presupposed*;—they are *supposed* (or posited) by the self-repulsion of the One, and *pre*-supposed as *not* posited; the fact of their being posited is transcended; they are related to one another as Beings as being related each only to itself.

Thus plurality does not appear as otherness, but as a determination quite external to One. One, which repels itself, remains a relation to itself, like that which first is taken as repelled. Thus it does not concern the Ones that, comprehended into the determinateness of plurality, they are related to one another as Others. If plurality were a relation of the Ones themselves to one another, they would limit one another and would contain Being-for-Other affirmatively. Their relation—theirs by virtue of their self-subsistent unity—as posited here, is determined as none: it is once more the Void already posited. It is their limit, a limit which, however, is external to them, in which they must not be for one another. The limit is that in which that which is limited is and also is not; but the Void is determined as pure not-being, and this alone constitutes their limit.

The self-repulsion of the One is the explication of that which the One is in itself; but infinity, as split-up, is here infinity which has passed beyond itself: and this it has done through the immediacy of the infinite entity, the One. It is a simple relation of One to One, and equally, or rather, the absolute unrelatedness of the One; it is the former according to the simple affirmative self-relation of One, and the latter according to the same as negative. In other words, the plurality of the One is its self-positing; the One is its own negative self-relation and nothing else, and this relation (the One itself) is many Ones. But, equally, plurality is merely external to the One; for the One also is the transcending of Otherness, Repulsion is its self-relation and simple self-identity. The plurality of Ones is infinity, as contradiction which unconcernedly produces itself.

Observation

Mention was made above of Leibniz's idealism. We may here add that from the imagining monad, determined as being for itself, it proceeded only to the Repulsion just considered, and in fact only to plurality as such, wherein each One is for itself only, remaining indifferent to the Being Determinate or for-Self of Others, or Others do not exist at all for the One. The monad is for itself the whole and complete world; none has need of the other; but this inner multiplicity which it has in its imagination in no way alters its determination of being "for itself." Leibniz's idealism takes plurality immediately as given, and does not conceive of it as of a Repulsion of the monad; with him, therefore, plurality is taken only in its aspect of abstract externality. Atomistic philosophy does not know the concept of ideality; it does not take One as comprehending within itself the two moments of Being-for-Self and Being-for-It (that is, as of ideal nature), but as being simply and bluntly for-Self. However, it proceeds beyond mere indifferent plurality; the atoms enter into a further determination relatively to one another, though in a really inconsistent manner; whereas in the former indifferent independence of the monads, plurality remains the rigid and fundamental determination, so that their relation belongs to the monad of monads, or to the contemplating philosopher.

C

REPULSION AND ATTRACTION

(*a*) Exclusion of the One

The many Ones have Being; their Determinate Being, their relation, is non-relation and external to them;—it is abstract Void. But they themselves are this negative relation to self now as to existent Others;—demonstrated contradiction, infinity, posited in the immediacy of Being. Thus here Repulsion immediately finds that which is repelled from it. In this determination it is Exclusion: One repels only the many Ones which were neither created nor posited by it. This mutual, or universal, Repulsion, is relative, limited by the Being of the Ones.

Plurality is, first, not-posited Otherness. The limit is only the Void, or that where the Ones are not. But also they *are* in the limit: they are in the Void, or, their Repulsion is their common relation.

This mutual Repulsion is the posited Determinate Being of the many Ones: it is not their Being-for-Self,—in which they would be distinct, as many, only for a third party,—it is their own distinguishing, which preserves them.—They negate one another, posit one another as being only for-One. But they equally negate this quality of being only for-One: they repulse this their ideality, and are.—Thus the moments, absolutely united in ideality, are separated. In its Being-for-Self the One also is for-One, but this One for which it is is itself: its distinction from itself is immediately transcended. But the One which is distinguished has a Being in plurality; Being-for-One (as it is determined in Exclusion) hence is a Being-for-Other. Thus each is repulsed by an Other, is transcended, and is made into something which is not for itself but for-One, and in fact is another One.

The Being-for-Self of the many Ones thus shows itself to be their self-preservation, due to the mediation of their mutual Repulsion, in which they cancel one another and posit each the others as mere Being-for-Other; but it also consists in the repulsion of this ideality and in positing the One as not being for-Other. But this self-preservation of the Ones through their negative relation is rather their dissolution.

The Ones not only have, but also preserve, their Being by mutual Exclusion. Now first that which ought to give them their fixed support against negation (namely, their distinctness), is their Being, in fact their Being-in-Self, as opposed to their relation to Other: this Being-in-Self is, that they are Ones. But this they all are: in their Being-in-Self they are the same, instead of having in it a definite distinction. Next, their Determinate Being and attitude to one another, that is, the fact that each posits itself as One, is their mutual negation; but this also is one and the same determination of all; and through it they rather posit themselves as identical. Similarly, the fact that in themselves they are the same makes the ideality, which is supposed to be posited in them through Others, their own; it also they equally, therefore, do not repel.—Thus, according to their Being and positing, they are but one affirmative unity.

Our comparison is just this view of the Ones—that, in their two determinations, both as being and as mutually related, they manifest themselves as identical, and necessarily identical. —But we must further see what is posited of them in their mutual relation.—They are; this is presupposed in this relation; and they are only in so far as they negate one another, at the same time repelling this their ideality or negatedness; that is, in so far as they negate mutual negation. But they are only in so far as they negate; and thus in negating their negation they negate their being. In so far as they *are*, indeed, this negation would not negate them, for it is only external to them: the negation of the Other rebounds from them and strikes their surface only to touch it. But they return upon themselves only through the negation of the Other: they are only by virtue of this mediation, and this return is their self-preservation and their Being-for-Self. In so far as their negation effects nothing, by reason of the resistance offered by the existents, as such or as negating, they do not return upon themselves, they neither preserve themselves, nor are.

We observed above that the Ones are the same, that each one is the same as the other. This is not only our relation—a mere external juxtaposing; rather, the Repulsion is itself relation: the One which excludes the Ones relates itself to them (the Ones), that is, to itself. Thus the negative attitude of the Ones to each other is only a collapse into themselves. This identity, into which their repulsion passes, is the transcending of their distinctness and externality, which, as Excluding, they ought mutually to assert.

This self-positing of the many Ones in one One is Attraction.

Observation

Independence having reached its quintessence in the One which is for itself, is abstract and formal, destroying itself; it is the highest and most stubborn error, which takes itself for highest truth;—appearing, more concretely, as abstract freedom, pure ego, and, further, as Evil. It is freedom which goes so far astray as to place its essence in this abstraction, flattering itself that, being thus by itself, it possesses itself in its purity. Determined more closely, independence is that error which regards as negative, and maintains a negative attitude towards, that which is its own essence. It is thus a negative attitude towards itself, which seeking to possess its own Being destroys it; and this its activity only manifests the vanity of its activity. The reconciliation consists in acknowledging rather that that which is counter to the negative attitude is its essence; it is a renunciation of the negativity of its Being-for-Self in place of a clinging to it.

It is an old proposition that One is Many, and more especially that the Many are One. We may here repeat the observation that the truth of the One and the Many, expressed in propositions, appears in an inadequate form; and that this truth can be seized and expressed only as a Becoming and as a process, as Repulsion and Attraction, and not as a Being; but it is as the latter, as a stable unity, that it is taken in a proposition. Above, we mentioned and recalled the dialectic of Plato in the *Parmenides* in deriving the Many from the One, namely, from the proposition "One is." The inner dialectic of the concept has been indicated; and the dialectic of the proposition "that Many are One," is most easily taken as external reflection. External it here may be allowed to be, since its object too, the Many, are those entities which are external to one another. This comparison of the Many, each with the other, shows immediately that One is determined just like Other: each is One, is One of the Many, and is by excluding the others;—they are simply the same: there is only one determination present. This is the fact, and just this simple fact must now be grasped. Understanding is stubborn in its refusal to accept it only because there is present to it also (and rightly) the distinction; yet the fact mentioned does not eliminate the distinction, and equally the fact exists in spite of the distinction. And one can comfort understanding for the naïve manner in which it grasps the fact of the distinction by reminding it that the distinction will return again.

(*b*) The One One of Attraction

Repulsion is the schism which the One makes of itself into Many, the negative attitude of which is powerless because they each presuppose the others as existent. It is only the Ought of ideality; but the latter is realized in Attraction. Repulsion passes over into Attraction, and the many Ones into One One. At first both Repulsion and Attraction are distinct; the former as the reality of One, and the latter as its posited ideality. The relation of Attraction to Repulsion is this, that the latter is its presupposition. Repulsion furnishes the material for Attraction. If there were no Ones there would be nothing to attract: the idea of continuous Attraction or consumption of Ones presupposes an equally continuous creation of Ones; the sensuous representation of spatial Attraction allows the series of attracted Ones to continue; the place of the atoms which vanish into the point which attracts them is taken by a new multitude surging out of the void; and so, if it is desired, to infinity. If attraction were to be imagined as completed—that is, the many Ones brought to the point of One One—there would be but one sluggish One and no Attraction. The ideality which determinately exists in Attraction also has in it the determination of self-negation, that is, has in it the many Ones to which it is the relation; and Attraction is inseparable from Repulsion.

First, Attraction is equally the property of each of the immediate Ones; not one is preferred beyond another; and thus we really have an equipoise in Attraction, or rather an equipoise of Attraction and Repulsion itself, a sluggish rest in which there is no determinate ideality. But there can here be no question of preference of one One beyond others, for this would imply a determinate distinction between them: Attraction rather is the positing of the already present indistinguishability of the Ones. It is only Attraction itself which is the positing of a One distinct from the others; these are only the immediate Ones, which are supposed to preserve themselves by means of Repulsion; but, from their posited negation, the One of Attraction arises, which is hence determined as One mediated and posited as One. The first Ones, as being immediate, do not return upon themselves in their ideality, but have it in another One.

But the One One is ideality realized and posited in the One. It attracts by the mediation of Repulsion, and contains this mediation as its determination. Thus it does not absorb the attracted Ones in itself as in a point, that is, it does not abstractly cancel them. It contains Repulsion in its determination, and Repulsion preserves the Ones, as Many, in it; through its Attraction it makes a profit, so to speak, gaining in width and content. Thus there is in it unity of Repulsion and Attraction in general.

(*c*) The Relation of Repulsion and Attraction

The distinction of One and Many has determined itself to be that of their relation to one another; this is divided into two relations, Repulsion and Attraction. Each of these remains outside the other and independent for the present; and yet essentially they are connected. Their unity, as yet indeterminate, must reveal itself more definitely.

Repulsion as the fundamental determination of the One appears first and immediately, like its Ones, which though produced by it are yet posited as immediate; they are thus unaffected by Attraction, which is added to them, thus presupposed, from without. Repulsion, on the other hand, does not presuppose Attraction, so that the latter ought to have no part in its positing and Being; that is, so that Repulsion is not in itself the negation of itself, nor the Ones in themselves negated. Thus we have Repulsion abstractly by itself; and indeed Attraction too, relatively to the Ones as existents, has the aspect of an immediate Determinate Being; leaving its place it comes to them as an Other.

Thus if we take pure Repulsion by itself it is the scattering of many Ones into the indeterminate, outside the sphere of Repulsion: for it is just the negation of the mutual relation of the Many; if it is taken in the abstract, unrelatedness is its determination. But Repulsion is not merely the Void; the Ones, as unrelated, do not repulse or exclude one another; and this constitutes their determination. Repulsion, though negative, still essentially is a relation; mutual exclusion and avoidance do not mean liberation from that which is excluded and avoided, and that which excludes is still in connexion with that which is excluded. Now this moment of relation is Attraction, and thus is in the midst of Repulsion itself: it is the negation of that abstract Repulsion according to which the Ones would be but self-related existents, without the moment of excluding.

But when we start from the Repulsion of the determinately existing Ones, and in doing so posit Attraction as being added to them from without, then, though inseparable, both are kept apart as distinct determinations; we have seen, however, that Attraction does not only presuppose Repulsion, but that there also is a reverse relation of Repulsion to Attraction, so that the former equally has its presupposition in the latter.

According to this determination they are inseparable, and also each is determined as Ought and Barrier relatively to the other. Their Ought is their abstract determinateness as being in themselves; thereby, however, it is just referred beyond itself and relates itself to the other, so that each is mediated by the other as other. Their independence consists in this, that in this mediation they are posited as *another* determining for each other (Repulsion is the positing of the Many, Attraction of the One; the latter is also negation of the Many, the former, the negation of their ideality in the One), and in that Attraction is Attraction only by the mediation of Repulsion, and Repulsion Repulsion only by the mediation of Attraction. The further fact that mediation through Other is here in fact rather negated and each of these determinations is self-mediation, results from closer contemplation and carries them back to the unity of their concept.

First, then, the attitude of Repulsion and Attraction (which as yet are only relative) already implies that each presupposes itself, and in so doing relates itself only to itself.

Relative Repulsion is the mutual repelling of the many given Ones, which are supposed to exist immediately. But that there are many Ones, this assertion is just Repulsion; and any presupposition of Repulsion is only its own positing. Further, the determination of Being, which might belong to the Ones apart from the fact that they are posited—a determination which would give them pre-existence—also belongs to Repulsion. It is by means of Repulsion that the Ones manifest and preserve themselves as Ones, and have Being as such. Their Being is Repulsion itself, which thus is not relative to any other Determinate Being, but stands related only to itself.

Attraction is the positing of the One as such, of the real One, in opposition to which the Many are determined, in their Determinate Being, as only of ideal nature, and as vanishing. And thus Attraction also presupposes itself, namely in the determination of the other Ones to be of ideal nature, which elsewhere are supposed to be repellent for-themselves and for-others, and therefore also for whatever attracts. They gain ideality as opposed to this determination of repulsion, not in the end and only through their relation to Attraction; rather it is presupposed, and is the self-existent ideality of the Ones, since, as Ones (and here we include the One which is imagined as attracting), they are undistinguished one from another, they are one and the same.

If then the two determinations thus presuppose themselves, *each for itself*, this further means that each contains the other as moment. Self-presupposition is the positing of self in One as its own negative,—here we have Repulsion; and that which is presupposed is the same as that which presupposes,—here we have Attraction. The fact that each is in itself mere moment constitutes the transition of one into the other, negating itself in itself and positing itself as its own Other. The One as such is self-transcendence, and its only meaning is to posit itself as its Other, the Many; and the Many similarly is only collapse upon itself,—it posits itself as its Other, the One, and in it alone relates itself to itself; so that each continues itself in its Other; and thus we have already here self-transcendence (Repulsion) and self-positing-as-One (Attraction) present unseparated. That which is posited in a Repulsion and Attraction which are relative (that is, which presuppose immediate Ones existing determinately) is this, that each is in itself its own negation, and therefore the continuation of its self into its Other. The Repulsion of Ones having Determinate Being is the self-preservation of each one by means of the mutual repulsion of the others: thus (1) the other Ones are negated in it; this is the aspect of its Determinate Being, or Being-for-Other; this therefore is Attraction as being the ideality of the Ones:—and (2) the One is in itself, without relation to others; but "in-itself" passed long ago into "Being-for-self," and further the One in itself is by its determination that Becoming of Many.—The Attraction of determinately existing Ones is their ideality, and is the positing of the One, wherein as negation and production of the One it thus transcends itself, and, as positing of the One, is its own negative, or Repulsion.

Thus Being-for-Self has completed its development and has reached its result. The One as in infinite self-relation (infinite because it is the posited negation of negation) is mediation: itself, as its own absolute (that is, abstract) otherness (the Many), it repulses from itself; and, entering into a negative relation to this its not-Being, it transcends it and thus again is self-relation; and One is no more than this Becoming, in which the determination has vanished which states it to be Beginning, that is, posits it as immediate and having Being, and, further, as reconstituting itself, in the result, as One, a One equally immediate and exclusive. The process, which is the One, everywhere posits and contains it only as transcended. This transcendence first is determined only as relative, as a relation to other Determinate Being, which thus itself is a different Repulsion and Attraction; next, it shows itself as passing over into the infinite relation of mediation through negation of the external relations of Beings immediate and Determinate, having for result this Becoming which, through the instability of its moments, is the collapse, or rather self-gathering, into simple immediacy. Having acquired this new determination, this Being is Quantity.

If we review in brief the moments of this transition of Quality into Quantity, the qualitative has for fundamental determination Being and immediacy, where limit and determinateness are identical with the being of the Something in such a manner that, these being altered, the Something itself vanishes. Being posited in this manner it is determined as finite. By virtue of the immediacy of this unity, in which distinction has disappeared, though it is in itself present in the unity of Being and Nothing, the distinction becomes Otherness in general and is outside this unity. This relation to Other is in contradiction to the immediacy in which qualitative determinateness is self-relation. This Otherness transcends itself in the infinity of Being-for-Self; by the latter the distinction, which it has in and by itself in the negation of negation, is made real into the stages of One and Many and their relations: the qualitative is elevated to the unity which is true because it is no longer immediate but posited as in harmony with itself.

This Unity then is (α) Being, purely affirmative, that is, immediacy mediated with itself through the negation of negation; Being is posited as the unity which runs through each determinateness, through limit and so on; these are posited in it as transcended:

(β) Determinate Being; according to such a determination it is negation or determinateness as moment of affirmative Being; but it is such no longer as immediate, but as intro-reflected, related not to Other but to Self; it is Determinedness-in-itself, or One; Otherness as such is itself Being-for-Self:

(γ) Being-for-Self, as this Being which endures through determinateness; here One and Determinedness-in-itself are themselves posited as transcended. The One is now determined as having passed beyond itself and as Unity; and hence the

One, the absolutely determinate Limit, is posited as the Limit which is not Limit, and as indifferent to Being although it is posited in it.

Observation

Attraction and Repulsion are generally, of course, regarded as forces. This determination, and the relations connected with it, should be compared with the concepts which they have been found to be.—In the view mentioned they are regarded as independent, as if they were not related to one another by their own nature; that is, they are taken not as two moments passing each into its opposite, but rather as standing one against the other in fixed opposition. It is further imagined that they meet in a third element, namely, matter; but this takes place in such a manner that this unification is not held to be their truth: rather, each is a first principle and is for itself, while matter and its determinations are posited and created by them. When it is said that matter *contains* the forces, this unity is intended to mean a connexion, while at the same time they are presupposed as having independent self-existence.

Kant, as we know, has constructed matter from Repulsive and Attractive force, or, at least, he has erected (as he calls it) the metaphysical elements of this construction.—It will not be without some interest to cast light upon this construction. This *metaphysical* exposition of an object which, not only itself but also in its determinations, appeared to belong only to experience, is remarkable: first, because, as an essay of the concept, it furnished at least the impulse to the modern philosophy of nature, which does not make nature as a sense-datum the foundation of science, but proceeds from the absolute concept to the cognition of its determinations; next, because often a halt is made at this Kantian construction, and it is taken as the philosophic beginning and foundation of physics.

Now an existent such as empirical matter is not the object of Logic any more than are Space and the determinations of Space. But Repulsive and Attractive force, too, in so far as they are looked on as forces of empirical matter, are based upon the pure determinations of One and Many here considered, and on those relations of them which (because these names were most obvious) I have called Repulsion and Attraction.

Closely considered, Kant's method in the deduction of matter from these forces, which he calls a construction, does not deserve this name, unless indeed every kind of reflection, even analytical reflection, be called a construction; and, indeed, later natural philosophers have given the name of construction to the shallowest reasoning and the most baseless concoction of arbitrary imagination and thoughtless reflection—especially if it employed and everywhere displayed the so-called factors of attractive and repulsive force.

For at bottom Kant's method is analytical and not constructive. He presupposes the idea of matter, and next asks what forces are necessary to maintain its determinations thus assumed. Thus first he postulates Attractive force, because really matter cannot exist through Repulsion alone without Attraction (*Anfangsgründe der Naturwissenschaft*, pp. 53 *et seq.*). Next he derives Repulsion, too, from matter, and gives as reason the fact that we imagine matter as impenetrable, since it presents itself under this determination to the sense of feeling, by which it manifests itself to us. Consequently (he proceeds) Repulsion is immediately thought in the concept of matter, since it is immediately given therein; whereas Attraction is syllogistically added to it. But these syllogisms, too, are based on what has just been said, namely, that matter having only repulsive force does not exhaust our image of matter.—It is evident that this is the procedure of knowledge reflecting on experience, which first perceives determinations in the phenomenon, next makes them the basis, and finally assumes for their so-called explanation corresponding fundamental materials or forces which are supposed to produce these determinations of the phenomenon.

In view of the different fashion in which repulsive and attractive force are thus discovered by cognition in matter, Kant further observes that attractive force really equally belongs to the concept of matter, "although it is not contained in it." Kant lays emphasis on this last expression. Yet it is not clear what the distinction may be; for a determination which belongs to the concept of a thing must be truly contained in it.

What causes the difficulty and gives rise to this vain subterfuge is that Kant from the beginning arbitrarily counts impenetrability alone as proper to the concept of matter; this, he says, we perceive by the senses, and therefore repulsive force —the repelling of other from one—is immediately given. But if he goes on to say that matter cannot *exist* without attraction, then this assertion is based on a view of matter which is taken from experience; perception therefore must yield us the determination of attraction, too. Indeed it is easy to perceive that beyond its Being-for-Self, which transcends Being-for-Other (or offers resistance), matter contains a relation of entities which are for themselves, and thus has spatial extension and cohesion, which latter, in the qualities of rigidity and solidity, is very strong. Explanatory physics demands, in order (for instance) that a body may be torn apart, a force which must be stronger than the mutual attraction of the parts of that body. And from this truth reflection can just as immediately deduce attractive force, or take it as given, as it did with repulsive force. Indeed, if we consider Kant's syllogisms from which he tries to deduce attractive force (the proof of the proposition, that the possibility of matter demands attractive force as a second basic power, *loc. cit.*), they are found to contain nothing save that mere repulsion would not make matter spatial. When matter is presupposed as filling space, continuity is ascribed to it, and the basis of continuity is supposed to be attractive force.

But though this so-called construction of matter would at most have value as analysis (a value diminished by the impure exposition), still the fundamental thought which derives cognition of matter from these two opposed determinations as from its basic forces, must always be highly esteemed. Kant chiefly is at pains to banish the vulgar and mechanistic idea which stops at the one determination, namely, impenetrability, or self-existent puncticity, and makes into something external the opposite determination, the relation of matter in itself or in other matters, which in turn are regarded as separate Ones; a manner of presentation which, as Kant says, will allow no other motive forces except pressure and impact, that is, action from without. This externality of cognition always presupposes motion as already existing externally in matter, and never thinks of taking it as something internal and of comprehending it in matter, which thus is assumed in itself as motionless and inert. This point of view looks only on common mechanics, and not on immanent and free movement.—It is true that Kant transcends this externality in so far as Attraction (or the relation of one matter to another in so far as they are taken as separate, or else of matter in general, in its existence beyond itself) is with him made into a force of matter itself; still, on the other side, his two basic forces remain, within matter, opposed to one another, external and independent.

The independent distinction between these two factors, attributed to them from the point of view of this cognition, was void, and equally void every other distinction must show itself to be if it is made with regard to the determination of their content as something which ought to be fixed; for, as they were considered above in their truth, they are no more than moments, which pass into one another.—I will consider these further determinations of distinction as indicated by Kant.

Kant determines attractive force as a penetrative force, by means of which one piece of matter can act immediately upon parts of another even beyond the surface of contact; repulsive force, on the other hand, he determines as a superficial force, by means of which parts of matter can act upon one another only at the common surface of contact. The reason cited for this determination of the latter is as follows:—"Parts in contact with one another limit each the sphere of activity of the other, and repulsive force cannot move any more distant part except through those parts which intervene; an immediate action of one part of matter on another, passing straight through these, by means of forces of expansion (that is, repulsive forces), is impossible." (See ibid. *Erklärung und Zusätze*, p. 67.)

And here the reader must remember this: when "nearer" and "more distant" parts of matter are assumed, the distinction would also arise with regard to Attraction, in this way, that though one atom might act upon another, yet a third and more distant atom (between which and the attracting atom the "other" atom would be) would next enter into the sphere of attraction of the intermediate (and more proximate) atom; thus the action of the first atom on the third would not be immediate and simple. Thus mediated action would result for attractive as well as for repulsive force. Further, true penetration of attractive force would then have to consist solely in the fact that all parts of matter had power to attract in themselves, and not in the passivity of a certain number and the activity of just one atom.—With immediate regard to repulsive force itself we may remark that in the passage quoted there occur "parts in contact with one another"; which means a homogeneity and continuity of finished matter not admitting of repulsion. But this homogeneity of matter, where parts, no longer separated by void, are in contact with one another, already presupposes the transcendence of repulsive force; according to the sense-image of Repulsion which governs here, parts in contact must be taken as such as do not repel one another. The tautological conclusion therefore is that where the not-being of Repulsion is assumed there can take place no Repulsion. But nothing further follows from this by way of a determination of repulsive force.—If, however, it is considered that parts in contact touch one another only in so far as they still keep one another apart, then repulsive force no longer resides merely in the surface of matter, but within that sphere which was intended to be the sphere of Attraction only.

Next Kant assumes the determination that "through attractive force matter occupies but does not fill space" (ibid.); "and since matter does not through this force fill space, it can act across empty space, since there is no intervening matter which could limit it."—This distinction resembles the one mentioned above, where a determination is supposed to belong to but not to be contained in the concept of a thing; similarly here matter is to occupy space but not to fill it. It is then Repulsion (if we remain at its first determination) by means of which the Ones repulse one another and are related to one another only negatively, that is, here, across empty space. But here it is attractive force which keeps space empty; it does not fill space by means of the relation between atoms which here prevails; that is, it keeps the atoms in a negative relation to one another. —Thus Kant is unconsciously brought to do what is implicit in the nature of the case, that is, he ascribes to attractive force precisely what he ascribed (according to the first determination) to the opposite force. While the distinction between the two forces was being laid down, one had passed over into the other.—By Repulsion, on the other hand, matter is intended to fill space, that is, through it the empty space left over by Attractive force is intended to vanish. And in fact, in cancelling empty space it cancels the negative relation of the atoms or Ones, that is, the Repulsion which takes place between them; in other words, Repulsion is determined as its own opposite.

Thus the distinctions are blurred; and to this is added the confusion arising from the fact that (as was observed at the beginning) Kant's exposition of the opposed forces is analytic, and in the whole course thereof matter, which should be derived first from its elements, appears fully and completely constituted. In the definition of superficial and of penetrative force both are taken as motive forces, by means of which matter can act in one or in the other manner.—They are thus here set out as forces by which matter is not created but, being already given, is only moved. But in so far as we speak of forces by means of which matter acts upon matter, or moves itself, this is a very different thing from the determination and relation which they were supposed to have as moments of matter.

In a further determination centripetal and centrifugal force form the same antithesis as attractive and repulsive force. The former seem to offer an essential distinction, since in their sphere one One, or centre, is fixed, relatively to which other Ones behave as not self-existent, so that the distinction between the forces may be connected to this presupposed distinction between a central One and others which are not independent relatively to it. But in so far as they are used for explanation, for which purpose (like Repulsive and Attractive force) they are taken to be in inverse quantitative ratio, one increasing as the other decreases, the phenomenon of movement and its inequality, which they are accepted in order to explain, is supposed only to result from them. However, it is only necessary to examine any exposition of a phenomenon, like the unequal velocity of a planet in its orbit around the central body, to see that, when based on the opposition of these forces, it is full of confusion and that it is impossible to disentangle the magnitudes; so that the one, which in the exposition is assumed to be diminishing, may just as well be assumed to increase, and conversely. To make this evident would require an exposition more lengthy than can be offered here; what is essential occurs below, under the category of Inverse Ratio.

SECTION TWO

MAGNITUDE (QUANTITY)

We have indicated the difference between Quantity and Quality. Quality is primary and immediate determinateness; quantity is such determinateness as has become indifferent to Being. It is a limit which also is no limit, it is Being-for-Self which is simply identical with Being-for-Other, the Repulsion of many Ones which immediately is their Non-Repulsion or continuousness.

Now Being-for-Self has been so posited as not to exclude its Other but rather affirmatively to continue itself into it; hence it is Otherness (in so far as Determinate Being again arises in this continuity) and its determinateness simultaneously ceases to be simply self-related and is no longer the immediate determinateness of the determinately existing Something; instead, it is posited as self-repelling and as having its relation to itself, as determinateness, rather in some other Determinate Being (which is for itself); and, since also they exist as indifferent, intro-reflected, and relationless limits, determinateness is simply beyond itself; it is external to itself and is a Something as equally external; such a limit, together with the indifference of itself to itself and to Something, constitutes its *quantitative* determinateness.

Pure Quantity must here be distinguished from determinate quantity, or Quantum. As the former, Quantity is, first, real Being-for-Self, which has returned upon itself and as yet has no determinateness; as infinite, homogeneous unity which continues into itself.

Secondly, this passes over into a determinateness, which is posited of it as one which is none, or, rather, is only external. It becomes Quantum. Quantum is indifferent determinateness, that is, it passes beyond and negates itself; as this otherness of otherness it is caught in the infinite progress. Infinite Quantum, however, is the transcendence of indifferent determinateness, it is the rehabilitation of Quality.

Thirdly, Quantum in qualitative form is quantitative relation. Quantum merely passes beyond itself; in Relation it passes over into its otherness in such a way that the latter, which forms its determination, is posited simultaneously, and is another Quantum; we then find that it has returned to itself and that it is related to itself (namely in its otherness).

Further, this relation is based on the externality of Quantum; the quanta which are related (that is, whose self-relation consists in their being beyond themselves) are indifferent. Thus this relation is a merely formal unity of Quality and Quantity. The dialectic of this relation is its transition into their absolute unity, into Measure.

Observation

In the Something, its limit, as Quality, essentially is its determinateness. But if by limit we mean quantitative limit, and a field (for instance) changes this its limit, it remains what it was—field. Whereas if its qualitative limit is changed, then its determinateness, which makes it a field, is changed, and it becomes meadow, forest, and so on.—A red which becomes darker or lighter still is red; but if it were to change its quality, it would cease to be red and would become blue or some other colour.—This determination of magnitude as Quantum as deduced above—namely, an unchanging substratum of Being which is indifferent to its determinateness—can also be deduced from every other example.

By magnitude Quantum is meant, as in the examples cited, and not Quantity; this is the chief reason why this alien term must be borrowed.

The definition of magnitude which mathematicians employ also refers to Quantum. Magnitude is generally defined as something which may be increased or diminished. But to increase means to make something more great (*maius*), and to diminish to make it less great (*minus*). Thus there is a distinction between magnitude in general and itself, and magnitude appears as something of which the magnitude may be varied. The definition thus proves improper, since it employs the determination which it would define. In so far as the same determination must not be employed in it, "more" and "less" must be analysed into an affirmative addition (which, according to the nature of Quantum, is likewise external) and a subtraction (an equally external negation). Indeed, it is to this external kind both of Reality and of Negation that the nature of change determines itself in Quantum. The governing moment, therefore (which is the main matter) must not be mistaken in this incomplete expression; namely, the indifference to change, so that its own More and Less, its indifference to itself, lies in its very concept.

CHAPTER I

QUANTITY

A

PURE QUANTITY

Quantity is Being-for-Self transcended; the repelling One, which (as we saw) remains merely negative towards the excluded One, has passed over into relation to it, and, becoming identical with Other, has lost its determination; Being-for-Self has passed over into Attraction. The absolute brittleness of the repelling One has melted down into this unity; and this, since it contains this One and also is determined by the immanent Repulsion, is unity with itself as being unity with Being-beyond-self. Attraction thus is the moment of continuity in Quantity.

Continuity thus is simple and self-identical self-relation, interrupted by no limit or exclusion; not, however, an immediate unity, but a unity of the Ones which are for themselves. The externality of plurality is here still contained, but as something undifferentiated and uninterrupted. In continuity, plurality is posited as it is in itself; each of the many is what the others are, each is equal to the other, and hence plurality is simple and undifferentiated equality. Continuity is this moment of the self-identity of Being-external-to-one-another, the self-prolongation of different Ones into other and distinct Ones.

Thus Magnitude in continuity has the moment of discreteness immediately—Repulsion, which now in Quantity is bare moment.—Stability is self-identity, self-identity of the many which do not become exclusive; it is only Repulsion which expands self-identity into continuity. And thus on its side discreteness is coalescent discreteness, where the Ones are related not to the void (or negative) but to their own stability, and do not, in the Many, interrupt this self-identity.

Quantity is the unity of these moments, of continuity and discreteness; but at the outset it is so in the form of one of them, namely of continuity; and this is the result of the dialectic of Being-for-Self, which has collapsed into the form of self-identical immediacy. Quantity as such is just this simple result, in so far as its moments are not yet developed, nor posited in it.—At present, posited as Being-for-self as it truly is, it only contains them. It was determined as self-transcendent self-relation, a perennial egress beyond itself. But that which it repels is itself; and thus Repulsion is its own creative onward flow. Because of this identity with that which is repulsed, this discreteness is an uninterrupted continuity; and because it passes beyond itself, this continuity, without being interrupted, is at the same time plurality, which equally immediately remains in its identity with itself.

Observation 1

Pure Quantity as yet has no limit, or in other words is not yet Quantum; and even in so far as it becomes Quantum, limit does not bar it in; it rather consists in the fact that limit does not bar it in, and that it contains Being-for-Self as transcended. Discreteness is moment in it, which may be expressed by saying that Quantity always and everywhere is the real potentiality of One, but that also conversely One is equally absolutely continuous.

Imagination operating without concept easily changes Continuity into Combination, that is, into an external relation of the Ones to one another, the One remaining in its absolute and brittle exclusiveness. But we saw when examining the One that, in its own true nature, it passes over into its ideality, which is Attraction, and that thus continuity is not external but peculiar to it, and founded in its essence. It is this externality of continuity for the One in which atomism remains entangled; it is this which imagination finds it so hard to leave.—On the other hand, mathematics rejects a metaphysic which should be content to allow time to consist of points of time, space in general (or, as a first step, the line) of points in space, the plane, of lines, and the whole of space, of planes; it allows no validity to such discontinuous Ones. And although it determines, for instance, the magnitude of a plane as consisting of the sum of an infinity of lines, yet this discreteness is taken only as a momentary image; and the infinite plurality of lines implies, since the space which they are meant to constitute is after all limited, that their discreteness has already been transcended.

It is the concept of pure Quantity as opposed to the mere image of it that Spinoza (to whom it was of especial importance) has in mind when he speaks of Quantity as follows (Eth. P. I. Prop. XV, Schol.):—

> Quantitas duobus modis a nobis concipitur, abstracte scilicet sive superficialiter, prout nempe ipsam imaginamur; vel ut substantia, quod a solo intellectu fit. Si itaque ad quantitatem attendimus, prout in imaginatione est, quod saepe et facilius a nobis fit, reperietur finita, divisibilis, et ex partibus conflata, si autem ad ipsam, prout in intellectu est, attendimus, et eam, quatenus substantia est, concipimus, quod difficillime fit,—infinita, unica, et indivisibilis reperietur. Quod omnibus, qui inter imaginationem et intellectum distinguere sciverint, satis manifestum erit.

Should more definite examples of pure Quantity be demanded, Space and Time, Matter in general, Light, and so forth, the Ego itself, will furnish them; only by Quantity we must not understand (as was remarked above) Quantum. Space, Time, and so on, are expansions or pluralities where egress beyond self, and a constant flow, take place; though what is reached is not the opposite (Quality, or the One), but, passing beyond themselves, they are engaged upon a perpetual self-production of their unity.

Space is such an absolute being-beyond-self; it is also absolutely uninterrupted, a still-repeated otherness which is identical with itself; and Time is an absolute passing-beyond-self, a creation of the One (the point of time or Now), which immediately becomes the annihilation of it, and also the perpetual annihilation of this destruction: so that this self-creation of Not-being is also simple self-equality and self-identity.—As regards Matter as Quantity, among the seven Prepositions surviving of the first dissertation of Leibniz there is one (the second) which deals with this (left-hand page of Part One of his works); it runs thus: *Non omnino improbabile est, materiam et quantitatem esse realiter idem.*—And, indeed, there is no difference between these concepts, save that Quantity is a pure determination of thought, while Matter is the same as existing externally.—The Ego, too, is determined as pure Quantity, since it is an absolute alienation, an infinite removal or universal repulsion towards the negative freedom of Being-for-Self, though it remains absolutely simple continuity—the continuity of universality or Being-with-Self, which is not interrupted by the infinite variety of limits, the content of sensations, intuitions, and so forth.—Those who refuse to take plurality as simple unity, and want an image of this unity, apart from the concept (namely, that each of the Many is the same as every other, that is, One of the Many,—since we are not here speaking of many further determined, of green, red, and so on, but of the Many considered in itself), will find plenty of examples among those static existences which offer as present the deduced concept of Quantity in simple intuition.

Observation 2

It is the nature of Quantity to be this simple unity of Discreteness and Continuity; and to this appertains the conflict or Antinomy of the infinite divisibility of Space, of Time, of Matter, and so on.

This antinomy consists solely in the necessity of asserting Discreteness as much as Continuity. The one-sided assertion of Discreteness gives an infinite or absolute division (and thus something indivisible) for principle; and the one-sided assertion of Continuity, infinite divisibility.

Kant's Critique of Pure Reason sets up, as is well known, four (cosmological) Antinomies, the second of which deals with the opposition which is constituted by the moments of Quantity.

These Kantian antinomies still remain an important part of the critical philosophy; they, principally, effected the fall of the previous metaphysics, and may be looked on as a chief transition to modern philosophy; for they in particular assisted to produce a conviction of the invalidity of the categories of finitude by examining their content;—and this is a more correct method than that formal method of a subjective idealism according to which their only fault is supposed to be that they are subjective, and not that which they are in themselves. But, great as are its merits, this exposition is very incomplete, being in part self-hampered and cross-grained, and in part faulty in view of its result which assumes that cognition has no other form of thought than finite categories.—In both respects these antinomies deserve a more exact criticism, as well investigating their standpoint and method more closely as liberating the main and essential point from the useless mould into which it has been forced.

First I remark that Kant tried to give an appearance of completeness to his four cosmological antinomies by his method of classification, which he borrowed from his scheme of categories. A deeper insight into the antinomous or, rather, into the dialectic nature of Reason shows, however, that *every* concept is a unity of opposite moments, which could therefore be asserted in the shape of an antinomy. Thus, Becoming, Determinate Being, and so on, and other concepts, could each furnish its particular antinomy, and as many antinomies could be set up as concepts were yielded.—The old scepticism did not shrink from the labour of demonstrating this contradiction or antinomy in every concept which it found in the sciences.

Next, Kant took the antinomy not in the concepts themselves, but in the already concrete form of cosmological determinations. In order to have a pure antinomy which might be dealt with in its simple concept, it was necessary not to take the determinations of thought in their application to and admixture with the ideas of World, Space, Time, Matter, and so on, but to consider them purely for themselves without this concrete material, which here has no force nor avail; for these determinations alone constitute the essence and base of the antinomies.

Kant offers this concept of the antinomies, that they are no tricks of sophistry, but contradictions which Reason must necessarily hit on (as Kant puts it): a view which is important. —"The natural appearance of the antinomies no longer actively deludes Reason when it perceives its base; yet still Reason is deceived by it."—Naturally; for the critical solution by means of the so-called transcendental ideality of the world of perception has no other result than this, that it makes the so-called conflict into something subjective; and there, of course, it remains appearance, that is, it remains unresolved, as much as before. The true solution can only be this, that two determinations, being contradictory, and yet necessary to the same concept, cannot be valid each by itself in its one-sidedness, but have their truth only in their transcendence, in the unity of their concept.

More closely considered, the Kantian antinomies contain no more than the simple categorical assertion of each one of the two opposed moments of a determination, apart from the other. But while this is done, the simple categorical or rather assertorical statement is enveloped in a scaffolding of reasoning, awry and crooked, intended to produce a show of proofs and to hide and disguise its merely assertorical character. This will become clear on closer examination.

The antinomy proper to this place is that of the so-called infinite divisibility of matter, and is based on the opposition between the moments of continuity and discreteness contained in the concept of Quantity.

In Kant's exposition the Thesis is as follows:—

"Every composite substance in the world consists of simple parts, and there exists everywhere nothing but what is simple or is composed of simple parts."

Here to the simple, the atom, is opposed the composite, which as compared with the static or continuous is a rudimentary determination.—The substratum given to these abstractions (namely, substances in the world) means no more than things as perceived by the senses, and is irrelevant to the antinomy; Space or Time might equally well have been taken. —Since now the thesis deals only with compositeness instead of continuity, it is really an analytical or tautological proposition. That the composite is not One in itself, but is only something put together externally and consisting of Other, is its immediate determination. But the Other of the composite is the simple. It is therefore a tautology to say that the composite consists of the simple.—If it is asked of what something consists, it is demanded that something other be indicated, the compounding of which constitutes this something. If ink is to be made up just of ink, then the meaning of the question which asks of what other bodies it consists is missed and, not being answered, merely repeats itself. It is then a further question whether the object of enquiry is to consist of something or not. But the composite is just that which is supposed to be a conglomeration and to consist of others.—If the simple (which is allowed to be Other to the composite) is taken as merely relatively simple, but again composite in itself, then the question still remains. Imagination has before it only this or that concrete composite, of which this or that Something might be indicated as its simple unit, though composite in itself. But here we are speaking of the composite as such.

As regards Kant's proof of the Thesis, it—like all Kant's proofs of the remaining antinomous propositions—makes the detour (which will prove highly superfluous) of being apagogic.

"Assume" (he begins) "that the composite substances do not consist of simple parts; then, if all composition were mentally removed, there would be no composite part; now we have assumed that there are no simple parts; there is therefore no simple part—and hence nothing at all—left; hence no substance is given us."—

This conclusion is quite correct: if there is only composite and we imagine everything composite removed, then nothing remains; we may grant this, but this tautological superfluity might be spared, and the proof might begin with what follows, namely:—

"Either all composition cannot be mentally removed, or, it being removed, something subsisting without composition—namely, the simple—must remain."

"But in the first case again the composite would not consist of substances (for with these compositeness is only a contingent relation of the substances,[1] and they must be capable of existing as persistent beings without it).—But this case is contradictory to the hypothesis, and therefore only the second remains: namely, that all composite substance consists of simple parts."

The reason which is the main point, compared with which all that has been said is quite superfluous, is relegated to a parenthesis. The dilemma is this:—That which persists is either the composite or else the simple. If the first (the composite) were that which persists, then that which persists is not the substances, for in these compositeness is a contingent relation; but the substances are that which persists, therefore that which persists is the simple.

It is evident that the apagogic detour might be avoided, and that to the Thesis ("Composite substance consists of simple parts") this reason might immediately be added as proof, namely that compositeness is merely a contingent relation of the substances, external to and not concerned with them.—If compositeness is in truth contingent, then the essence is of course the simple. But this contingency, which is the sole point at issue, is not proved but simply assumed (in parenthesis) as though it were obvious or beside the point. It is, of course, self-evident that compositeness is the determination of contingency and externality; but, if we were only about to deal with a contingent juxtaposition instead of continuity, it was not worth while to set up an antinomy about it, or, rather, it was impossible; in that case it is mere tautology (as was said) to assert that the parts are simple.

---

[1] Here redundance of language is added to redundance in proof,—"for with *these* (to wit, the substances) compositeness is only a contingent relation of *the substances*."

Thus in the apagogic detour we see that the very assertion which is to be its result already occurs. Thus the proof might more briefly be put as follows:—

Let it be assumed that substances do not consist of simple parts, but are composite. Now all composition can be thought away (since it is a merely contingent relation); it being removed, therefore, no substances remain unless they consist of simple parts. But we must have substances, since we assumed their existence; everything must not vanish, something must remain; for we have presupposed that some such persistent entity (which we called substance) exists. Therefore this something must be simple.

To complete matters, we must consider the conclusion, which runs as follows:—

"Hence it follows immediately that all the things in the world are simple essences, that compositeness is a condition merely external to them, and that reason must consider the elementary substances to be simple essences."

Here the external, that is, the contingent, nature of compositeness is cited as a consequence, after having already been introduced parenthetically and used in the proof.

Kant protests much that he is not looking for sophisms in the conflicting propositions of the antinomy, in order to effect (as the phrase is) an advocate's proof. Indeed, this proof is not really to be accused of sophistry so much as of a forced and useless tortuosity, which serves only to produce the outer form of a proof, not allowing it to be perfectly transparent that that which should stand out as conclusion is the (parenthetic) hinge of the proof, and that there is no proof but only an assumption.

The Antithesis is as follows:—

"No composite thing in the world consists of simple parts, and nothing simple anywhere exists in it."

The proof again has the apagogic turn, and in another manner is as faulty as the previous.

"Assume," it runs, "that a composite thing as substance consists of simple parts. Now all external relation (and therefore also the composition of substances) is possible only in space, and therefore the space which it occupies must consist of as many parts as those of which the composite thing consists. Now space does not consist of simple parts, but of spaces. Therefore each part of the composite thing must occupy one space."

"But the absolutely first parts of every composite thing are simple."

"Hence the simple thing occupies a space."

"Now every reality which occupies a space comprehends a manifold of mutual externalities; it is therefore composed (of substances), and thus the simple would be a substantial composite, which is self-contradictory."

This proof may be called a perfect nest (to use an expression which is elsewhere employed by Kant) of faulty procedure.

First, the apagogic turn is a baseless fabric. The assumption that whatever is substantial is spatial, while space does not consist of simple parts, is a direct assertion which is made the immediate foundation of that which is to be proved; after this there is no need for further proof.

Next, this apagogic proof begins with the proposition "that all composition of substances is an external relation," but, strangely enough, immediately forgets it. For the further conclusion is made that composition can take place only in space, that space does not consist of simple parts, and that therefore the reality occupying a space is composite. If once compositeness has been taken as an external relation, then spatiality itself (since composition is supposed to be possible in it alone) is for that very reason a relation external to the substances, which does not concern them nor touch their nature any more than does all the rest that can be inferred from the determination of spatiality. For this very reason the substances should not be placed in space.

Further, it is assumed that the space into which the substances are here transplanted does not consist of simple parts; since it is an intuition (that is, according to Kant's determination, a sensuous representation) which can be given only by one single object, and is not a so-called discursive concept.—This distinction of Kant's between intuition and concept has, of course, led to the abuse of the former, and, in order to save the labour of achieving the latter, its value and sphere have been extended to all cognition. What is relevant here is this, that space, like intuition itself, must be conceived if we would have any concept at all. And thus the question would arise whether space, though simple continuity for intuition, might not conceptually have to be taken as composed of simple parts, since otherwise space would be affected by that same antinomy into which substance only was transplanted. Indeed, if the antinomy is taken in the abstract, it extends to all Quantity (as was said above) and therefore also to Space and Time.

Now in the proof it is assumed that space does not consist of simple parts; and this should have been sufficient reason for not transplanting the simple into this element, which is inadequate to the determination of the simple.—Here also the continuity of space comes into collision with compositeness; these two are confused, and the former is inserted in the place of the latter (which results in a *Quaternio terminorum* in the conclusion). Kant expressly determines space as being *one*, its parts being based only on limitations, so that they do not precede space, sole and all-comprehending, as its component parts from which it could be put together. (*Critique of Pure Reason*, Second Ed., p. 39.) Here continuity is predicated of space, very justly and definitely, in opposition to its compositeness out of parts. In the proof, on the other hand, the transplanting of the substances into space is to involve "a manifold of mutual externalities," which, further, is "therefore composite." Yet we have quoted what shows that the manner in which a manifold exists in space is expressly designed to exclude compositeness and parts which precede its unity.

In the Note to the proof of the Antithesis, the other fundamental notion of the critical philosophy is further expressly adduced, namely, that we have a *concept* of bodies only as *phenomena*, which, as such, necessarily presuppose space as the condition of the possibility of all external manifestation. If by substances mere bodies are meant, such as are seen, felt, tasted, and so forth, then there is really no question of their conceptual meaning: sense-perception only is in question. The proof of the Antithesis might then be put in brief thus:—All visual, sensory, and other experience shows us only what is composite; the best microscopes and the keenest knives have never yet allowed us to hit on anything simple. Therefore Reason, too, should not want to hit on anything simple.

And thus when now we regard more closely this opposition of Thesis and Antithesis, liberating their proofs of all idle superfluity and tortuousness, we find that the proof of the Antithesis, by transplanting the substances into space, contains a dogmatic assumption of Continuity, and that the proof of the Thesis, by assuming Compositeness as the class of relation subsistent between the substances, contains the dogmatic assumption of the contingency of this relation—that is, the assumption that the substances are absolute Ones. Thus the whole antinomy reduces itself to the separation and direct assertion of the two moments of Quantity as being absolutely separate. Looked at from the point of view of mere discreteness, substance, matter, space and time, and so on, are absolutely divided, and the One is their principle. From the point of view of continuity, this One is merely suspended: division remains divisibility, the possibility of dividing remains as possibility, without ever actually reaching the atom. Now even if we do not move beyond the determination implied in what has been said about these contradictions, still continuity itself contains the moment of the atom, since continuity exists simply as the possibility of division; just as accomplished division, or discreteness, cancels all distinction between the Ones—for each simple One is what every other is,—and for that very reason contains their equality and therefore their continuity. Each of the two opposed sides contains its other in itself, and neither can be thought of without the other; and thus it follows that, taken alone, neither determination has truth, but only their unity. This is the true dialectic consideration of them, and the true result.

The dialectic examples of the old Eleatic school, especially those which deal with motion, are incomparably deeper and richer in meaning than the antinomy of Kant which we have considered. They, too, are based on the concept of Quantity, and in it have their solution. To consider them here would lead too far; they concern the concepts of Space and Time, and can be dealt with in connexion with these subjects and with the history of philosophy.—They do the highest honour to the reason of their inventors; their result is the pure Being of Parmenides, since they demonstrate the solution into itself of all determinate being: in themselves, therefore, they are the Flux of Herakleitos. They therefore deserve a fuller consideration than the ordinary explanation that they are just sophisms; —an assertion which clings to empirical perception, following that method of Diogenes (so convincing to common sense), who, when a dialectician demonstrated the contradiction contained in motion, is said to have put no further strain on his reason, but, by mutely walking up and down, to have referred to the evidence of the eyes;—an assertion and a refutation which of course it is easier to make than to enter upon thought, to seize the confusions into which thought, quite unforced, leads when it formulates itself in ordinary consciousness, and to solve them by means of thought.

The solution of these dialectic formations which Aristotle effects is highly to be praised: it is implied in his truly speculative concepts of Space, Time, and Motion. Infinite divisibility (which, being imagined as actually completed, is equivalent to infinite division, or the atoms), as being the principle on which are based the most famous of these proofs, is by him opposed to Continuity (of Time as well as of Space), in such a manner that infinite—that is, abstract—plurality is contained in continuity only in itself, or potentially. The actual, as opposed to abstract plurality (or to abstract continuity), is their concrete form, it is Time and Space itself; and these in turn are opposed to Motion and Matter. The abstract exists only in itself, or potentially; it exists only as a moment of the Real. Bayle in his *Dictionnaire* (Article, Zenon) considers Aristotle's solution of Zeno's dialectic "*pitoyable*": he does not understand the meaning of the *potentially* infinite divisibility of matter. He replies that if matter is infinitely divisible, it actually contains an infinite number of parts, so that this is not an infinite *en puissance* but an infinite really and actually existing.—No: divisibility itself is only a potentiality, not an *existence* of parts; and in any case, in Continuity, plurality is posited only as a moment already transcended.—Keen understanding—though in this, too, probably Aristotle has not been excelled—is not sufficient to seize and judge his speculative concepts; nor is the clumsy method of sensuous representation which we cited sufficient to refute such arguments as Zeno used. Such an understanding makes the mistake of taking these objects of thought or abstraction, such as an infinite number of parts, for Something, for true and actual; and this sensuous consciousness will never let itself be raised from the sphere of the empirical to that of Thought.

Kant's solution of the antinomy, too, consists only in demanding that Reason must not rise above sense-perception, but must take the phenomenon as it is. This solution leaves on one side the content of the antinomy; it does not reach the nature of the concept of its determinations, each of which, isolated by itself, is null, being no more than a transition to its Other, while Quantity is their unity and therefore their truth.

B

CONTINUOUS AND DISCRETE MAGNITUDE

1. Quantity contains the two moments of Continuity and Discreteness. Each is its determination, and it must therefore be posited in each.—Primarily it is their immediate unity, that is, primarily it is posited only in one of its determinations, namely, continuity; and thus is Continuous Magnitude.

Or again, continuity is one of the moments of Quantity which requires the other moment, discreteness, to complete it. But still Quantity is concrete unity only in so far as it is the unity of *distinct* moments. The moments therefore must be taken as distinct, but must not be resolved back into Attraction and Repulsion; they must be taken in their truth, as each forming the whole in its unity with the other. Continuity is only coherent and homogeneous unity as unity of discrete elements, and, posited thus, it is no longer mere moment but complete Quantity: this is Continuous Magnitude.

2. Immediate quantity is Continuous Magnitude. But in fact Quantity is not immediate; immediacy is a determinateness, and Quantity itself is this immediacy transcended. It must therefore be posited in the determinateness immanent in it, and this is the One. Quantity is Discrete Magnitude.

Discreteness, like continuity, is a moment of Quantity, but is also the whole of Quantity just because it is a moment of this whole, and thus, as distinct from it, does not relinquish its unity with the other moment.—Quantity is externality in itself, and Continuous Magnitude is this externality as propagating itself without negation, as a context which remains at one in itself. Discrete Magnitude is this externality as non-continuous or interrupted. But though we thus have a multitude of Ones, this is not the multitude of atoms and the void, or Repulsion in general, meeting us once again: Discrete Magnitude is Quantity; and for that very reason their discreteness is continuous. The continuity in discreteness consists in the fact that the Ones are equal to one another, or have the same unity. Discrete Magnitude, then, is the externality of much One posited as the same, and not of the many Ones in general; it is posited as the Many of one unity.

Observation

Ordinarily, when an image is formed of Continuous and of Discrete Magnitude, it is overlooked that each of these magnitudes has both the moments, continuity and discreteness, and that their difference is only constituted by which of the two moments is *posited* determinateness, and which that which only is in itself. Space, Time, Matter, and so on, are fixed magnitudes because each is a repulsion from itself, where there is a stream of extrogression without any transition or relation to what is qualitatively Other. They have the absolute potentiality of the One being posited relatively to them, and not merely as the empty potentiality of a mere otherness (as we say that a tree might occupy the space occupied by a stone): they contain in themselves the principle of the One, it is one of the determinations which constitute them.

Conversely, continuity must not be overlooked in Discrete Magnitude: this moment, as has been shown, is the One as unity.

Continuous and Discrete Magnitude may be considered as species of Quantity, but only in so far as magnitude is posited not under any external determinateness, but solely under the determinatenesses of its own moments; the ordinary transition from genus to species allows external determinations to be applied to these according to some external principle of classification. And so far Continuous and Discrete Magnitude are not yet Quanta: they are Quantity itself in each one of its two forms. If they are called magnitude, this is because they have this in common with Quantum, to be a determinateness applied to Quantity.

C

LIMITATION OF QUANTITY

Discrete Magnitude, first, has the One for principle; secondly, it is a plurality of Ones; thirdly, it is, essentially, static; it is One, but One transcended, or as unity; it is self-continuation as such in the discreteness of the Ones. It is therefore posited as one magnitude, and its determinateness is the One, which, in this positedness and determinate being is exclusive One, or Limit, applied to unity. Discrete Magnitude as such is not to be limited immediately; but, as distinct from Continuous Magnitude, it exists as a Determinate Being and a Something whose determinateness is One, and, as being in a Determinate Being, is also first negation and Limit.

This Limit is referred to unity and applied to it as negation, and, besides this, as One is also referred to itself; it is thus including and comprehending Limit. Here the Limit is not distinguished from the Something of its Determinate Being, but, as One, is immediately this negative point itself. But the Being which here is limited exists essentially as continuity, by virtue of which it passes beyond the Limit and beyond this One, and is indifferent to them. Real discrete Quantity is therefore *one* Quantity, or Quantum,—Quantity as Determinate Being and Something.

Since the One, which is Limit, comprehends in itself the many Ones of discrete Quantity, it in so doing also posits them as transcended in itself. It is Limit as applied to continuity in general as such, and hence the distinction between Continuous and Discrete Magnitude is here immaterial; or, rather, it is Limit to the continuity of the one as much as of the other: both pass over into the state of Quantum.

CHAPTER II

QUANTUM

QUANTUM—which, *first*, means Quantity having any determinateness or limit at all—is, in its complete determinateness, Number. It is divided into

secondly (*a*) Extensive Quantum, where the Limit is the Barrier of a plurality there existing; (*b*) Intensive Quantum or Degree, the plurality having passed over into being-for-self. This Barrier being for itself, and therefore as Limit indifferent, is equally immediately outside itself and has its determinateness in an Other. Thus Quantum is a posited contradiction, it is simply determined for itself and also has its determinateness outside itself and thus refers beyond itself; it therefore

thirdly passes over, as that which is itself posited as external to itself, into Quantitative Infinity.

A

NUMBER

Quantity is Quantum; it has a Limit, whether it be taken as continuous or as discrete magnitude. The distinction between these two species is here of no importance.

As transcended Being-for-Self, Quantity in itself already is indifferent to its Limit. For that very reason Limit (or the fact that it is a Quantum) is not indifferent to it; for it contains the One, or absolute determinateness, in itself as its own moment, which thus posited in its continuity or oneness is its Limit, but yet remains that One which it has become.

This One is thus the principle of Quantum; but it is the One of Quantity. Hence, first, it is continuous, or Unity; secondly, it is discrete, plurality of Ones which is in itself (as in continuous magnitude) or posited (as in discrete magnitude): these Ones are equal to one another, they have this continuity, this same unity. Thirdly, this One is also negation of the many Ones as simple Limit, an exclusion of otherness, or a determination of itself in opposition to other Quanta. One is thus Limit (α) referring to itself, (β) inclusive, and (γ) exclusive of Other.

Quantum completely posited in these determinations is Number. Complete positedness is implied by the existence of the Limit as plurality and its consequent distinctness from unity. Hence Number appears as discrete magnitude, but in unity it also has continuity. Hence too it is Quantum in complete determinateness: for in it Limit is determinate plurality, having for principle One, the absolutely determinate. Continuity, where One is only in itself or as transcended (posited as unity)—is the form of indeterminateness.

Quantum, merely as such, is limited generally; its limit is its abstract and simple determinateness. But since it is Number, the Limit is posited as being manifold within itself. It contains the many Ones which constitute its Determinate Being, but not in a merely indeterminate manner: the determinateness of the Limit falls within it. The Limit excludes other Determinate Being, that is, other Manies: the Ones included in it are a determinate heap, an Amount. This is discreteness as it is in Number, and hence its other is Unity, the continuity of this. Amount and Unit constitute the moments of Number.

With regard to Amount, it must further be considered in what manner the many Ones of which it consists are in the Limit; it is correct to say of Amount that it *consists* of the Many, for in it the Ones do not exist as transcended, but in it they *are*, posited only with the excluding Limit, to which they are indifferent. The Limit is not, however, indifferent to them. The relation of Determinate Being to Limit (as was seen when that category was treated) was such that Determinate Being, as the affirmative, remained on this side of its Limit, while the Limit (or negation) remained outside, at the edge; similarly, with the many Ones, the abrupt exclusion of other Ones appears as a determination falling without the included Ones. But it was seen above that Determinate Being is penetrated by Limit and is coextensive with it, and that thus something is inherently limited, that is, finite.—Thus Number—which is Quantitative—is imagined in such a way that with one hundred, for instance, the hundredth One alone shall limit the Many in such a manner as to make them one hundred.

This is correct; yet among the hundred Ones none is preferred individually, since all are equal: each equally is the hundredth; they all equally belong to the Limit which makes the number one hundred, and, for the determinateness of the number, not one can be spared. Thus the others do not constitute relatively to the hundredth One a Determinate Being external to the limit, or distinct from it within it. Amount then is not a plurality as opposed to the inclusive and limiting One, but rather itself constitutes the limitation which is a determinate Quantum: the Many constitute one number, *One* two, *One* ten, *One* hundred, and so forth.

Now the limiting One is determinateness against Other, the distinguishing of one number from another. But this distinguishing does not become qualitative determinateness; it remains quantitative and appertains only to external comparing reflection: Number, as One, has returned upon itself, and at this resting-place it remains indifferent to Others. This indifference of Number to Others is its essential determination: by virtue of it, Number is determinate-in-itself, but also in itself external.—It is thus a numerical One, as absolutely determinate; it has the form of simple immediacy, and therefore relation to Other is quite external to it. And as being One which is Number, it further has determinateness (in so far as it is relation to Other) as its moments in itself, in its distinction between Unit and Amount; and Amount itself is plurality of Ones, that is, in itself it is this absolute externality.—This contradiction of Number, or of Quantum in general, in itself, is the quality of Quantum: this contradiction develops itself in the further determinations of the latter.

Observation 1

Spatial and numerical magnitude are generally considered as two species, each being a determinate magnitude as much as the other; and it is thought that, the distinction being only in the different determinations of continuity and discreteness, they rank equally as Quantum. Geometry, in spatial magnitude, generally has continuous magnitude for object; and arithmetic, in numerical, has discrete magnitude for object. But, the objects being thus different, they are not limited or determined in the same manner or with the same completeness. Spatial magnitude is merely limited in the abstract; if it must be considered as a definitely determinate Quantum, Number is required. Geometry as such does not *measure* spatial figures —it is not the art of measuring: it only compares. In its definitions too the determinations partly are derived from equality of sides and angles, or from equidistance. Thus the circle, since it is based solely upon the equidistance of all possible points in it from a centre, can be determined without Number. These determinations, based on equality and inequality, are true geometry. But they are not sufficient, and others, such as triangle or quadrilateral, require Number, which in its principle, the One, contains determinedness-for-self, not determinedness by the aid of an Other, hence not by comparison. It is true that, in the point, spatial magnitude has a determinateness corresponding to the One; but the point, in so far as it passes out of itself, becomes an Other, becomes the line; and, being essentially the One of Space, it becomes, when related, a continuity, in which its puncticity (or determinedness-for-self, or the One) is transcended. In so far as determinedness-for-self is to preserve itself in self-externality, the line must be imagined as a multitude of Ones, and the Limit must contain the determination of *many* Ones; that is, the magnitude of the line—like other spatial determinations —must be taken as Number.

Arithmetic contemplates Number and its figures; or, rather, it operates with them and does not contemplate. For Number is indifferent determinateness and inert; it must be actuated and brought into relation from without. The different methods of relation are the species of calculation. In arithmetic they are enumerated in series, and it is evident that they are mutually dependent. But arithmetic does not give prominence to the thread which guides their progress. However, the systematic arrangement which the exposition of these elements in the text-books justly claims, results easily from the conceptual determination of Number itself. We will here briefly notice these cardinal determinations.

The principle of Number is the One, and for this reason it is a purely analytic figure; its elements are connected externally, not internally. It is thus only an external creation; for which reason, too, all arithmetical operations are the production of numbers,—numbering, or, more closely, adding together. Variations in this external production, which ever proceeds in the same manner, can depend only on relative diversity between the numbers which are to be added together; such diversity itself must come from another source, and from external determination.

The qualitative distinction which constitutes the determinateness of Number is, as we have seen, that which subsists between Unit and Amount; hence all conceptual determinateness which can arise in the species of calculation reduces itself to this. But the distinction proper to numbers as Quanta is external identity and external difference, or equality and inequality; and these are moments of reflection, and must be dealt with when we deal with the determinations of Essence (under the heading of Difference).

And further we must premise that numbers can in general be produced in two ways—by joining, or by disjoining what was already joined. Each takes place in a manner of numbering determined in the same way; and thus to the joining of numbers there corresponds what may be called a positive kind of arithmetic; and to disjoining, a negative; the determination of each species of calculation itself is independent of this contrast.

1. We proceed to indicate the species. The first creation of Number is the joining together of Manies as such, that is, such as are posited each as *One* only,—Numbering. The Ones are external to one another, and are thus imagined under a sense-image: the operation which creates the number becomes a process of counting fingers, dots, and so forth. We can only *point* to an example of four, or of five, and so on. And the point where the process of joining is broken is (since the Limit is external) contingent and arbitrary.—The distinction between Amount and Unit which arises in the process of the species of calculation, becomes the basis of a system of numbers, such as the dyadic, decadic, and so on; in general such system rests on the choice of the amount which is consistently to be taken as unit.

The numbers produced by numbering are once more numbered: posited thus immediately they are determined as mutually quite unrelated, and indifferent to equality and inequality; their relative magnitude is contingent; they are hence altogether unequal. This is Addition. We discover that 7 and 5 make twelve by adding (by counting on our fingers or otherwise) 5 more Ones to the 7; the result is afterwards learnt by heart: but there is nothing internal here. We similarly learn that $7 \times 5 = 35$ by counting on the fingers, and so forth, one seven being added to another, this process being gone through five times, and the result again being learnt by heart. The trouble of numbering, the discovering of sums or products, is abolished by the finished "one and one" or multiplication-tables, which have only to be learnt by heart.

In the Introduction to the Critique of Pure Reason, V, Kant has considered the proposition that $7+5=12$ as a synthetic proposition. "At first," he says, "one should think [of course!] that it is merely an analytical proposition, resulting from the concept of a sum of seven and five by the Law of Contradiction." The concept of a sum is just this abstract determination that these two numbers *ought* to be joined together, and that too (since they are numbers) in an external and conceptless manner,—that we are to count on from seven until the Ones which are to be added (their amount being determined as five) are exhausted; the result bears the name independently known as twelve. "But," so Kant continues, "if we look more closely, we find that the concept of the sum of seven and five contains no more than the joining of these two numbers into one; it is not thought at all what this single number is which comprehends these two;" "—however much I may analyse my concept of such a possible sum, I shall never find twelve therein." The transition from the problem to the result has indeed nothing to do with the thinking of the sum and the analysis of the concept; and he adds, "We must pass beyond these concepts and take to intuition, to five fingers and so on, adding the five ones given by *intuition* to the *concept* of seven." Of course five is given intuitively, that is, it is an entirely external agglomeration of the thought of one, arbitrarily repeated; but equally seven itself is no concept; there are no concepts beyond which a transition here is made. The sum of 5 and 7 is the conceptless connexion of these two numbers, and the conceptless process of numbering from seven on, until five are exhausted, can literally be called a synthesis, which name can also be given to the process of numbering from one onwards;—but this synthesis is wholly analytical in nature, for the connexion is quite artificial and nothing is introduced that is not at hand externally. The postulate of adding 5 to 7 is to the postulate of numbering at all as is the postulate of producing a straight line to that of drawing a straight line.

The expression "synthesis" is void, and equally void is the determination that it takes place *a priori*. Counting is not, of course, a determination of sensation (which, according to Kant's determination of intuition, is all that remains to be predicated of whatever is *a posteriori*), and it is an occupation on the basis of abstract intuition: that is, it is determined through the category of the One, and abstraction is here made from all other determinations of sensation, as well as from concepts. Altogether, the *a priori* is quite vague; and determination of feeling, in the shape of impulse, tendency, and so forth, contains the moment of apriority as much as space and time are determined as existing, and the temporal and the spatial are determined, *a posteriori*.

We may add in this connexion that Kant's assertion of the synthetic nature of the foundations of pure geometry equally lacks solidity. He asserts that several are really analytic, but cites in favour of this idea only the proposition that the straight line is the shortest between any two points. "My concept of the straight does not contain anything about magnitude, but only a Quality; thus the concept of shortest is a pure addition, which can be extracted by no analysis from the concept of the straight line: we must have recourse to intuition, and then alone synthesis is possible."—But here again we are not dealing with a concept of the straight in general, but with that of a straight line, and this is already spatial and intuited. The determination (or, if you will, the concept) of the straight line is surely none other than this, that it is the absolutely simple line, that is, that, in passing beyond itself (the so-called movement of the point) it is simply self-related; in its extension no sort of differentiation of the determination, or relation to any other point or line without it, is posited;—it is just simple direction in itself. This simplicity is indeed its Quality; and, if it should seem hard to define the straight line analytically, this is so only because of this determination of simplicity or self-relation, and solely because, when determination is thought of, a multiplicity, or determining through others, is primarily imagined. In itself, however, it is not difficult to seize on this determination of the simplicity in itself of extension, and of its indeterminedness by Other;—Euclid's definition contains nothing else than this simplicity.—Now the transition of this Quality to quantitative determination (namely, that of being the shortest), which is supposed to constitute the synthetic element, is purely analytical. Being spatial, the line is Quantity in general; the most simple thing, if predicated of Quantum, is "least"; and this, predicated of a line, is "shortest." Geometry can take these determinations as corollary to the definition; but Archimedes in his books on Sphere and Cylinder (translation Hauber, p. 4) did more suitably in making this determination of the straight line fundamental: and therein he was right, as was Euclid when he placed the determination relating to parallel lines among the fundamentals; for, in order to become a definition, the development of this determination too would have required qualitative determinations not properly spatial but more abstract, namely (as simplicity above, so here) similarity of direction and the like. The ancients gave a plastic character even to their sciences; they kept their exposition strictly within the peculiarities of their material, and therefore excluded what would have been heterogeneous.

The concept advanced by Kant in the synthetic judgments *a priori*—the concept of differentiation with inseparability, and of identity which in itself is unseparated differentiation—belongs to the great and immortal part of his philosophy. In intuition this concept—since it is the concept itself, and everything in itself is concept—likewise is certainly present; but the determinations which have been taken in those examples do not represent it; Number, and numbering, rather is an identity and the production of an identity which is merely external, or merely a superficial synthesis; a unity of Ones, such as are posited as not identical with one another, but rather as mutually separate and external. With the straight line, the determination of being the shortest line between two points is based rather on the moment of abstract identity, without differentiation in itself.

From this digression I revert to addition. The corresponding negative arithmetical process, subtraction, likewise is the purely analytical severing of numbers, which, as in addition, are determined merely as mutually altogether unequal.

2. The next determination is the equality of the numbers which are to be numbered. This equality makes them into a unity; and thus there is applied to Number the distinction between Unit and Amount. Multiplication is the problem of adding an amount of units of which each is an amount. It is here indifferent which of the two numbers is taken as unit and which as amount,—whether we say 4 times 3 (where 4 is the amount and 3 the unit) or, conversely, 3 times 4.—We have already indicated above that the original discovery of the product is effected by simple numbering, by counting on the fingers and so forth; the product, later, can be indicated immediately because there is a collection of these products (the multiplication table) which is known by heart.

Division is the negative arithmetical process with the same determination of the distinction. It is equally indifferent which of the two factors, divisor or quotient, is determined as unit or as amount. The divisor is determined as unit and the quotient as amount when the problem in division is expressed in the words that it is desired to see how many times (amount) one number (unit) is contained in a given number; conversely the divisor is taken as amount and the quotient as unit, when it is said that a number is to be divided into a given amount of equal parts, and that the magnitude of such a part (the unit) is to be found.

3. The two numbers which are determined as being related to one another as unit and amount, are still immediately opposed as numbers, and therefore are altogether unequal. The equality which is attained is next that of unit and of amount themselves: thus the progress towards the equality of the determinations immanent in the determination of Number, is completed. According to this complete equality, counting is the raising to a certain power (the negative arithmetical process being to take the root)—and, first, the squaring of a number;—numbering is here completely determinate in itself, when (1) the many numbers which are to be added are the same, and (2) their plurality, or amount, is the same as the number which is posited so many times, that is, which is unit. There are no

further determinations in the concept of Number which could offer a differentiation; nor can the differentiation immanent in Number be made more homogeneous. Where a number is raised to a higher power than the square, there is a formal continuation; partly (with the even exponents) there is a mere repetition of squaring; partly (with the odd powers) inequality returns; for, although the new factor is formally equal (for example, in a first instance with the cube) both to amount and to unit, yet it stands, as unit, in a relation of inequality to the amount (the square, 3 against 3 times 3); still more in the cube of four, where the amount, 3, according to which the number which is unit should be multiplied by itself, is actually different from the latter.—Where these determinations—amount and unit—are present, they constitute in themselves the essential differentiation of the concept: in order that that which has gone outside itself may completely return upon itself, these two must be equalized. The above exposition contains the reason why partly the solution of higher equations must consist in the reduction to quadratic equations, and partly why the equations of odd exponents can only be formally determined, and, just when the roots are rational, they cannot be found otherwise than by an imaginary expression, that is, by the opposite of that which the roots are and express.—From what has been said it appears that the arithmetical square alone contains absolute self-determinedness in itself; hence the equations with further formal powers must be reduced to it; just as the right-angled triangle in geometry contains that absolute self-determinedness in itself which is set forth in the theorem of Pythagoras, and so all other geometrical figures must be reduced to it for their complete determination.

A method proceeding under the guidance of a judgment logically constituted treats of powers before it treats of proportions; it is true that the latter are connected with the difference between unit and amount, which constitutes the determination of the second arithmetical process; but they step beyond the One of immediate Quantum, in which unit and amount are only moments; and the further determination according to it still remains external to it. In Ratio, Number no longer is *immediate* Quantum: there it has its determinateness, as mediation. Qualitative Relation will be considered in what follows.

We may say of this exposition of the progressive determination of the arithmetical processes, that it is no philosophy of them nor demonstration of their inner meaning, since in fact it is not an immanent development of the concept. But philosophy must know how to distinguish what is in its nature a material external to itself, so that here the progress of the concept can take place only in an external manner, while its moments can have only the peculiar form of their externality, such as oddness and evenness in this example. It is an essential condition, if one would philosophize about real objects, that the spheres be distinguished in which a definite form of the concept belongs, or, in other words, is present as existence: else what is external and contingent will be disturbed in its peculiarity by ideas; and also these ideas, through the inadequacy of the material, will be distorted and formalized. Now this externality in which the moments of the concept appear in this external material—Number—is here the adequate form; they represent the object in its understanding, and further contain no speculative demands and thus appear easy: they therefore deserve to be employed in text-books of the elements.

Observation 2

It is well known that Pythagoras represented intellectual relations (or philosophemata) by numbers; in more modern times too numbers and the forms of their relations (such as powers and so on) have been used in philosophy, in order to regulate thoughts or to express them.—From an educational point of view Number has been considered the most suitable object of inner intuition, and the arithmetical manipulation of its relations has been considered that activity of the mind where it manifests its most peculiar relations and altogether the fundamental relations of Essence.—The concept of Number, as it has here yielded itself, shows how far Number may claim this high worth.

We have seen in Number the absolute determinateness of Quantity, and in its element differentiation which has become indifferent;—determinateness in itself, which also is posited as wholly external. Arithmetic is an analytic science, because all connexions and distinctions which occur in its object are not inherent in it, but are applied to it wholly from without. It has no concrete object containing internal relations which, at first hidden from knowledge and not given in the immediate idea of the object, have to be elaborated by the efforts of cognition. Not only does it not contain the concept and therewith the task for conceptual thought: it even is its opposite. The connected terms are indifferent to the connexion, which lacks necessity: the activity of thought, therefore, is here one which is the extremest self-renunciation, an activity which forces it to move in thoughtlessness and to connect terms which admit of no necessity. The object is the abstract thought of externality itself.

Number thus being the very thought of externality, is also the abstraction from the manifold of the senses; it has retained nothing of the sensuous except the abstract determination of externality itself; and the latter therefore approaches nearest to it in thought; it is the pure thought of the self-renunciation of thought.

Mind, which rises above the world of the senses and understands its own essence, when it seeks an element for its pure imagination, for the expression of its essence, may therefore strike upon that inner and abstract externality, Number, before it seizes thought itself as this element and achieves the purely spiritual expression fit to represent it. Hence we see, early in the history of Science, Number being used to express philosophemata. It constitutes the last step of that incomplete method which apprehends the universal as affected by the sensuous. The ancients were clearly conscious that Number is midway between the sensuous and thought. Aristotle (*Metaphys.* I. 5) quotes Plato as saying that the mathematical determinations of things stand apart from and between the sensuous and the Ideas, distinguished from the former because they are invisible (eternal) and unmoved, and from the latter because they are a manifold and have similarity, while the Idea is simply self-identical and one within itself.—A more detailed and profound consideration of this matter, by Moderatus[1] of Cadiz, is quoted in *Malchi Vita Pythagorae ed. Ritterhus*, pp. 30 sq. He thinks that the Pythagoreans hit upon numbers because they were not yet able clearly to seize in Reason the fundamental ideas and first principles, these principles being hard to think and hard to express. Numbers serve the teacher well as designations; and the Pythagoreans imitated, herein as elsewhere, the geometers, who cannot express the corporeal in thought, and therefore use figures, saying "this is a triangle" when, however, they do not mean that this optically visible figure is to be taken for a triangle, but that only the concept is to be represented. And thus the Pythagoreans used the expression "One" for the concept of unity, of identity and equality, the basis of agreement, of connexion, of the preservation of all, of the self-identical and so forth.—It is superfluous to observe that the Pythagoreans passed beyond the expression of Number to the expression of Thought, to the express categories of equal and unequal, to limit and infinity; indeed, at the place where these numerical expressions are considered (*ibid.*, note to p. 31, l. s., from a *Life of Pythagoras* in Photius, p. 722), it is mentioned that the Pythagoreans distinguished between the Monas and the One, taking the Monas as concept and the One as number, and, similarly, the Two for the arithmetical term, but the Dyas (for this seems the correct reading) for the concept of the indeterminate.—Thus these ancient writers correctly perceived the inadequacy of these numerical forms as determinations of thought, and equally rightly they demanded, in place of this first substitute for thought, its characteristic expression; how much further had they progressed in thought than those who in our day, when some put in the place of determinations of thought numbers and determinations of numbers (like powers), next the infinitely great and the infinitely small, one divided by infinity, and other such determinations, which often are a perverted mathematical formalism, take the return to this impotent childishness for something praiseworthy and even for something thorough and profound.

---

[1] A Neopythagorean who lived in the time of Nero.

We quoted above the expression that number stands between the *sensuous* and thought, since it shares with the former the quality of being in itself the *Many*, or separate existence: it is here to be noted that this Many itself, the sensuous which is taken up into thought, is that category of the self-external which is proper to it. Further, concrete, veritable thoughts, which of all things are the most quick and flexible, conceived only where there is relation, when translated into this element of being beyond self become dead and rigid determinations. The richer in determinateness, and hence in relation, thoughts become, the more confused and the more arbitrary and senseless becomes their representation in such forms as numbers. The One, the Two, the Three, and the Four, Henas or Monas, Dyas, Trias, Tetraktys, still approximate to the simple and abstract concepts; but when numbers are required to pass over into concrete relations it is a vain attempt which would keep them close to the concept.

But the hardest thing is asked of thought, when the determinations of thought by One, Two, Three, Four are designated as that movement of the concept through which alone it is concept. It moves in the element of its opposite, which is unrelatedness; its work is the work of derangement. It is a hard suggestion to conceive that One is Three (for instance) or Three One, because the One is the unrelated and, therefore, does not show in itself the determination by virtue of which it passes over into its opposite, since its very being is the absolute exclusion and negation of such a relation. Conversely, Understanding makes use of this against speculative truth (for instance against that which is laid down in the doctrine called that of the Trinity), and *counts* those determinations of it which constitute One Unity, in order to demonstrate it to be a clear contradiction;—that is, itself contradictorily makes unrelated that which essentially is relation. It was not expected, when the name of Trinity was coined, that Understanding would consider One and Number as the essential determinateness of the content. This name expresses contempt for Understanding, which has nevertheless confirmed itself in its vanity in clinging to One and Number as such, and has set it up against Reason.

It is, in a manner, harmless, when numbers or geometrical figures are taken as mere symbols, as has often been done with the circle, the triangle, and so on, the circle for instance standing for eternity and the triangle for the Trinity; but it is foolish to think that more is thus expressed than can be comprehended or expressed by thought. These symbols, like others created by fancy in the mythologies of peoples and in poetry generally,—compared with which the bare geometrical figures are in any case meagre,—are supposed to contain a profound wisdom and a profound meaning; if it is so, then the sole purpose of thought is, to extract the wisdom which lies only *here*, and not merely *in* symbols but also *in* Nature and *in* Spirit. In symbols truth is darkened and veiled by the sensuous element; it is wholly revealed to consciousness only in the form of thought: Meaning is only the thought itself.

But when mathematical categories are used to determine something bearing upon the method or content of philosophic science, such a procedure proves its preposterous nature chiefly herein, that, in so far as mathematical formulae mean thoughts and conceptual distinctions, such meaning must first report, determine, and justify itself in philosophy. In its concrete sciences, philosophy takes the logical element from logic and not from mathematics; it must be a mere refuge of philosophic impotence when it flies to the formations which logic takes in other sciences, of which many are only dim presentiments and others stunted forms of it, in order to get logic for philosophy. The mere employment of such borrowed forms is in any case an external attitude: a knowledge of their worth and of their meaning should precede their use; but such knowledge results only from thoughtful contemplation, and not from the authority which mathematics gives them. Logic itself is such a consciousness; and such consciousness strips them of their particular form, making it superfluous and useless, and, while correcting them, alone gives them justification, meaning, and worth.

The value of the use of Number and of Arithmetic, in so far as it is supposed to be a main basis of education, is evident from the above. Number is a non-sensuous object, and an occupation with it and its combinations a non-sensuous business: thus the mind is urged to reflect in itself and to do inner and abstract work; which is of great though one-sided importance. For, on the other hand, Number being based only on external and non-conceptual differentiation, this occupation becomes thoughtless and mechanical. The effort chiefly consists in seizing units void of concept and combining them without the use of concept. The content is the empty One: that rich content of moral and spiritual life, and its individual growth, on which, as its noblest nourishment, education should rear the young mind, would here be ousted by the empty One; and when such exercises form the main matter and the main occupation, the effect can be no other than to hollow and blunt the spirit in form and concept. Arithmetic being so extremely external and therefore mechanical a matter, it has been possible to construct machines which execute the arithmetical operations in the most perfect manner. If this one circumstance were known about the nature of arithmetic, it alone would contain a judgment about the value of the notion which would make arithmetic the chief instrument for educating the mind by putting it on the rack which would perfect it into a machine.

B

EXTENSIVE AND INTENSIVE QUANTUM

(*a*) THEIR DIFFERENCE

1. We saw that Quantum has its determinateness as limit in Amount. It is a something discrete and manifold in itself, and has no Being distinct from its limit and having this outside itself. Quantum and its limit (and the limit in itself is a manifold) together thus are Extensive Magnitude.

Extensive Magnitude must be distinguished from continuous magnitude: the former is opposed to intensive and not to discrete magnitude. Of extensive and intensive magnitude each is a determinateness of the quantitative limit itself, while Quantum is identical with its limit; continuous and discrete magnitude, on the other hand, are determinations of magnitude in itself, that is, of Quantity as such, in so far as in Quantum abstraction is made from limit.—The moment of continuity is attached to extensive magnitude and to its limit, for its Many is simply continuous; the limit as negation thus appears in this levelling of the Many, as limitation of unity. Continuous magnitude is self-repeating Quantity without regard to any limit, and, in so far as it is imagined as having a limit, this is limitation in general, without discreteness being posited in it. Quantum taken merely as continuous magnitude is not yet truly determinate for itself, for it lacks Number and the One which implies that it is determinate for itself. And similarly discrete magnitude is, immediately, only differentiated Many in general, which, in so far as it had a limit as such, would be just a multitude, that is, a something limited but limited indeterminately; in order to make it into a determinate Quantum, the Many must be subsumed into One, by which process they are posited as identical with the limit. Continuous and discrete magnitude, taken as Quantum generally, have each posited in it only one of the sides by which it is completely determined and made into Number. The latter is immediate Extensive Quantum,—*simple* determinateness, existing essentially as *amount*, though only as an amount of one and the same unit, being distinguished from Number only in this, that in the latter determinateness is expressly posited as plurality.

2. However, when the magnitude of something is determined by number, no contrast with another magnitude is needed: the determinateness of this magnitude does not depend upon itself and another magnitude, for the determinateness of magnitude in general is self-determinate, indifferent and simply self-related Limit; which, in Number, is posited as enclosed in the self-existing One, externality and relation to Other being within itself. This Many of the Limit, further, (like the Many in general) is not differentiated within itself, but continuous; each of the Many is what every other is; its determinateness as such does not consist in its quality of mutual externality of parts, or discreteness. Thus this Many for itself collapses into its continuity and becomes simple unity.—Amount is only a moment of Number, but does not constitute the determinateness of Number as a multitude of numerical Ones: the Ones, as indifferent and self-external, are transcended when Number has returned upon itself. Externality, which constituted the Ones of plurality, vanishes in the One as self-relation of Number.

Quantum, as extensive, had its existing determinateness as self-external Amount: its limit thus passes over into simple determinateness. In this simple determination of the limit, Quantum is intensive magnitude; and the limit or determinateness which is identical with Quantum is now also posited as simple—it is Degree.

Degree thus is determinate magnitude or Quantum, but it is not also a multitude or multiplicity within itself; it is only a "more"; and the quality of "more" is the many taken together in the simple determination, Determinate Being which has returned to Being-for-Self. Its determinateness requires a number (which is the complete determinedness of Quantum) to express it; but the number is not Amount but, simply, a Degree. If we speak of ten or twenty degrees, then the Quantum which has so many degrees is the tenth or twentieth degree, and not the amount and sum of them; for, if so, it would be extensive; but in fact it is only one—the tenth or twentieth—degree. It contains the determinateness implied in the amount ten or twenty, but does not contain it as many; it is Number as transcended amount or simple determinateness.

3. In Number, Quantum is posited in its complete determinateness; but as Intensive Quantum, as in the Being-for-Self of Number, it is posited as it is according to its notion, or in itself. For the form of self-relation which it has as Degree is also its self-externality. As Extensive Quantum, Number is numerical plurality, and thus contains externality within itself. The latter, as being Many in general, collapses into undifferentiatedness and transcends itself in the One of Number, which is its self-relation. Now Quantum has determinateness when taken as Amount: as shown above, it contains amount though amount is not posited in it. And so Degree, as being simple in itself, no longer has this external otherness within itself; it has it outside itself, and it stands related to it as to its determinateness. A plurality external to it constitutes the determinateness of the simple limit, which Degree is for itself. If, therefore, Amount (so far as, in Extensive Quantum, it existed within Number) there transcended itself, it passes into being posited outside Number. Number being posited as One, as introreflected self-relation, excludes from itself the indifference and externality inherent in Amount, and is self-related because through itself it is related to something external.

Herein Quantum has the reality proper to its notion. The indifference of determinateness constitutes its Quality; that is, determinateness which in itself exists as self-external determinateness.—Degree accordingly is a simple determination of magnitude among several such intensities, which are different, each being a simple self-relation but also being in an essential relation to the others, so that each has its determinateness in this continuity with the others. Since, then, Degree is related to its Other through itself, the ascent or descent in a scale of degrees becomes a steady progress or flow, which is an uninterrupted and indivisible mutation; not one of the several entities which are distinguished therein is separated from the others, but each has its determinateness only in them. As self-related determination of magnitude each degree is indifferent to the others; but also it is implicitly related to this externality, and is what it is only through it; in a word, its self-relation is the non-indifferent relation to the external, and in this fact consists the quality of the degree.

(*b*) IDENTITY OF EXTENSIVE AND INTENSIVE MAGNITUDE

Degree is not self-external within itself. But it is not indeterminate One, or that general principle of Number which is no amount except that negative amount implied in the fact that it is not amount. First, Intensive Magnitude is a simple One of the heap; there are several degrees; but they are determined neither as simple One nor as heap, but only in the relation of this self-externality, or in the identity of One and plurality. So that, if members of a plurality, as such, are outside the simple degree, its determinateness consists in its relation to them: it thus contains Amount. Twenty, as Extensive Magnitude, also contains twenty discrete Ones; on the other hand, the determinate degree contains them as a continuity; and this determinate plurality is this continuity simply. It is the twentieth degree, and is such only by virtue of this amount, which as such is external to it.

The determinateness of Intensive Magnitude must therefore be considered from two aspects. It is determined through other intensive Quanta, and it is continuous with its otherness, in such a manner that its determinateness consists of its relation to this. First, then, in so far as it is simple determinateness, it is determinate relatively to other degrees; it excludes them, and this exclusion constitutes its determinateness. Secondly, however, it is determinate in itself: it is determinate in amount, as in "its" amount, not in amount as excluded nor as taken as the amount of other degrees. The twentieth degree contains twenty in itself; it is determinate, not only as distinguished from the nineteenth, twenty-first, or other degrees; but, further, its determinateness is *its* amount. But, in so far as the amount is its own—and determinateness also essentially exists as amount—degree is Extensive Quantum.

Thus Extensive and Intensive Magnitude are one and the same determinateness of Quantum; they only differ in that one has amount within itself, and the other without. Extensive Magnitude passes over into Intensive Magnitude because the Many of Extensive Magnitude in itself collapses into the unity to which the Many is external. But conversely this simple result has its determinateness only in amount, and in fact in "its" amount; it is indifferent to intensities otherwise determined, and, therefore, has the externality of amount in itself; and thus Intensive Magnitude equally essentially is Extensive Magnitude.

Having reached this identity, we have reached the qualitative Something; for this identity is unity self-related by virtue of the negation of its distinctions; now the latter constitute the existing determinateness of magnitude: this negative identity thus is Something, Something moreover which is indifferent to its quantitative determinateness. Something is a Quantum, but now qualitative Determinate Being, as it is in itself, is posited as indifferent towards it. Quantum, Number as such, and so forth, could be spoken of without mention of a Something such as should be their substratum. Now, however, Something stands opposed to these its determinations as existing for itself; it has become mediated with itself through their negation, and, in having a Quantum, is opposed to them as that same Something which is defined as having Extensive and Intensive Quantum. Its *one* determinateness proper to it as Quantum is posited in the distinct moments of Unit and Amount. Now not only is this determinateness one and the same in itself, but, when it is posited in these distinctions as Extensive and Intensive Quantum, there is a return to that unity which, as negative, is the Something which is posited as indifferent to it.

Observation 1

In common imagination Extensive and Intensive Quantum generally are distinguished as different *kinds of magnitude*, as though some objects had only intensive and others only extensive magnitude. To this has been added the view proper to a philosophy of nature, where the many, or that which is extensive (as happens, for instance, in the fundamental determination of matter—namely, that it occupies space—and in other concepts), is changed into something intensive in this sense, that the true determination was supposed to be the equation of the intensive with the dynamic; it was thought that, for instance, density (or the specific occupation of space) must be taken essentially as a certain degree of the space-occupying force of matter, and not as a certain multitude and amount of material parts in a quantum of space.

Here two determinations must be distinguished. In the so-called transvaluation of the mechanistic into the dynamic view, there occurs the concept of independent parts existing apart from one another (the nexus which makes them a whole being merely external), and the distinct concept of force. That which, as occupying space, was considered merely as a multitude of mutually external atoms, is now regarded as the manifestation of a simple latent force.—But these relations of Whole and Parts, of Force and its Manifestation, which are here opposed to one another, do not belong to this place, and will be considered later. But this much may be said, that the relation of Force and its Manifestation, which corresponds to intensity, is more valid than the relation of Whole and Parts; but that in spite of this force is equally as one-sided as intensity, and that manifestation (the externality of extensiveness) is equally inseparable from force, so that one and the same content is found alike in both forms, in the intensive as well as in the extensive.

The other determinateness which here occurs is quantitative determinateness as such, by which is meant a determinateness which is transcended as Extensive Quantum and is changed into Degree, which is supposed to be the valid determination. But it has been shown that Degree too contains the former, so that one form is essential to the other, and each Determinate Being represents its determination of magnitude as Extensive and as Intensive Quantum equally.

Everything, so far as it appears as a determinate magnitude, will serve as example for this. Number itself necessarily bears immediately in it this double form. It is amount, and in so far it is extensive magnitude; but also it is One, one Ten, one Hundred, and in so far it stands at the point of transition to intensive magnitude, since in this unity the multiple collapses into simple. One is extensive magnitude *in itself*, and may be imagined as any desired amount of parts. Thus the tenth or hundredth part is this simple intensity which has its determinateness in the Many which fall outside it, that is, in extensity. A number is ten or one hundred, and also is the tenth or one hundredth in the numerical system; and each determinateness is identical with the other.

In the circle the One is called Degree, because any part of a circle essentially has its determinateness in the Many external to it, and is determined as merely one of a limited amount of such Ones. As a mere spatial magnitude the degree of the circle is a common number; regarded as degree it is an intensive magnitude, which only has meaning as determined by the amount of degrees into which the circle is divided; just as Number in general has its meaning only in the numerical series.

The magnitude of a more concrete object also has this twin aspect of extensive and intensive, and manifests this in the twin determinations of its Determinate Being, in one of which it appears as external, while in the other it appears as internal. Thus, for instance, a mass, as weight, has extensive magnitude, since it constitutes an amount of pounds, hundredweights, and so forth; and intensive magnitude in so far as it exerts a certain pressure: the magnitude of pressure is simple, it is a degree which has its determinateness in the scale of degrees of pressure. Mass, as exerting pressure, appears as a Being-in-Self or Subject, which has an intensive differentiation of magnitude.—Conversely, that which exerts this degree of pressure has the power to displace a certain amount of pounds (and so on), and therein lies the measure of its magnitude.

Again, heat has a degree, and this degree (whether the 10th, 20th, or any other) is a simple sensation, is subjective. But this degree equally exists as extensive magnitude, as the expansion of a liquid, of mercury in the thermometer, of air, of sound, and so on. A higher degree of temperature expresses itself as a longer column of mercury, or as a narrower sound-cylinder;

it heats a greater space in the same manner as a lesser degree heats a smaller space.

The higher note, as the more intensive, is also a greater multitude of vibrations; and a louder note (to which we ascribe a higher degree) is audible in a greater space.—A more intensive colour will serve to stain a larger surface than a weaker colour can similarly stain; that which is brighter (again a different kind of intensity) is visible farther than the less bright; and so forth.

And in the spiritual sphere high intensity of character, of talent, or genius, has a similarly comprehensive being, extended action, and manifold contact. The profoundest concept also has the most universal meaning and application.

Observation 2

Kant has employed the application of the determinateness of Intensive Quantum to a metaphysical determination of the soul in a peculiar manner. In his critique of the metaphysical propositions about the soul (which he calls Paralogisms of Pure Reason), he comes to consider the deduction of the permanence of the soul from its simplicity. To this conclusion he objects in these terms (*Critique of Pure Reason*, p. 414):—"Although we grant that the Soul is of this simple nature, since it contains no manifold of separate entities, and therefore no extensive magnitude, yet we cannot deny to it, any more than to any other existent, intensive magnitude, that is, a *degree* of reality with regard to all its powers and in general with regard to everything that exists; this degree can decrease through an infinity of lesser degrees, so that this would-be substance can be changed into nothing by gradual relaxation (*remissio*) of its forces if not by division; for consciousness itself always has a degree which can still be diminished; this therefore must apply also to the faculty of self-consciousness and to all the other faculties."—In rational psychology—and this abstract metaphysic was such—the soul is considered not as Spirit but as a merely immediate Being, as a soul-thing. Kant thus has the right to apply the category of Quantum to it "as much as to any other existent;" and, in so far as it is determined as simple, to apply the category of Intensive Quantum. It is true that Being is a part of Spirit, but a Being of an intensity wholly different from that of Intensive Quantum; an intensity rather where the form of mere immediate Being and every category of it have been transcended. Not only the removal of the category of Extensive Quantum should have been admitted: Quantum should have been removed altogether. But it is yet a further matter to understand how existence, consciousness, and finitude are in the eternal nature of Spirit and emerge from it, while it does not thereby become a thing.

(*c*) THE ALTERATION OF QUANTUM

The distinction between Extensive and Intensive Quantum is indifferent to the determinateness of Quantum as such. But, altogether, Quantum is determinateness posited as transcended, it is the indifferent limit and a determinateness which is also its own negation. This distinction is developed in Extensive Magnitude; Intensive Magnitude is the presence of this externality which Quantum is in itself. It is posited as its own contradiction in itself, where it is simple self-related determinateness and also that self-negation which has its determinateness not in itself but in another Quantum.

According to its Quality, therefore, a Quantum is posited in absolute continuity with its externality and otherness. Thus, it is not merely *possible* to pass beyond every determinateness of magnitude, and *possible* to alter it; but it is posited that it *must* alter. The determination of magnitude continues itself into its otherness in this manner, that it has its being only in this continuity with an Other: it is a limit which is not, but becomes.

The One is infinite, or is self-related negation, and therefore self-repulsion. Quantum too is infinite, being posited *as* self-relating negativity; it repels itself from itself. But it is a determinate One, a One which has passed over into Determinate Being and Limit: it is the self-repulsion of determinateness and not the creation of the self-equal (as happens in the repulsion of One) but of its otherness: it is now posited in itself to refer beyond itself and to become an Other. It consists in increase or diminution: it is in itself the externality of determinateness.

Thus Quantum refers itself beyond itself: it becomes an

Other, and this also is, primarily, Quantum; but it is this as a limit which is not, but forces itself beyond itself. Thus the limit which arises again in this outward passage is simply one which again transcends itself and refers to yet another, and so on to infinity.

C

THE QUANTITATIVE INFINITY

(*a*) Its Notion

Quantum alters and becomes another Quantum; the further determination of this alteration, that it proceeds to infinity, is implied in the fact that Quantum is posited as self-contradictory.—Quantum becomes an Other; but it continues itself into its otherness; the Other too is, therefore, a Quantum. But this latter is the Other not only of *a* Quantum, but of *Quantum itself*: it is the negative of Quantum taken as limited; it is, therefore, its unlimitedness, or Infinity. Quantum is an Ought: it is determined for itself, and this is a part of it, and this very determinedness is, rather, a determinedness in an Other. And, inversely, it is transcended determinedness in an Other, it is indifferent self-subsistence.

Finitude and Infinity thus forthwith each receive a double, and in fact an opposite, meaning. Quantum is finite first because it is limited, and secondly because it refers beyond itself, and has its determinedness in an Other. And Quantum is infinite, first, in its non-limitedness, and, secondly, because, having returned upon itself, it is indifferent Being-for-Self. If, now, we compare these moments, we find that the determination of finitude in Quantum, that reference beyond itself to an Other which constitutes its determination, is likewise determination of the infinite: the negation of the limit is a similar passing beyond determinateness, so that Quantum is to have its last determinateness in this negation, the infinite. The other moment of Infinity is that Being-for-Self which is indifferent to limit, but Quantum itself is limited in such a manner that relatively to its limit (and, therefore, to other Quanta and to its beyond) it is in itself indifferent. Finitude and Infinity (of the sort which tries to be separate from the finite—the bad infinite) in Quantum each already contain the other for moment.

Qualitative and Quantitative Infinity are distinct because in the former the opposition between finite and infinite is qualitative, and the transition from the finite into the infinite, or the relation of either one to the other, subsists only in the "in-itself," in their notion. Qualitative determinateness is immediate, and is related to otherness essentially as to another Being; and it is not posited as having its negation, its Other, in itself. But magnitude is, as such, transcended determinateness: it is posited as being both unequal and indifferent to itself, and, therefore, as being that which is alterable. Qualitative Finitude and Infinity are, therefore, opposed absolutely, that is, abstractly; their unity is the fundamental inner relation; and, therefore, the finite continues itself into its Other only in itself and not in the infinite. But the quantitative finite refers itself to its infinite in the latter itself, claiming to have its absolute determinateness in it. This relation, then, is represented by the Quantitatively Infinite Progress.

(*b*) THE QUANTITATIVE INFINITE PROGRESS

In general an infinite progress is the expression of a contradiction: here it expresses the contradiction contained in the quantitatively finite, or in Quantum in general. It is that reciprocal determination of finite and infinite which was considered in the sphere of Quality, with this difference that, as has just been pointed out, in the quantitative the limit in itself refers and continues beyond itself, and that therefore, conversely, the quantitatively infinite is posited as containing Quantum in itself,—for Quantum, in its self-externality, is also itself; its externality is part of its determination.

Now the Infinite Progress expresses but does not solve this contradiction; but, by reason of the merging of one determinateness into the other, it produces an apparent solution by uniting the two. As it is primarily posited, it makes the infinite the goal, which, however, is not reached: it is a perpetuated creation of it, while yet Quantum is never left behind and the infinite never becomes positive and present. It is part of the notion of Quantum to have a beyond. This beyond is, first, the abstract moment of the not-being of Quantum; this resolves itself in itself; and thus it relates itself to its beyond as to its infinity, according to the qualitative moment of the opposition. Secondly, however, Quantum is continuous with this beyond: it is the very being of Quantum to be its own Other, to be self-external; and thus this external part is equally nothing else than Quantum: the beyond, or the infinite, itself is a Quantum. Thus the beyond has been recalled from its flight, and the infinite has been reached. But now this "beyond" which has become "hither" once more is a Quantum; and, therefore, only a new limit has now been posited. This is a Quantum, and itself has again fled from itself; being a Quantum, it is beyond itself, and has repelled itself into its own not-being and its own beyond, which in turn becomes Quantum in the same perpetual manner in which Quantum repels itself from itself into beyond.

The continuousness of Quantum with its Other gives rise to the combination of the two in the expression of the infinitely great and the infinitely small. Both still contain the determination of Quantum, and, therefore, are variable; and the absolute determinateness, which would be a Being-for-Self, is thus not reached. This self-externality of the determination is posited in the double infinite, whose terms are in the opposition of "more" and "less," namely, the infinitely great and the infinitely small. In each, Quantum is preserved in perpetual opposition to its beyond. However much the great is extended, it shrinks into insignificance; it refers to the infinite as to its non-being, and thus the opposition is qualitative; the extended Quantum has wrested nothing from the infinite, which remains its not-being as before. Or again, Quantum by being enlarged does not approach the infinite, for the distinction between Quantum and its infinity has this for essential moment, that it is not a quantitative distinction. This is only the expression of the contradiction brought under narrower consideration: the infinitely great is defined as great (that is, Quantum) and infinite (that is, not Quantum).—Similarly, the infinitely small, as small, is Quantum, and, therefore, remains qualitatively (that is, absolutely) too great for the infinite, and is opposed to it. In both is preserved the contradiction of the Infinite Progress, which was supposed to have found its goal in them.

This infinity which, as the beyond of the finite, is determined as stable, may be described as the "bad quantitative infinity." Like qualitative bad infinity, it is an infinite alternation between the two members of a persistent contradiction, from the limit to its not-being, and from there back once more to the limit. In the quantitative progress the term towards which the progress moves is not indeed abstractly and altogether Other, but is merely a Quantum posited as different; but it remains in opposition to its negation in a similar manner. The progress, too, therefore, is no forward advance but a repetition of the same cycle, of positing and transcending still repeated; an impotence of the negative which by the very act of transcending is ever again faced with that which it had transcended. Two members are connected in such a way that they must fly apart; and in so doing they cannot part company, but even in flying from one another remain connected.

Observation 1

Bad infinity, more especially in its form of the progress to infinity of the quantitative, is often considered sublime, and this continual surpassing of the limit (which really means an incapacity to transcend it), together with the perpetual relapse into it, has been held a kind of divine service. Philosophy has considered it as ultimate. And this progress has often furnished material for outbursts which have been admired as lofty achievements. Yet this modern sublimity does not exalt the object, for that escapes, but only the subject, which consumes such vast quantities. The meagre nature of this exaltation, which, still subjective, climbs up the ladder of the quantitative, is made clear when it is admitted that it vainly labours to reach an unattainable goal, which indeed to be gained must be quite otherwise attacked.

In the following outbursts of this nature it also appears whither this exaltation moves and where it ceases. Thus Kant cites it as sublime (*Critique of Practical Reason:* end)

"when the subject rises in thought above the place which it occupies in the world of sense, magnifying into the infinitely great its relation,—a relation with stars beyond stars, worlds beyond worlds, systems beyond systems, where periodic movement, beginning, and duration are infinite in time as they are in space.—Imagination fails before this progress into the immeasurably far, where the furthest world has one yet further, and past and future, however distant, have a more distant past and a more distant future beyond; thought fails before this image of the immeasurable; as a dream, where one goes down a long passage ever further and interminably further, without an end in sight, ends in a fall or in dizziness."

This description condenses into rich imagery the content of quantitative exaltation: it also deserves praise for the truthfulness with which it shows in what this exaltation ends: thought fails, the end is a fall and dizziness. And what causes thought to fail and produces the fall and the dizziness, is nothing else than the weariness of repetition, where the limit is made to vanish, return, and vanish again perpetually, and the hither to arise and perish in the beyond, and the beyond in the hither, one after the other and the one in the other; which causes a feeling of the impotence of this infinite and this *ought*, which would master the finite and cannot.

Haller's description of eternity, too (called "the horrible description" by Kant), is often admired, though not for that aspect which constitutes its real merit:—

> "I pile up monstrous numbers,
> Mountains of millions,
> I pile time upon time and world upon world;
> When from the horrible height
> Dizzy I look again on you,
> All power of number, taken a thousandfold,
> Is no part even of you.
> I discard it, and you are wholly revealed."

When this heaping and piling of numbers and worlds is considered valuable as a description of eternity, it is overlooked that the poet himself declares this so-called horrible process vain and void, concluding that the true infinite reveals its presence only after this empty infinite progress has been abandoned.

There have been astronomers who took pleasure in priding themselves upon the sublime nature of their science because it dealt with an immeasurable multitude of stars and immeasurable extents of space and time, where distances and periods, already great in themselves, serve as units which being taken however many times over themselves shrink into insignificance. The shallow amazement to which they here abandon themselves, and those silly hopes of travelling from one star to another in a second life, and of thus acquiring into infinity similar new knowledge, were proclaimed by them one of the chief accounts of excellence in their science,—which in truth is admirable, not because of this quantitative infinity, but rather because of those relations of measure and those laws which reason recognizes in these objects; for they are the infinity of reason and those others the infinity of unreason.

Kant opposes another infinity to that which relates itself to external sensuous intuition, when

"the individual returns upon his invisible ego, and opposes the absolute freedom of his will as pure ego to all the terrors of fate and of tyranny, and, beginning with his nearest surroundings, allows them to vanish before him, next causes what seem most durable, worlds upon worlds, to collapse in ruins, and, alone, knows himself equal to himself."

The ego in this loneliness has indeed achieved the Beyond, it has gained itself, is gathered upon itself, and is on the hither side: in pure self-consciousness absolute negativity, which ever flies in that progress above sensuous Quantum, is affirmed and made present. But while this pure ego fixes itself in its abstraction and lack of content, it has opposed to and beyond itself existence in general, or the totality of the natural and spiritual universe. The same contradiction appears on which the infinite progress is based: namely, a term has returned upon itself and also, immediately, is self-external, is related to its Other as to its not-being; and this relation remains a longing, for the ego has, on the one hand, fixed before itself, its own hollow and unstable void, and, on the other, the full content of its beyond which remains present even in the negation.

Kant subjoins to these two examples of the sublime the remark "that astonishment (for the first and external kind) and reverence (for the second and inward) stimulate investigation but cannot replace the lack of it."—He thus declares these sublimities to be unsatisfactory to reason, which cannot remain with them and the feelings connected with them, and cannot allow the name of ultimate to the beyond and the void.

The infinite progress has been considered ultimate chiefly in its application to morality. The opposition just quoted between finite and infinite as standing respectively for the manifold world and the ego exalted in its freedom, is at this point qualitative. The ego in determining itself attempts simultaneously to determine Nature and to free itself from it: it thus relates itself to its Other through itself: and the Other, being an external existence, is a manifold and quantitative. The relation to what is quantitative itself becomes quantitative; and the negative relation of the ego to it, the power which the ego exercises over the non-ego (the world of the senses, and external Nature), is therefore imagined as meaning that morality can and ought to grow, and the power of sensuousness to wane. But the perfect adequacy of the will to the moral law is relegated to the progress into the infinite, that is, it is imagined as an absolute and unattainable Beyond; and this its unattainable nature is supposed to be the true sheet-anchor and proper consolation; for morality should be a fight; and it is such only where will is not adequate to the law, and the law therefore is its absolute Beyond.

In this opposition ego and non-ego or pure will and moral law, and Nature and the sensuousness of will are presupposed as entirely independent and indifferent to one another. Pure will has its peculiar law, which is in essential relation with sensuousness; and sensuousness and Nature have their own laws, which neither come from nor are adapted to will, nor yet, although different from it, have any essential relation to it: they are determined in themselves, finished and complete. But also both are moments of one and the same simple essence, namely, the ego: will is determined as standing in the relation of negative to Nature, so that it exists only in so far as this different sphere exists, which it transcends, but is itself touched and affected thereby in so doing. Nature, taken as the sensuous sphere of man, is an independent system of laws: it is therefore indifferent to limitation by any Other; even while it is being limited it preserves itself, enters independently into relation, and limits the will of law as much as the will of law limits it.—When will determines itself and transcends the otherness of a nature, and again, when this otherness is posited as existing, as continued into a state of becoming-transcended, and yet as not transcended—these two are one act. This contradiction is not resolved in the infinite progress, but is on the contrary represented and affirmed as unresolved and incapable of resolution: the struggle between morality and sensuousness is imagined as the self-existent, absolute relation.

Where it is thus impossible to master the qualitative opposition between finite and infinite and to seize the idea of true will, or substantial freedom, recourse is had to magnitude as mediation, for magnitude is the qualitative element transcended, or distinction become indifferent. But both the members in opposition remain, qualitatively distinct, at bottom, so that when they are mutually related as quanta each immediately is made indifferent to this change. Nature is determined by ego and sensuousness by the will to the good, and the change thus produced therein is only a quantitative difference and one which allows it to remain as it is.

In the more abstract exposition of the Kantian philosophy, or at least of its principles, which is found in Fichte's *Doctrine of Science*, the infinite progress similarly is the basis and the ultimate. The first principle of the exegesis, ego = ego, is followed by another which is independent of it, namely, the opposition of the non-ego; and the relation of these two is immediately taken as being a quantitative difference, non-ego being partly determined by ego and partly not. Thus the non-ego prolongs itself into its not-being, in such a manner that it remains opposed to this latter in it as being not yet transcended. Thus after the contradictions herein contained have been developed in the system, the final result is the same relation as was the beginning; the non-ego remains an infinite "resistance-principle" (*Anstoss*), an absolute Other; the ultimate relation between it and the ego is the infinite progress, longing and yearning,—the same contradiction with which a beginning was made.

The quantitative is determinateness posited as transcended: for this reason it was thought that much or rather everything had been gained for the Unity of the Absolute, for Monosubstantiality, when contrariety in general was reduced to a merely quantitative difference. For a time it was a main thesis of modern philosophy, that all contrariety is only quantitative; opposite determinations are of the same essence and content, they are real sides of the opposition, in so far as each contains both the latter's determinations and both its factors; only, one factor, matter or energy, preponderates on one side, one on the other, or one is present in greater quantity or more intense degree on the one side than on the other. In so far as different kinds of matter and energy are assumed, the quantitative difference rather confirms and completes their externality and indifference both in relation to one another and to their unity. The distinction of absolute unity is supposed to be merely quantitative; the quantitative is transcended immediate determinateness, but still, as negation, it is incomplete and first negation only, not infinite nor negation of negation.—Being and Thought are imagined as quantitative determinations of absolute Substance, and therefore, being quanta, they become quite external and unrelated to one another; as, in a subordinate sphere, do carbon, nitrogen, and so on. It is a third factor, an external reflection, which abstracts from the difference between them and recognizes their unity, which is inner and only in itself, not for itself. Thus in fact this unity is imagined merely as of the first and immediate kind, or as Being which in its quantitative difference *remains* equal to itself, but does not through itself *posit* itself as equal to itself: it is thus not conceived as negation of negation, or infinite unity. It is only in the qualitative opposition that posited infinity or Being-for-Self appears; and quantitative determination passes over—as will be seen immediately—into the qualitative.

Observation 2

It was mentioned above that Kant's antinomies are representations of the opposition between finite and infinite in a more concrete shape and applied to more specialized substrata of imagination. The antinomy there considered contained the opposition of qualitative finitude and infinity. In another (the first of the four cosmological antinomies) it is rather the quantitative limit of which the self-conflicting nature is considered. I shall therefore proceed here to the examination of this antinomy.

It deals with the spatial and temporal limitation or unlimitedness of the world.—Equally well the contradiction might have been considered with reference to time and space themselves, for the antinomy of limitation and unlimitedness remains in them unchanged, whether they be relations of things or only forms of intuition.

The closer analysis of this antinomy will also show that the two propositions with their proofs (which, like those considered above, are apagogic) come to nothing except these two simple and contrary assertions: "there is a limit," and "the limit must be overpassed."

The Thesis is as follows:—

"The world has a beginning in time, and spatially, too, is limited."

One part of the proof, which deals with time, assumes the opposite:—

"If the world has no beginning in time, then an eternity has passed up to any given point of time; hence an infinite series of successive world-states has passed. But the infinite nature of a series consists precisely in this, that it can never be completed by successive synthesis. Thus an infinite world-series which also has elapsed, is impossible, and, therefore, a beginning of the world is an essential condition of its existence,—which was to be proved."

The remaining part of the proof, which deals with space, is based upon time. It would require an infinite time to comprehend the parts of a spatially infinite world, and this time would have to be regarded as expired in so far as the world in space is to be regarded not as a process but as a given whole. But it was shown of time in the first part of the proof that it is impossible to assume the expiration of an infinite period of time.

But it is immediately evident that it was unnecessary to make the proof apagogic, or, indeed, to give any proof, since it contains immediately for basis an assertion of that which was to be proved. For each, or any, point of time is here assumed, up to which an eternity has expired (—eternity here meaning merely infinite time in the bad sense). Now a given point of time means nothing else than a definite limit in time. In the proof, therefore, a limit of time is assumed to exist; which is precisely what was to be proved. For the thesis is, that the world has a beginning in time.

There is only this one distinction, that the limit of time which is assumed is "now" as end of an already expired period, while that which is to be proved is "now" as beginning of the future. But this distinction is immaterial. "Now" is taken as the point where an infinite series of consecutive world-states has ended: it is taken as end, or qualitative limit. If this "now" were considered merely as quantitative limit, which remains fluid and not only must be overpassed but itself *is* nothing but an overpassing of itself, then the infinite time-series in it would not be expired but would continue to flow; and the argument of the proof would vanish. However, the point of time is taken as qualitative limit for the past, but this is also the beginning of a future (for in itself every point of time is the relation of past and future), and, further, is its absolute or abstract beginning, that is, that which was to be proved. It is irrelevant that this future and this its beginning are preceded by a past; in so far as this point of time is qualitative limit—and this is implied in the determination of expired or completed, that is, not self-continuing—time in it is broken off short, and that past is without relation to this time which could be called future only with reference to that past, and consequently when unrelated becomes time in general, which has an absolute beginning. But if—as is the case—through the "now" (or given point of time) it were linked to the past, and so determined as future, then this point of time too, seen from the other side, would be no limit; the infinite time-series would continue itself in that which was before called future and would not be, as was assumed, completed.

In truth time is pure Quantity: the point of time at which it is supposed to be interrupted, which is employed in the proof, is simply the self-transcending Being-for-Self of the Now. The proof effects nothing, except to present to imagination the absolute limit of time, which is asserted in the Thesis, as a given point of time, and to take it boldly as a complete or abstract point;—a popular determination which sensuous imagination easily allows to pass as limit, and thus allows the proof to assume what before was set forth as to be proved.

The Antithesis is this:—

"The world has no beginning and no limits in space, but is infinite both with regard to time and to space."

The proof likewise assumes the opposite:—

"Let the world be assumed to have a beginning. Now the beginning is an existence preceded by a time in which the thing does not exist; therefore a time must have preceded in which the world did not exist; that is, an empty time. Now in an empty time nothing can arise, for no part of such a time has in itself, and in preference to any other, the differentiating condition of its existence or non-existence. Therefore any series of things can begin in the world, but the world itself cannot begin and with regard to past time is infinite."

Like the others this apagogic proof contains the direct and unproved assertion of that which it ought to prove. At first it assumes a Beyond of the existence of the world, namely, empty time; but next it also continues the existence of the world beyond itself into this empty time, thus cancelling it, and continuing this existence into infinity. The world is an existence; the proof presupposes that this existence arises, and that the arising has a temporally precedent condition. But the Antithesis just consists in this, that there is no unconditioned existence and no absolute limit, but that the existence of the world always demands a precedent condition.—What was to be proved is thus already present in the proof as an assumption.—Further, the condition is now looked for in empty time, which means that it is taken as temporal and therefore as an existence and as limited. Thus altogether the assumption is made that the world as existence presupposes another conditioned existence in time, and so to infinity.

The proof with regard to the spatial infinity of the world is the same. The spatial finitude of the world is posited in the apagogic manner: "the world would thus be in an empty and unlimited space and would stand in a relation to it; but such a relation of the world to no object is Nothing."

There again the proof directly presupposes what was to be proved. It is directly assumed that the limited spatial world must be in an empty space and stand in a relation to it, that is, that it must be overpassed;—a process leading on the one hand into the void, the beyond and not-being of the world, and, on the other, meaning that it stands in a relation to the void, that is, that it continues itself into it and therefore that the beyond is to be imagined as filled with world-existence.

The spatial infinity of the world which is asserted in the Antithesis is just (*a*) empty space and (*b*) the relation of the world to it, that is, the continuity of the world in it, or the filling of empty space; and this contradiction—that space is empty and also is full—is the infinite progress of Existence in space. And this contradiction itself, the relation of the world to empty space, is directly made the basis of the proof.

The Thesis and the Antithesis and their proofs thus represent nothing but the contrary assertions that there is a limit, and that this limit equally is cancelled, that the limit has a beyond, but that it is in relation with the latter and must pass over to it; which again gives rise to such a limit which is no limit.

The solution of these antinomies, as with the first, is transcendental, that is, it consists in the assertion of the ideality of space and time as forms of intuition,—in this sense, that the world in itself is not in contradiction with itself and does not transcend itself, but that *consciousness* in its intuition and in the relation of intuition to understanding and reason is a self-contradictory essence. But it is an excessive tenderness towards the world to remove from it the contradiction and to transfer it into Spirit or Reason, allowing it to remain there unresolved. In point of fact it is Spirit which is strong enough to support the contradiction, but it is also Spirit which knows how to resolve it. As for the so-called world (whether it be called objective or real world, or, according to the transcendental idealism, subjective intuition and sensuousness determined by the category of understanding), it never and nowhere is without the contradiction, but, since it cannot support it, is subject to arising and passing away.

(*c*) THE INFINITY OF QUANTUM

1. Infinite Quantum, as infinitely great or infinitely small, is, in itself, the infinite progress; being either great or small, it is Quantum; and it is also the not-being of Quantum. Hence the infinitely great and the infinitely small are pictures of the imagination, which, closely considered, prove mere mist and shadow. But in the infinite progress this contradiction is present explicitly, and with it that which is the nature of Quantum which has reached its reality as intensive magnitude and is

now posited in its determinate existence as it is in its concept. This identity must now be considered.

Quantum as degree is simple, self-referred, and self-determinate. Through this simplicity, otherness and determinateness are cancelled in it, and therefore the latter is external to it: it has its determinateness outside it. This self-externality is, first, the abstract not-being of Quantum in general, or bad infinity. But, further, this not-being is itself a magnitude; Quantum continues itself into its not-being, for it is in its externality that it has its determinateness; therefore this externality, too, is itself Quantum; and its not-being, or infinity, is thus limited, that is, this beyond is transcended and is determined as Quantum, which thus, in its negation, is gathered to itself.

But this is just what Quantum as such is in itself. For it is itself because it is outside itself: externality constitutes that by virtue of which it is Quantum and is with itself. Thus in the infinite progress the *concept* of Quantum is posited.

If we take it first in its abstract determinations as we find them, it contains the transcendence of Quantum, but also of its Beyond; there is thus the negation of Quantum as well as the negation of this negation. In its truth it is the unity in which these are contained, but as moments only.—This truth is the solution of the contradiction of which the progress is the expression, and its nearest meaning is therefore the rehabilitation of that concept of magnitude where it is indifferent or external limit. In the infinite progress as such the only reflection made generally is that every Quantum, however great or small, can vanish, that progress may take place beyond it; but the reflection, that this its transcendence, the Beyond, and the bad infinite also vanish, is generally not made.

The first transcendence, the negation of Quality in general, whereby Quantum is posited, is already in itself the transcendence of negation, Quantum being transcended qualitative limit and therefore transcended negation; but also it is this in itself only; it is posited as a Determinate Being, and hence its negation is established as the infinite, as the Beyond of Quantum, which takes up the place of Hither or of immediate: thus the infinite is determined only as first negation, and as such it appears in the infinite progress. But it has been shown that this contains more, namely, the negation of negation or that which is in truth the infinite. This was looked at before in such a way that the concept of Quantum was thus restored; and this restoration means, first, that its Determinate Existence has received a closer determination: for Quantum determined according to its concept has now arisen, and this differs from immediate Quantum; externality is now its own opposite; it is posited as a moment of magnitude itself, and Quantum is posited as having its determinateness in another Quantum through the mediation of its not-being, namely, of infinity; that is, it is *qualitatively* that which it is. However, this comparison of the concept of Quantum with its existence belongs rather to our reflection, to a relation which is not yet here at hand. The nearest determination is this, that Quantum having returned to Quality is now determined qualitatively. For Quality, which is its characteristic, is externality or indifference of determinateness; and it is now posited as, in its externality, *being* itself, and as self-relating, having its determination in simple self-unity, that is, qualitatively.—This Qualitative is also more closely determined, it is determined as Being-for-Self; for the relation to itself which it has reached has been evolved from mediation, from the negation of negation. Quantum is infinite and is determined for itself: and this is no longer external to it, but within itself.

The infinite, which in the infinite progress had the mere empty meaning of a not-being, a Beyond not reached but searched for, is, in fact, just Quality. Quantum as indifferent limit passes beyond itself into the infinite, and in so doing seeks after nothing else than determinateness-for-self, the qualitative moment which hitherto, however, is no more than Ought. Quantum is Quantum, because of its indifference to the limit and because, therefore, it has no self-subsistent determinateness and so overpasses itself: this its overpassing is to be negated and its absolute determinateness is to be found in the infinite.

And, quite generally, Quantum is Quality transcended; but Quantum is infinite, overpasses itself, and is its own negation: thus this overpassing is in itself the negation of negated Quality, and its restitution; and so it is posited that externality, which appeared as a Beyond, is determined as own moment to Quantum.

Quantum thus is posited as self-repelled: there thus are two Quanta, which, however, are transcended and exist only as moments of one unity: this unity is the determinateness of Quantum.—Quantum then, related to itself as indifferent limit in its externality and therefore qualitatively posited, is the Quantitative Ratio.—In Ratio, Quantum is external to and different from itself: this its externality is the relation of one Quantum to another, each of which has value only in this its relation to its Other; and this relation constitutes the determinateness of Quantum, which exists as such a unity. Here its determination is not indifferent, but qualitative: in this its externality it has returned upon itself, and in it is that which it is.

Observation 1

THE CONCEPTUAL DETERMINATENESS OF THE MATHEMATICAL INFINITE

The mathematical infinite is interesting, first, because its introduction has widened the scope of mathematics and has led to important results therein; next it is remarkable because this science has not yet succeeded in vindicating this use of it conceptually ("concept" being here taken in its proper meaning). Ultimately, such vindications are based on the correctness (*demonstrated on other grounds*) of the results achieved by means of this determination, and not on the clarity of the object and of the process which produces the results: indeed, the process itself is admitted to be faulty.

This in itself is a flaw: such a procedure is unscientific. But it also involves the drawback that mathematics, not knowing the nature of this its instrument (and this, because it cannot cope with the metaphysics and critique of it), could not determine the sphere in which it must be applied, and could not safeguard itself against misapplication.

In its philosophical aspect, however, he tmathematical infinite is important because the concept of the true infinite is, in fact, its basis: it stands on a much higher level than the so-called metaphysical infinite, whence objections against the former are launched. The science of mathematics often has no refuge from these objections except in denying the competence of metaphysics, asserting that it has nothing to do with this science and need not trouble about its concepts if only it acts consistently within its own sphere. It need not consider truth in itself, but only what is true in its own domain. And metaphysics, though it opposes them, cannot deny nor invalidate the brilliant results of the employment of the mathematical infinite; while mathematics cannot achieve clearness about the metaphysics of its own concept, nor, therefore, about the derivation of the methods which the employment of the infinite necessitates.

If it were this single difficulty of the *concept* in general which oppressed mathematics, it could without ceremony neglect this, at least in so far as the concept is more than an expression of the essential determinations of a thing, that is, those which determine it for the mere Understanding: and mathematics has not allowed such determinateness to be lacking in *precision*; for it is not a science which is concerned with the concepts of its objects or has to create its content by the development (even if this were only through a process of argumentation) of the concept. But in the method of the mathematical infinite mathematics finds a radical contradiction to that very method which is characteristic of itself, and on which it rests as a science. For the calculation of the infinite admits of, and demands, modes of procedure which mathematics, when it operates with finite magnitudes, must altogether reject, and at the same time it treats these infinite magnitudes as finite Quanta, seeking to apply to the former those same methods which are valid for the latter. The most important step in the development of this science is to have imposed on *transcendent* determinations and the treatment of these the form of ordinary calculation.

In spite of this conflict in its operations, mathematics shows results which agree exactly with those found by the true mathematical method, the geometrical and analytic method. However, this does not apply to all results; and the end for which the infinite is introduced is not to shorten the beaten road, but to reach results to which it cannot lead. And also the success does not justify the manner of road. Now, this manner of calculating the infinite is plainly oppressed by the apparent inexactitude in which it involves itself when, first, it increases finite magnitudes by an infinitely small amount, and then partly retains these during the further processes, but partly also neglects. This procedure has this strange quality, that in spite of the admitted inexactitude a result is achieved which is *exact*,—not merely tolerably or to such a degree that the difference might be neglected, but *absolutely*. But in the process itself which precedes the result we cannot do without the suggestion that a certain quantity is not equal to nil but is so inconsiderable that it may be neglected. But where mathematical exactitude is meant, the distinction between greater and less accuracy quite disappears, just as in philosophy there can be no question of greater or less probability, but of truth alone. If the method and employment of the infinite are justified by success, it is not as superfluous to demand notwithstanding this a justification as to ask in the case of the nose for a demonstration of the right to use it. For mathematical cognition is a scientific cognition, and therefore the proof is essential; and as regards the results, too, it is the fact that a strictly mathematical method does not stamp all with the mark of success,—which mark in any case is only external.

It is worth while more closely to consider the mathematical concept of the infinite, together with the more remarkable among the attempts designed to justify its employment and to remove the difficulty by which the method feels itself oppressed. The consideration of these justifications and determinations of the mathematical infinite, which I shall undertake at some length in this Observation, will also throw the clearest light on the nature of the true concept itself, and will show how a confused notion of this latter was present as a foundation for those processes.

The mathematical infinite is commonly determined as a magnitude than which there is no greater (when it is determined as the infinitely great) or no smaller (when it is determined as the infinitely small); or as one which is greater (in the former) or smaller (in the latter instance) than any given magnitude.—It is true that in this definition the true concept is not expressed, but only, as has already been remarked, the same contradiction as is contained in the infinite progress; however, let us see what it contains in itself. A magnitude is defined in mathematics as something which can be increased and decreased: that is, then, as an indifferent limit. Now since the infinitely great or small is such as cannot be increased nor decreased, it is in fact no longer a Quantum as such.

This consequence follows necessarily and immediately. But it is just the reflection that Quantum has been transcended which is generally not made (and in this Observation I designate as "finite Quantum," Quantum in general, as we find it): and it is just this which constitutes the difficulty for ordinary thought; for it is demanded that Quantum, in so far as it is infinite, be thought of as transcended and as something which is not a Quantum, although its quantitative determinateness remains.

We may here quote Kant's judgment about this determination:[1] he finds that it does not agree with the common notion of an "infinite whole." "According to the ordinary concept, that magnitude is infinite beyond which there can be no greater magnitude (that is, beyond the *multitude* of a given unit which it contains). But no multitude is the greatest, because it is always possible to add one or more units.—But the notion of an infinite whole does not imply any magnitude, and therefore the concept of an infinite whole is not the concept of a maximum (or minimum): this concept rather expresses the relation of the whole to any given unit, with respect to which it is greater than any number. The infinite would be greater or less according as this unit was taken to be greater or smaller; but infinity, which consists only in the relation to this given unit, will always remain the same, although of course the absolute magnitude of the whole cannot be known through it."

Kant disapproves when infinite wholes are regarded as a maximum, as the entire multitude of a given unit. Maximum or minimum as such still appears as a Quantum, or a multitude. And this idea cannot avert the conclusion, mentioned by Kant, which leads to a greater or lesser infinite. And generally, so long as the infinite is imagined as Quantum, the distinction between greater and less applies to it. But this criticism has no force against the concept of the true mathematical infinite, the infinite difference, for this no longer is a finite Quantum.

---

[1] Observation to the Thesis of the First Cosmological Antinomy, in the *Critique of Pure Reason.*

Opposed to this is Kant's concept of infinity, which he calls the true and transcendental,—namely, "that the successive synthesis of the unit, in measuring a Quantum, can never be completed." A Quantum in general is supposed to be given: it is demanded that, by synthetizing the unit, this be made into an amount, into a Quantum which can definitely be indicated; and it is asserted that this synthetizing can never be completed. Clearly, it is only the progress to infinity which has here been posited; only it is figured in a transcendental, that is really in a subjective and psychological, manner. It is demanded that in itself Quantum be complete; but (it is said) in the transcendental manner, that is, in the subject, which gives it a relation to a unit, the only determination of the Quantum which arises is such as is incomplete and altogether infected with a Beyond. The contradiction which magnitude contains still remains the limit of our progress; but it is distributed between object and subject, limitedness being attributed to the former, and to the latter the passing beyond every determinate measure which it may have reached at any stage into the bad infinite.

On the other hand it was stated above that the determination of the mathematical infinite, as it is used in the higher analysis, corresponds to the concept of the true infinite: the comparison of the two determinations will now be undertaken in its more detailed development.—And, first, as regards true infinite Quantum, it determined itself as intrinsically infinite; and it is so because, as was seen, finite Quantum (or Quantum in general) and its Beyond, the bad infinite, are alike transcended. Quantum transcended has thus returned into simplicity and self-relation, and this not merely as did Extensive Quantum when it passed over into Intensive Quantum; for this latter has its determinateness only *in itself* in an external multiplicity, towards which it is required however to be indifferent and from which it must be distinct. But Infinite Quantum contains in itself, first externality, and then the negation of this; and thus it no longer is a finite Quantum or determinateness of magnitude having a determinate existence as Quantum, but is simple, and therefore exists only as moment: it is determinateness of magnitude in a qualitative form, and its infinite nature consists in the fact that it is qualitative determinateness.—As moment, then, it is essentially at one with its Other only as determined through this its Other, that is, it has significance only with reference to something which stands in a relation towards it. Apart from this relation it is nil,—just because the Quantum as such has to be indifferent *towards* the relation, whilst *in* the relation it is an immediate constant value. In the relation, as a mere moment, it is not indifferent for itself; since it is at the same time a quantitative determinateness, it exists in infinity (regarded as Being-for-Self) only as a "For-One."

It will appear that the concept of the infinite as here abstractly expounded is the basis of the mathematical infinite: the concept itself will become clearer if we consider the several steps of the expression of Quantum, as a moment of relation, beginning with the lowest step where it is still also Quantum as such, and proceeding to the higher where it receives the meaning and expression of real infinite magnitude.

Let us first take Quantum in a ratio, such as is a fraction. Such a fraction, for instance $\frac{2}{7}$, is not a Quantum like 1, 2, 3 and so on; it is an ordinary finite number, but not absolute, like integers: as fraction it is determined mediately by two other numbers which stand in the relation of Amount and Unit, the Unit itself being a determinate Amount. However, if we abstract from this closer mutual determination of these, and consider them only according to their fate as Quanta, in the qualitative relation in which they here are, 2 and 7 are Quanta otherwise indifferent: here, however, they act only as moments, one of the other, and, therefore, of a third (namely, that Quantum which is called the Exponent): here it is not their status as 2 and 7, but their mutual determinateness, which matters. In their place 4 and 14, 6 and 21, and so on to infinity, might equally well be put. They thus herewith assume a qualitative character. If they counted as mere Quanta, then 2 and 7 are, the one just 2 and the other just 7; and 4, 14, 6, 21 and so on are simply other than those numbers, and, in so far as they would be absolute Quanta, they could not be substituted for one another. But, since 2 and 7 are not counted according to their determinateness as such Quanta, their indifferent limit is transcended; and thus in this aspect they contain the moment of infinity; for not only is each number not itself, but also their quantitative determinateness remains: it remains, however, as qualitative in itself, and depends on their value as related. An infinite number of others could be put in their place, and yet the value of the fraction, owing to the determinateness possessed by the relation, is not varied.

But infinity is still not adequately represented by a fraction, and this because the two sides of the fraction, 2 and 7, can be taken apart from the ratio, and are ordinary indifferent Quanta: the relation which subsists between them (namely, that of being in ratio, and of being moments) is external and indifferent to them. And their relation too is an ordinary Quantum, the exponent of the ratio.

The letters with which general arithmetic operates form the next universality into which numbers rise: they have not the peculiarity of bearing a determinate numerical value; they are only general symbols and indeterminate potentialities of any determinate value. And therefore the fraction $\frac{a}{b}$ seems to be a more fitting expression of the infinite, since a and b, taken apart from their relation to one another, remain indeterminate, and severally too have no special and peculiar value.—But, though these letters are posited as indeterminate magnitudes, their significance is that they are some finite Quantum. Thus they are the general image, but the image only of determinate number, and therefore with them too it is indifferent that they should be in a ratio, and they retain this value also when they are no longer in such ratio.

If we consider more closely what is present in Ratio, we find therein these two determinations, first that it is a Quantum, and secondly that this Quantum is not immediate, but contains the qualitative opposition; and also there remains in it that determinate and indifferent Quantum, because it returns upon itself from its otherness (the opposition), and therefore is infinite itself. These two determinations are seen clearly developed in their distinctness from each other in the following well-known form.

The fraction $\frac{2}{7}$ may be expressed as $0{\cdot}285714\ldots$, $\frac{1}{1-a}$ as $1 + a + a^2 + a^3 + \ldots$. It thus appears as an infinite series;

the fraction itself is called the sum or finite expression of it. If these two expressions be compared, then one, the infinite series, no longer represents it as a ratio, but in its aspect as Quantum (since it is a multitude of added units) or as Amount. —It is here irrelevant that the magnitudes of which it is supposed, as Amount, to consist are in turn themselves made up of decimal fractions, that is, of ratios; for this fact applies only to the special kind of unit of which these magnitudes are composed, and not to the magnitudes as constituting an Amount; and, similarly, a multi-figured integer of the decimal system is counted essentially as Amount, and the fact is neglected that it consists of the products of a number and of the number ten and the powers of ten. And, similarly, it is here irrelevant that there are other fractions besides the one taken of $\frac{2}{7}$, which, if converted into decimals, do not yield an infinite series; every fraction can be so expressed in a numerical system having a different unit.

Thus, in the infinite series which is to represent the fraction as Amount, its aspect of ratio vanishes; and simultaneously there vanishes that aspect which—as was shown above—gave to it infinity. Infinity, however, has now been introduced in another manner; for the series itself is infinite.

Now it is self-evident what is the nature of the infinity of the series: it is the bad infinity of the progression. The series contains and illustrates the contradiction which would represent as a mere Quantum or Amount, or as non-relational, that which is a ratio and includes a *qualitative* nature. The consequence of this is that the Amount expressed in the series is ever incomplete, so that what is posited must still be surpassed if the determinateness demanded would be reached. The law of progress is known, and is implicit in the determination of Quantum which is contained in the fraction, and in the nature of the form in which this determination has to be expressed. The Amount can indeed be made as exact as is necessary by a continuation of the series; but the representation through the series still remains an Ought; it is infected with a Beyond which cannot be transcended, because to express as Amount that which is based on *qualitative* determinateness, is an abiding contradiction.

This inexactitude, of which there is only an appearance in the true mathematical infinite, is actually present in this infinite series. These two kinds of mathematical infinite are as little to be confounded as the two kinds of philosophical infinite. In representing the true mathematical infinite the form of a *series* was originally used, and has again lately been invoked. But this form is not necessary for it; on the contrary the infinite of the infinite series differs from it essentially, as the sequel will show. Indeed, it is inferior even to the fraction as a form of expression.

For the infinite series contains bad infinity, since that which the series is designed to express remains as Ought; and what it does express is infected with a Beyond that never vanishes, and is *distinct* from what it is desired to express. It is infinite, not because of the number of its terms, but because they are incomplete, because the Other, which essentially belongs to them, is beyond them; that which it really contains (let the terms posited be as many as they will) is but something finite, in the proper sense, posited as finite—that is as something which is not what it ought to be. That, on the contrary, which is called the finite expression or sum of such a series, has nothing lacking: it contains fully the value which the series only seeks after; the Beyond is recalled from its flight; what it is, and what it ought to be, are not severed, but the same.

The differentia of these two is, more precisely, this: in the infinite series the negative is *outside* its terms, which are present only in so far as they count as parts of the Amount. On the other hand, in the finite expression, which is a ratio, the negative is immanent as being the mutual determinateness of the sides of the ratio; and this, being a return upon self or self-relating unity as negation of negation (for both the sides of the ratio exist only as moments), contains the determination of infinity.—Thus, in fact, what is ordinarily called sum $\left(\text{like } \frac{2}{7} \text{ or } \frac{1}{1-a}\right)$ is a ratio; and this so-called finite expression is the truly infinite expression. The infinite series, on the other hand, is in truth a sum: it is designed to set forth in the form of sum what is in itself a ratio, and terms of the series exist not as terms of a ratio but of an aggregate. And, further, it is, more properly, the finite expression; for it is an incomplete aggregate and essentially has something lacking. According to its actual content it is a determinate Quantum, but also a Quantum less than it should be; and that which is lacking is then also a determinate Quantum. This lacking part is in fact that which is called the infinite part of the series, in this merely formal meaning, that it is a something which is lacking, or a not-being; according to its content it is a finite Quantum. It is only that which the series contains, plus that which is lacking, which constitutes the fraction,—that determinate Quantum which it too ought to, but cannot, be.—In the infinite series, too, the word "infinite" generally enjoys the reputation of something lofty and venerable; this is a kind of superstition, the superstition of the understanding; we have seen that it reduces itself rather to the determination of a deficiency.

We may still remark that the existence of infinite series which cannot be summed is a circumstance altogether external and contingent with respect to the form of the series. These contain a higher kind of infinity than do the series which can be summed: they contain incommensurability,—that is to say, the quantitative ratio which they contain cannot be represented as Quantum even in the shape of a fraction. However, the *form* of series as such, which is theirs, contains the same determination of bad infinity that exists in the series which admits of being summed.

The terminological inversion remarked above in connexion with fractions and their equivalent series is found also in so far as the mathematical infinite—not that just mentioned, but the true one—has been called the *relative* infinite, and the ordinary metaphysical infinite on the other hand (by which is meant the abstract and bad infinite), the *absolute*. But, in fact, this metaphysical infinite is merely relative, because the negation which it expresses remains in the opposition of a limit only in such a way that the limit persists outside it and is not transcended by it; whereas the mathematical infinite has veritably in itself transcended its finite limit, because its Beyond is incorporated with it.

It is in this sense, in which it has just been demonstrated that the so-called sum or finite expression of an infinite series is in fact to be regarded as its infinite expression, that Spinoza opposes the concept of true infinity to that of bad, and elucidates it by means of examples. His concept gains the greatest clarity if I join his observations on this subject to my exposition.

He begins by defining the Infinite as the absolute affirmation of the existence of any one Nature, and the Finite on the contrary as determinateness, or negation. For the absolute affirmation of an existence must be taken as meaning its self-relation, the fact that its existence does not depend on that of an Other. The finite, on the other hand, is negation, that is, cessation as meaning relation to an Other which begins beyond it. It is true that the absolute affirmation of an existence does not exhaust the concept of infinity; for this concept also implies that infinity is affirmation not as immediate, but only as restored through the intro-Reflection of the Other, or as negation of the negative. But with Spinoza, Substance and its absolute unity have the form of an inert, that is, of a not self-mediating, unity,—of a rigidity wherein the concept of the negative unity of the self (Subjectivity) has not yet found a place.

Spinoza explains the true infinite by means of a mathematical example (*Epist. XXIX*), namely, a space contained between two unequal circles which are not concentric, while one lies within the other without touching it. It seems that he thought much of this figure and of the concept for which he used it as example: he made it the motto of his Ethics.—"Mathematicians," he says, "conclude that the inequalities which are possible in such a space are infinite, not from the infinite multitude of parts (for its magnitude is determinate and limited, and I can posit similar spaces both greater and smaller), but because the nature of the matter surpasses any determinateness."—Evidently Spinoza rejects that idea of the infinite according to which it is imagined as multitude or as incomplete series, reminding us that here in the space of the example the infinite is not beyond, but is present and complete; this space is limited, and is infinite, "because the nature of the matter surpasses all determinateness," because, that is, the magnitudinal determination which it contains cannot also be represented as a Quantum, or (to use the expression of Kant quoted above) because the synthesis cannot be completed into a (discrete) Quantum.—How in general the opposition of continuous and discrete Quantum leads to the infinite, must be explained in a later Observation.—Spinoza calls this serial infinite "the infinite of imagination," but the infinite as self-relation "the infinite of thought," or *infinitum actu*. For it is actually (*actu*) infinite, because it is complete and present in itself. Thus the series 0·285714 . . . or $1 + a + a^2 + a^3 + \ldots$ is merely the infinite of fancy or opinion, for it has no actuality and has something altogether lacking; while $\frac{2}{7}$ or $\frac{1}{1-a}$ is the actual infinite, and not only is what the series is in those of its terms which are given, but has also that which the series lacks and *is* what the latter only *ought* to be. The fraction $\frac{2}{7}$ or $\frac{1}{1-a}$ is also a finite magnitude, like Spinoza's space enclosed between two circles, with its inequalities; and, like that space, it may be increased or decreased. But this does not result in the absurdity of an infinite which may be greater or less; for this Quantum of the whole does not touch the ratio between its moments, the "nature of the matter," that is, the qualitative determination of magnitude. That which the infinite series presents to us is also a finite Quantum; but, further, it is defective.—But fancy stands still at Quantum as such, and does not reflect upon the qualitative relation that is the basis of the incommensurability which faces it.

That incommensurability which is implied in Spinoza's example includes in a general way the functions of curves, and, more exactly, leads to the infinite which mathematics has introduced with such functions and, generally, with the functions of variable magnitudes; this is the true mathematical or qualitative infinite, which also Spinoza had in mind. We will now discuss this determination in greater detail.

First, as regards that important category of *variability*, which comprises the magnitudes which are related in these functions, these are supposed to be variable, but not with that meaning in which in the fraction $\frac{2}{7}$ the two numbers 2 and 7 are variable, since 4 and 14, 6 and 21, and other numbers to infinity, may be put in their place without altering the value denoted in the fraction. Similarly, in $\frac{a}{b}$ any number may be put in place of a or b without altering what $\frac{a}{b}$ is intended to express. It is only in this sense (that an infinite, that is, an inexhaustible, multitude of numbers may be put in the place of the x and y of a function) that a and b are variable magnitudes as much as that x and that y. And hence the expression "variable magnitudes" is very vague and ill-chosen for determinations of magnitude whose interest and manner of treatment lie in something quite different from their mere variability.

In order to make clear where lies the true determination of the moments of a function to which higher analysis devotes its attention, we must once more run through the stages to which attention has already been drawn. In $\frac{2}{7}$ or $\frac{a}{b}$, 2 and 7 are each a determinate Quantum, and their relation is not essential to them; and a and b are likewise meant to represent such Quanta as remain what they are even outside the ratio. And, further, $\frac{2}{7}$ and $\frac{a}{b}$ are each a fixed Quantum, a quotient; the ratio constitutes a number of which the unit is expressed by the denominator and the Amount of these units by the numerator, or conversely; and even when 4 and 14, or other numbers, are substituted for 2 and 7, the ratio, as a Quantum, remains the same. But in the function $\frac{y^2}{x} = p$, for example, this is essentially changed; for, though here x and y have the meaning that they can be determinate Quanta, it is not x and y but only x and y^2 which here have a determinate quotient. Hence these sides of the ratio, x and y, first are not determinate Quanta, and secondly the ratio subsisting between them is not a fixed quotient or Quantum (nor is such a Quantum intended as with a and b); but this quotient, simply as a Quantum, is itself variable. But all this is implied in the fact that x is related, not to y, but to the square of y. The relation of a magnitude to a power is not a Quantum, but is essentially qualitative: the power-relation is the circumstance which must be regarded as the fundamental determination.—But in the function of the straight line, $y = a x$, $\frac{y}{x} = a$ is an ordinary fraction and quotient; hence this function is only *formally* a function of variable magnitudes, and here x and y are what a and b are in $\frac{a}{b}$; that is, they are not in that determination in which the differential and the integral calculus consider them.—On account of the peculiar nature of variable magnitudes, in this point of view it would have been well done to have introduced a name for them and also symbols other than those used for ordinary unknown magnitudes in any finite equation determinate or indeterminate; and this because of their essential difference from such merely unknown magnitudes, which in themselves are wholly determinate Quanta, or a determinate range of determinate Quanta.—And it is only because the peculiarity of that which constitutes the interest of the higher analysis, and has led to the need for and invention of the differential calculus, is not fully understood, that functions of the first degree, like the equation of the straight line, have been included by themselves in the treatment of this calculus. This formalism results, further, from the misconception which thinks to fulfil the just demand for the universalization of a method by neglecting the specific determinateness on which the need is based, so that here the assumption is made that in this sphere only variable magnitudes in general are dealt with. Much formalism in the consideration and treatment of these objects would have been spared if it had been understood that it dealt, not with variable magnitudes as such, but with determinations of powers.

But the mathematical infinite shows itself in its peculiar nature at yet a further stage. In an equation where x and y are posited primarily as determined through a ratio of powers, x and y as such are still meant to denote Quanta: now this meaning is entirely lost in the so-called infinitesimal differences. dx and dy are no longer Quanta and are not supposed to signify such; they have a significance only in their relation, a meaning merely as moments. They no longer are Something (Something being taken as Quantum), nor are they finite differences; but also they are not Nothing or the indeterminate nil. Apart from their relation they are pure nil; but they are meant to be taken only as moments of the relation, as determinations of the differential coefficient $\frac{dx}{dy}$.

In this concept of the infinite, Quantum veritably is perfected into a qualitative Determinate Being; it is posited as actually infinite, and is transcended not only as being this or that Quantum, but as Quantum in general. And yet the quantitative determinateness as element of Quanta remains the principle, or, as has also been said, it remains part of their primary concept.

All the attacks which have been made upon the fundamental determination of the mathematics of this infinite, that is, on the differential and integral calculus, have been directed against this concept. The mistaken ideas of mathematicians themselves prevented it from being justly recognized; but mainly these attacks are due to the impossibility of justifying this object as a concept. But, as has been mentioned above, mathematics cannot here evade the concept; for, being the mathematics of the infinite, it does not confine itself to the finite determinateness of its objects (as does genuine mathematics, where space and number and their determinations are considered and related only according to their finitude); but, on the contrary, it transplants into identity with its opposite a determination already taken from that study and treated by it, as for instance it makes a curve into a straight line, or a circle into a polygon, and so on. The operations which it presumes to undertake as differential or integral calculus are, therefore, quite contradictory to the nature of merely finite determinations and their relations; their only justification, therefore, would be in the concept.

Now when the mathematics of the infinite still maintained that these quantitative determinations were vanishing magnitudes, that is, magnitudes which no longer are any Quantum, but also are not nothing, being merely a determinateness as opposed to Other, it seemed abundantly clear that such an intermediate state, as it was called, between Being and Nothing did not exist.—The nature of this objection and of the so-called intermediate state has been indicated above under the category of Becoming, Observation 4. The unity of Being and Nothing is not indeed a *state*; for a state would be a determination of Being and Nothing such as might have been reached by these moments only contingently, as it were through disease or external influence, and through erroneous thinking;

but, on the contrary, this mean and unity, this vanishing and, equally, Becoming, is, in fact, their only truth.

It has further been said that infinities cannot be compared as greater or less, and that, therefore, there could be no ratio between infinite and infinite in their ranks and orders, as in the distinctions of the infinite differences that occur in the science which concerns itself with them.—This objection, already referred to, is based upon the idea that Quanta are here in question, and are being compared as such; and that determinations which are Quanta no longer, have no longer the relation of ratio. But, on the contrary, that which is *only* in such a relation is no Quantum: Quantum is a determination which is supposed to have a perfectly indifferent existence apart from its ratio, and its distinctness from an Other is supposed to be indifferent to it: while that which is qualitative is that only which it is in its distinctness from an Other. These infinite magnitudes, therefore, are not only comparable, but exist only as moments of the comparison or ratio.

I proceed to enumerate the chief determinations which have been offered in mathematics about this infinite; it will appear that, although they are based upon the thought of the real matter, agreeing with the concept here developed, its authors did not fully probe it as concept, and in applying it were again forced to have recourse to means dissonant with their better cause.

No correcter determination of the thought can be made than that offered by Newton. I here set apart the determinations belonging to the idea of motion and velocity (from which latter chiefly he took the name of *fluxions*), for here the thought appears not in its due abstraction, but concrete and mixed with unessential forms. Newton explains these fluxions (*Princ. Mathem. Phil. Nat. L. 1. Lemma XI. Schol.*) by saying that he takes them not as indivisible (a form used by earlier mathematicians, Cavalieri[1] and others: it contains the concept of a Quantum determinate in itself), but as vanishing *divisibilia*; and, further, not as the sums and ratios of determinate parts, but as the limits (*limites*) of the sums and ratios. It will be objected, he says, that vanishing magnitudes have no final ratio, because the ratio before they vanish is not the last, and, after they have vanished, no longer exists. But by the ratio between vanishing magnitudes must not be understood the ratio which exists either before or after, but that *with* which, they vanish (*quacum evanescunt*). And, similarly, the first ratio of becoming magnitudes is that *with which* they arise.

---

[1] Cavalieri, Francesco Bonaventura, 1598 1647, Professor of Mathematics at Bologna: *Geomet ia indivisibilium continuorum nova*, 1635; *Exercitationes geometricae*, 1647

Newton did what was demanded by the stage which scientific method had reached in his day—he only explained what was to be understood by a certain expression; but really the demand for such a particular explanation is subjective, or historical, and it is not demonstrated that such a concept is necessary in itself and possesses inner truth. But what has been quoted shows that the concept as posited by Newton conforms with the development of infinite magnitude from the reflexion of Quantum, as it was set out above. He means magnitudes which are at the point of vanishing, that is, are Quanta no longer; and not ratios between definite parts, but limits of the ratio. Thus, the Quanta in themselves, the sides of the ratio, and also the ratio in itself (that is, in so far as it is Quantum), are supposed to vanish; the limit of the magnitudinal ratio is that point where it is and is not,—or, more precisely, where the Quantum has vanished, and the ratio, therefore, is preserved only as qualitative quantity-ratio, and its sides as qualitative quantity-moments.—Newton adds that it must not be concluded from the existence of final ratios between vanishing magnitudes that there are final magnitudes or *indivisibilia*. For this would be another leap from the abstract ratio to such sides of it as should have a value for themselves outside their relation, as being *indivisibilia* or something that is a One, a non-relational entity.

And further, to refute this error he reminds us that final ratios are not ratios between final magnitudes, but limits to which the ratios of the magnitudes decreasing without limit are nearer than any given (that is, finite) difference; but they never overstep this limit so as to become Nothing.—It has been said above that by final magnitudes *indivisibilia*, or Ones, might have been understood. But in the determination of final ratio the idea of indifferent One (non-proportional Quantum), as well as that of finite Quantum, is absent. But, if the suggested mode of determination had been developed into the concept of a form of magnitude which is simply a moment of ratio, there would be need neither of that decrease without limit into which Newton transposes Quantum (and this expresses only the progress to infinity), nor of the characteristic of divisibility, which here no longer has any immediate meaning.

In connexion with the preservation of the ratio at the vanishing of Quanta, the expression is elsewhere found (for instance in Carnot,[1] *Réflexions sur la Métaphysique du Calcul Infinitésimal*) that, by virtue of the law of continuity, vanishing magnitudes still retain the ratio whence they arise before they vanish.—This idea expresses the true nature of the matter in so far as it is not the continuity of Quantum which is meant, for this continuity of the Quantum would consist in the endless process by which it continues itself into its own vanishing in such a manner that in its Beyond there only arises again a finite Quantum, a new term of the series; while a continuous progress is always imagined as such that the values, which still are finite Quanta, are passed through. But where the transition made is to the true infinite, it is the ratio that is continuous; and it is so completely continuous and self-supporting that the transition just consists in presenting the ratio in its complete purity and causing the non-relational factor (that is, the concept of a Quantum which is one term of a ratio and yet also a Quantum apart from this relation) to vanish.—This purification of quantitative ratio is exactly analogous to what happens when an empirically given existent is grasped by conceptual thought. Such an existent is thereby raised above itself in such a manner that its concept contains the same determinations as itself, but taken in their essentiality and in the unity of the concept, wherein they have lost their indifferent and non-conceptual persistence.

The other form of Newton's exposition of the magnitudes in question—namely, generative magnitudes or principles—is equally interesting. A generated (*genita*) magnitude is a product or quotient, roots, rectangles, squares, and also the sides of rectangles and squares, and, in general, any finite magnitude.—"It is considered as variable in its incessant motion and flow of increase or decrease, and to its momentary augmentation and diminution he gives the name of *Moments*. These, however, must not be taken as particles of determinate magnitude (*particulae finitae*). They are not moments themselves, but magnitudes produced by moments; the generative principles or beginnings of finite magnitudes must here be understood." —An internal distinction is here made in Quantum; it is taken first as product or Determinate Being, and next in its Becoming, in its beginning and principle, that is, as it is in its concept or (which is here the same thing) in its qualitative determination. In the latter the quantitative differences, the infinitesimal incrementa or decrementa, are moments only; and it is only in what has been generated that we have Quantum or that which has passed over into the indifference of determinate existence and into externality.—But, although the philosophy of the true concept must acknowledge those determinations of the infinite just mentioned in the consideration of the incrementa and decrementa, it must also be observed that the forms themselves of incrementa (and so on) fall within the category of absolute Quantum and of the continuous progress to which we have alluded; indeed, the ideas of increment, of addition, of growth of x by dx or i, and so on, must be considered the fundamental evil inherent in these methods,—as an enduring obstacle which makes it impossible clearly to disengage the determination of the qualitative moment of quantity from the idea of ordinary Quantum.

---

[1] Carnot, Lazare Nicolas Marguerite, Count, 1753–1823, "organizer of victory" in the republican armies, equally important as politician and soldier until his banishment in 1815, died at Magdeburg: *Réflexions, etc.*, 1797.

The idea of infinitely small magnitudes (latent also in increment and decrement) is far inferior to the mode of conception indicated. This idea supposes them to be of such a nature that they may be neglected in relation to finite magnitudes; and not only that, but also their higher orders relatively to the lower order, and the products of several relatively to one.—With Leibniz this demand to neglect (which previous inventors of methods referring to this kind of magnitude also bring into play) becomes more strikingly prominent. It is this chiefly which gives an appearance of inexactitude and express incorrectness, the price of convenience, to this calculus in the course of its operation.—Wolf has attempted to make it intelligible in his manner of popularizing a problem, which is to confuse the concept and to put in its place incorrect sensuous images. He compares the neglect of infinitesimal differences of higher orders relatively to lower with the procedure of a surveyor who, in measuring the height of a mountain, is none the less exact if meanwhile the wind has blown away a grain of sand from the top; or with the neglect of the height of houses or towers in a calculation of eclipses of the moon (*Element. Mathes. univ. Tom. I. El. Analys. math. P. II. C. 1. s. Schol.*).

Common sense in fairness admits such an inaccuracy; but all geometers have rejected this idea. It is perfectly self-evident that in the science of mathematics such empirical accuracy is not at all in question, and that mathematical measurement by means of the operations of the calculus or the constructions and proofs of geometry differs altogether from surveying, from the measuring of empirical lines, figures, and so on. And this apart, analysts (as mentioned above) demonstrate, by a comparison of the result as reached in a strictly geometrical way and by the method of infinitesimal difference, that one is the same as the other and that there is, absolutely, no more nor less of exactness. And it is obvious that an absolutely exact result could not be obtained by an inexact procedure. And yet, on the other hand, the procedure itself cannot do without this neglect of the "insignificant," in spite of all protests against the method of justification just quoted. And this is the difficulty about which the analysts are at pains,—to make intelligible and to remove the inherent anomaly.

In this regard Euler's[1] idea especially must be cited. On the basis of Newton's general definition, he insists that the differential calculus considers the ratios of the incrementa of a magnitude, while the infinitesimal difference as such is to be regarded wholly as nil (*Institut Calc. different. P.I.C. III.*).—It will be clear from the above how this is to be understood: the infinitesimal difference is nil only quantitatively, it is not a qualitative nil, but, as nil of quantum, it is pure moment of a ratio only. There is no magnitudinal difference; but for that reason it is, in a manner, wrong to express as incrementa or decrementa and as differences those moments which are called infinitely small magnitudes. This point of view has for basis the thought that something is joined to or taken from the finite magnitude first given, that subtraction or addition—an arithmetical external operation—takes place. But the transition of the function of the variable magnitude into its differential must be considered; it is of quite different nature, and in fact, as decided above, it must be regarded as the reduction of the finite functions to the qualitative ratio of their determinations of quantity.—On the other hand, the difficulty is self-evident when it is said that for themselves the incrementa are each nil, and that only their ratios are being considered; for a nil is altogether without determinateness. Thus this image, although it reaches the negative aspect of Quantum and expressly asserts it, yet does not simultaneously seize this negative in its positive meaning of qualitative determinations of quantity, which, if torn away from the ratio and treated as Quanta, would each be but a nil.—Lagrange[1] (*Théorie des fonctions analytiques*, *Introd.*) judges, regarding the idea of limits or final ratios, that while the ratio between two magnitudes may easily be imagined so long as they remain finite, this ratio offers to understanding no clear and determinate concept when its terms simultaneously become nil.—And indeed understanding must pass beyond this merely negative side, where each member of the ratio is a nil as Quantum, and must take them positively, as qualitative moments.—But we cannot regard as satisfactory Euler's further remarks (*loc. cit.* § 84 sqq.) with regard to this theory of his, by way of proving that two so-called infinitesimal magnitudes, which ought to be nothing but nil, yet stand in a ratio to one another; on which account they are commonly denoted, not by the symbol for nought, but by others. He attempts to base this upon the distinction between arithmetical and geometrical relations: in the former we consider the difference, in the latter the quotient, and, although the former is the same between any two noughts, this is not true of geometrical ratio; if $2 : 1 = 0 : 0$, then it would follow from the nature of the proportion (since the first term is twice as great as the second) that the third term is twice as great as the fourth; thus, according to the proportion, $0 : 0$ is to be taken as being the ratio of $2 : 1$.—And even in common arithmetic $n.0 = 0$; and therefore $n : 1 = 0 : 0$.—But it is just because $2 : 1$ or $n : 1$ is a ratio between Quanta that no ratio nor designation like $0 : 0$ is adequate to it.

---

[1] Euler, Leopold, 1707–1783, Professor at S. Petersburg, at Berlin, and again at S. Petersburg: *Introductio in analysin infinitorum*, 1748; *Institutiones calculi differentialis*, 1755; *Instit. calc. integralis*, 1768–1794.

---

[1] Lagrange, Jos. Louis, 1736–1812, Euler's successor at Berlin, then Professor at the *Ecole Polytechnique* in Paris: *Théorie des fonctions analytiques*, 1797.

I refrain from citing any further attempts. Those which we have considered have shown sufficiently that they contain the true concept of the infinite, but that it is not disengaged nor seized upon in its determinateness. Therefore, when mathematicians proceed to practice, it is impossible for the true conceptual determination to assert itself therein: the finite determinateness of quantity returns, and the operation can no longer do without the idea of a Quantum which is merely relatively small. The calculus makes it necessary to subordinate the so-called infinite magnitudes to the ordinary arithmetical operations of addition and so on (which are based on the nature of finite magnitudes), and thus to count and treat them for a moment as finite magnitudes. And on the calculus rests the burden of justifying this method, where first, drawing them down into this sphere, it treats them as incrementa or differences, and then neglects them as Quanta after having applied to them the forms and laws of finite magnitudes.

I now recount the chief heads of the attempts made by the geometers to remove these difficulties.

The older analysts had small scruples in this matter; the efforts of the moderns were chiefly directed towards bringing back the infinitesimal calculus to the evidence of a "properly geometrical method," and thereby attaining in mathematics to the "strictness of proof of the ancients." (These are Lagrange's expressions.) But since the principle of the analysis of the infinite is of higher nature than the principle of the mathematics of finite magnitudes, the former immediately was forced to renounce this kind of evidence; just as philosophy cannot claim the kind of obviousness proper to the sciences of the sensuous, such as natural history,—and indeed eating and drinking are counted a more intelligible business than thinking and understanding. We shall thus deal only with the attempts to reach the strictness of proof of the ancients.

Many have tried altogether to discard the concept of the infinite, and to effect their aim while dispensing with what seemed indispensable.—Lagrange speaks for example of the method invented by Landen,[1] and says that it is purely analytical and does not employ infinitesimal differences, but at first introduces different values of the variable magnitudes, and, in the course of the demonstration, makes them equal. But he considers that in other respects the advantages peculiar to the differential calculus—simplicity of method and ease in operation—are here lost.—Probably this procedure has something which corresponds to the starting point of Descartes' tangential method, of which further mention must be made below. But we may here remark that this at least is clear, that the general procedure which assumes different values of the variable magnitudes and afterwards makes them equal, belongs to a different sphere of mathematical treatment from that to which the method of the differential calculus itself belongs; and that the peculiarity of the simple relation (to be more fully considered later) to which the actual concrete determination of the calculus reduces itself, namely, the relation of the derivative to the original function, is not here emphasized.

The older of the moderns (such as Fermat,[2] Barrow,[3] and others), who first used the infinitesimally small in that application which was later developed into differential and integral calculus, and, subsequently, Leibniz and his successors, with Euler, always undisguisedly believed that they could neglect the products and the higher powers of infinitesimal differences only because they vanish relatively to the lower order. It is on this alone that with them the fundamental proposition is based, namely, the determination of what the differential of a product or a power is; for to this all their theoretical doctrine reduces itself. The rest is partly the mechanism of development and partly also application, in which, however (as must further be considered), in fact lies the higher or rather the only interest.—With respect to our present subject, only the elementary statement need here be made that, for the same reason of insignificance as caused the acceptance of the capital proposition, it is assumed, with regard to curves, that the elements of curves, the incrementa of abscissae and ordinates, are in the relation of subtangents and ordinates to one another; and for the purpose of obtaining similar triangles, the arc which constitutes the third side of a triangle together with the two incrementa of the characteristic triangle (as it used rightly to be called formerly) is regarded as a straight line and as part of the tangent, and thus one of the incrementa is regarded as reaching the tangent. By these assumptions those determinations are, on the one hand, raised above the nature of finite magnitudes; on the other hand a procedure is applied to the moments (now called infinitesimal) which is valid only for finite magnitudes; and where this is applied nothing may be neglected on the ground of insignificance. With such a mode of procedure, the difficulty which obsesses the method remains in all its force.

---

[1] Landen, John, English mathematician, 1719–1790: *Mathematical Lucubrations*, 1755, *etc.*

[2] Fermat, Pierre de, 1601–1665: *Varia opera mathematica*, 1679.

[3] Barrow, Isaac, 1630–1677, Professor at Cambridge: *Lectiones geometricae*, 1669 *Lectiones opticae*, 1674.

A remarkable procedure of Newton's may here be noted (*Princ. Math. phil. nat. Lib. II. Lemma II.* after *Propos. VII.*)—the invention of an ingenious trick designed to remove the neglect, arithmetically invalid, of the products of infinitesimal differences or of higher orders of these in the process of finding differentials. He finds the differential of the product (whence the differentials of quotients, powers and so on are then easily derived) in the following manner. When x and y are each taken as smaller by half its infinitesimal difference, the product passes over into $xy - \frac{x\,dy}{2} - \frac{y\,dx}{2} + \frac{dx\,dy}{4}$; but if x and y are increased by the same amount, it passes over into $xy + \frac{x\,dy}{2} + \frac{y\,dx}{2} + \frac{dx\,dy}{4}$. If the first product be subtracted from this second product, there remains over $y\,dx + x\,dy$ as surplus, and this is said to be the surplus due to an augmentation by a whole dx and dy, for this augmentation is the difference between the two products; it, therefore, is the differential of xy.—Clearly, in this procedure, the term which forms the chief difficulty, the product of the two infinitesimal differences, $dx\,dy$, drops out by itself. But, in spite of the name of Newton, it must be said that such an operation, though very elementary, is incorrect;

it is not true that $\left(x+\frac{dx}{2}\right)\left(y+\frac{dy}{2}\right)-\left(x-\frac{dx}{2}\right)\left(y-\frac{dy}{2}\right)$ $=(x \pm dx)(y+dy)-xy$. It can only have been the need to justify the fluxional calculus in its importance which could bring a Newton to deceive himself with such a proof.

Other forms which Newton employed in the derivation of the differential are bound up with concrete meanings (having reference to motion) of the elements and their powers.—In the case of the serial form, which elsewhere characterizes his method, the statement suggests itself that it is always possible to take the magnitude as exactly as required by the addition of further terms, and that those which are left out are relatively insignificant and the result only an approximation; and not that here too he was satisfied with that ground on which, in his method of solving equations of higher degrees by approximation, he omits the higher powers (which arise when any value, just found and as yet inexact, is substituted in the given equation)—namely for the crude reason that they are small; see *Lagrange*, *Équations Numériques*, p. 125.

The error into which Newton fell in the solution of a certain problem through neglecting higher powers which were essential to the problem, gave his opponents an opportunity of triumph for their method over his: Lagrange, in his recent investigation of it (*Théorie des fonctions analytiques*, 3*me P.*, *Ch. IV.*), has shown its true origin; and this mistake proves the formality and unreliability then still inherent in the use of this instrument. Lagrange shows that Newton made the mistake because he neglected the term of the series which contained that power which was essential in the given problem. Newton had remained faithful to the formal and superficial principle of omitting terms because of their relative smallness.—For it is known that in mechanics there is attributed a definite meaning to the terms of a series in which the function of a motion is developed, so that the first term (or the first function) refers to the moment of velocity, the second to accelerating force, and the third to the resistance of forces. Thus here the terms of the series must not only be regarded as part of a sum, but as qualitative moments of a conceptual whole. For this reason, if here the other terms of the series (infinite in the bad sense) are omitted, this has a significance wholly different from that which belongs to their omission on the ground of relative smallness.[1] The Newtonian solution contained this mistake, not because in it terms of the series were taken account of only as parts of a sum, but because the term containing the qualitative determination (and this was the essential matter) was neglected.

In this example, the procedure is made dependent upon the qualitative meaning. And here the general assertion can be made that the whole difficulty of the principle would be removed, if the qualitative meaning of the principle were indicated and the operation made dependent upon it, in place of the formalism which identifies the determination of the differential with the problem which gives it its name, the distinction generally between a function and its variation after its variable magnitude has received an increment. In this sense, it is clear that the first term of the series resulting from the development of $(x + dx)^n$ quite exhausts the differential of x^n. Thus the neglect of the other terms is not due to their relative smallness;—and no inexactitude, no mistake nor error is here assumed which is supposed to be compensated or rectified by another error,—a point of view from which many, and notably Carnot, justify the ordinary method of the infinitesimal calculus. A ratio, and not a sum, is here in question; and, therefore, with the first term the differential is fully found; and where further terms and differentials of higher orders are required, their determination does not imply the prolongation of a series as sum, but the repetition of one and the same ratio, which alone is aimed at and is complete already in the first term. The need for the serial form, its summation, and all therewith connected, must be quite separated from this interest in ratio.

---

[1] Lagrange in a simple manner sets side by side the two considerations in the application to mechanics of the theory of functions, in the chapter on rectilinear motion (*Théorie des fonct.*, *3me P.*, *Ch. I*, *art. 4*). The space passed through, considered as a function of the time elapsed, gives the equation $x = ft$; this developed as $f(t+\theta)$ gives: $ft + \theta f' t + \frac{\theta^2}{2} f'' t + \ldots$

Thus the space passed through in the period of time presents itself as $= \theta f' t + \frac{\theta^2}{2} f'' t + \frac{\theta^3}{2 \times 3} f''' t + \ldots$ The motion, therefore, by means of which passage through this space takes place, is composed, as has been said, of various partial motions (and this, because analytical development produces a plurality and in fact an infinite plurality of terms), the spaces of which—corresponding to the time—will be $\theta f' t$, $\frac{\theta^2}{2} f'' t$, $\frac{\theta^3}{2 \times 3} f''' t$, etc. When the motion is known, the first partial motion is formally uniform and has a velocity determined by $f' t$, the second uniformly accelerated and derived from an accelerative force proportional to $f'' t$. "Now the remaining terms are related to no known simple motion, and, therefore, they need not be specially taken into account; and it will be shown that we can abstract from them in determining the motion at the beginning of the point of time." There now follows the demonstration, which, however, is only a comparison of the series *all* of whose terms are required to determine the magnitude of the space passed through in the period of time, with the equation given in *art. 3* (for the motion of falling), namely $x = at + bt^2$, in which only these two terms occur. But this equation has itself received this form only because the explanation given to the terms evolved by analytical development is presupposed; and this presupposition is, that uniformly accelerated motion is composed of a formally uniform motion, continued at the velocity reached in the previous period of time, and of an increment (which is a in $s = at^2$, that is, the empirical coefficient) to which the force of gravity is ascribed;—a distinction which has no existence nor reason in the nature of the matter, and is only the expression—erroneously physicized—of the result of the analytical treatment assumed.

The explanations of Carnot concerning the method of infinitesimal magnitudes contain, most lucidly exposed, all that is soundest in the ideas cited above. But in the transition to practice the usual ideas about the infinite smallness of the terms omitted, relatively to the others, more or less emerge. And Carnot justifies his method by the fact that the results turn out correct, and by the advantage coming from the introduction of such imperfect equations (as he calls them—meaning those where there has been such an arithmetically incorrect omission) in the simplification and abbreviation of the calculus: he does not justify it from the nature of the matter itself.

It is well known that Lagrange resumed Newton's original method, the method of series, in order to be rid of the difficulties inherent in the idea of the infinitesimally small and in the method of first and last ratios and limits. The advantages of his functional calculus for exactitude, abstractness, and universality are generally acknowledged: here it is relevant to mention that it rests on the fundamental proposition that the difference, without becoming nil, may yet be taken as so small that any term of the series is greater than the sum of all the following terms.—In this method too a beginning is made from the categories of augmentation and the difference of the function, whose variable magnitude receives the augmentation from the original function, thereby introducing that tiresome series; while in the sequel the terms of the series which must be omitted are important only in the respect that they constitute a sum,—the reason of their omission being placed in the relative nature of their Quantum. Here too, therefore, the omission is not in a general way reconducted to that point of view which partly occurs in some applications, where (as was recalled above) the terms of the series are supposed to have a definite qualitative meaning, and terms are neglected because they are qualitatively insignificant, and not because they are insignificant in magnitude; but then partly the omission itself finds no place in the essential point of view, which, for the so-called differential coefficient, manifests itself only with Lagrange in the so-called application of the calculus, —as will be explained more fully in the next Observation.

That element of the form of magnitude under discussion which is called the infinitesimally small has been proved to partake of a qualitative character; and this is found most immediately in the category of the *Limit of Ratio* cited above, the consistent use of which in the calculus has become a characteristic and peculiar method. Lagrange criticizes this method as lacking ease in application and offering no definite idea in the expression "limit": we will take up the second criticism and consider more closely what is stated about its analytical significance. For the idea of limit does contain the valid category mentioned of qualitative determination of the ratio of variable magnitudes; for the forms which these introduce (dx and dy) must be regarded simply as moments of $\frac{dy}{dx}$, and $\frac{dx}{dy}$ itself as a single and indivisible symbol. We may disregard the fact that the practical application of the mechanism of the calculus loses the advantage which it derives from the separation of the sides of the differential coefficient. Now this limit is supposed to be Limit of a given function;—to indicate a certain value with reference to it, which value determines itself according to its derivation. The mere category of limit, however, would not take us beyond the scope of this Observation, which is to show that the infinitesimally small (which appears as dx and dy in the differential calculus) has not merely the negative and empty meaning of a not finite and not given magnitude (as when we say "an infinite multitude," "to infinity," and the like), but also the definite meaning of a qualitative determinateness of the Quantitative, or a moment of ratio as such. As yet, however, this category has no relation to what a given function is, and does not interfere in the treatment of it nor in the use made of this determination in it; and thus the idea of limit too, if confined to this demonstrated determinateness, would lead nowhere. The term "limit" itself, however, implies that it is the limit of something, that is, that it expresses a certain value contained in the function of variable magnitude; and we must here investigate the nature of this concrete behaviour.—It is supposed to be the limit of the ratio between the two increments by which the two variable magnitudes connected in an equation (one being regarded as a function of the other) were taken to be increased; the augmentation is here taken as just indeterminate, and, in so far, the infinitesimally small is not employed. But, first, the approach by which this limit is found involves the same inconsistencies as are implied in all the other methods. The approach is as follows. If $y = fx$, then fx, when y passes over into $y + k$, is to change into $fx + ph + qh^2 + rh^3 + \ldots$, and thus $k = ph + qh^2 + \ldots$, and $\frac{k}{h} = p + qh + rh^2 + \ldots$ Now if k and h vanish, the second member, except p, also vanishes: p thus is the limit of the ratio of the two increments. Clearly h (as Quantum) is equated with 0, while $\frac{k}{h}$ is nevertheless not supposed to equal $\frac{0}{0}$, but still to remain a ratio. Now the idea of the *limit* is supposed to afford the advantage of averting the implied inconsistency; and p at the same time is supposed to be, not the actual ratio $\left(\text{which would be} = \frac{0}{0}\right)$, but only the determinate value to which the ratio may approximate infinitely, that is, in such a manner that the difference can become smaller than any given difference. The closer meaning of approximation (with respect to the terms which are supposed to approximate) will be considered below.—But if anything can be self-evident in mathematics, then this is self-evident, that a quantitative difference which has the determination, not only that it can, but that it shall, be smaller than any given difference, has ceased to be a quantitative difference; and thus no progress has been made beyond $\frac{dy}{dx} = \frac{0}{0}$. If, on the other hand, $\frac{dy}{dx}$ is assumed to be equal to p, that is, is assumed as a determinate quantitative ratio —which in fact is the case—then there are difficulties for the assumption which makes $h = 0$; and by this assumption alone $\frac{k}{h} = p$ is found. Again, grant that $\frac{k}{h} = 0$;—and when $h = 0$, then, in fact, automatically $k = 0$; for the increase of k into y takes place only on the condition that the increment be h. But now the question arises, what p—which is a perfectly determinate quantitative value—is to be. A simple, barren answer immediately offers itself: it is a coefficient, and the derivation whence it arises is this,—the first function, derived in a determinate manner, of a primary function. It is possible to remain satisfied with this; in practice Lagrange did so; and then the general part of the science of the differential calculus, and, immediately, that part of it which is called Theory of Limits, would be freed from the incrementa, from their infinitesimal or arbitrary smallness, from the difficulty of removing again, beyond the first term, or rather only the coefficient of the first term, the further terms of a series which inevitably arrive with the introduction of these incrementa; it would be purged, too, of what follows further, the formal categories (chiefly that of the Infinite), of infinite approximation, and of the further categories (here equally void) of continuous magnitude,[1] and those which elsewhere (like nisus, or becoming,

[1] The category of continuous or flowing magnitude arises in the consideration of external and empirical variation of magnitudes,—which, by an equation, are brought into this relation, that one is a function of the other. Now the scientific object of the differential calculus is a certain ratio (generally expressed by the differential coefficient)—a determinateness which might also be called *Law*; and, therefore, for this specific determinateness mere continuity is partly a foreign aspect, and partly and in any case an abstract and here empty category, since nothing is here stated about the Law of Continuity.—The ingenious General Exposition of the fundamental determinations used in the deduction of the differential calculus by my respected colleague, Professor Dirksen,* which forms

---

* Dirksen, Enno Heeren, 1792–1850, Professor of Mathematics at Berlin: *Analytische Darstellung der Variationsrechnung*, 1823.

or opportunity for change) are deemed necessary. But now it would be necessary to show what p is apart from the bare determination (quite adequate for the theory) that it is nothing but a function derived from the development of a binomial, and what is its further meaning and value, that is, how it is connected with and employed for further mathematical requirements: this will be the subject of the second Observation.—But first we add an explanation of the confusion which has been introduced into the view of the proper and qualitative determinateness of Ratio by the common use, already exemplified, of the idea of approximation.

It has been shown that the so-called infinitesimal differences express the vanishing of the sides of the ratio as Quanta, and that what remains is their quantitative ratio, purely in so far as it is qualitatively determined; so little is the qualitative ratio here lost, that it is precisely the very result of the conversion of finite magnitude into infinite. And we have seen that this is the very nature of the matter.—Thus, for instance, in the ultimate ratio the Quanta of abscissae and of ordinates vanish; but the sides of this ratio essentially remain, the one an element of the ordinates, and the other of the abscissae. When that form of image is used in which one ordinate is allowed infinitely to approach the other, the one ordinate formerly distinct passes over into the other, and the previously distinct abscissa into the other abscissa; but it is an essential fact that ordinate does not pass over into abscissa nor abscissa into ordinate. The element of ordinate (to go no further than this example of variable magnitudes) must not be taken as the difference between one ordinate and another, but rather as the difference (or qualitative magnitudinal determination) relatively to the element of abscissa: the principle of one variable magnitude relatively to that of the other—these two are here in ratio with one another. The difference, no longer being a difference between finite magnitudes, has ceased to

---

an appendix to a criticism of certain new treatises on this science (*Jahrb. f. wissensch. Kritik*, 1827, Nr. 153 sqq.), will show what kind of formal definitions have been resorted to in this matter. The following definition (*op. cit.*, p. 1251) is actually quoted: "Every magnitude which is considered as in the state of becoming, provided that this becoming does not take place by leaps, but in an uninterrupted progress, is a constant or continuous magnitude, a Continuum." Surely this is a tautology, and the same as the *definitum*.

be a manifold within itself: it has collapsed into simple intensity, into the determinateness of one qualitative moment of the ratio relatively to the other.

This aspect of the matter is, however, obscured when that which we have just called element (element, for instance, of the ordinate) is taken as difference or increment in the sense that it is only the numerical difference of the Quantum of one ordinate from the Quantum of another ordinate. Thus Limit has not here the meaning of ratio: it counts only as the ultimate value which another and similar magnitude steadily approaches in such a manner, that the difference between them may be as small as desired, and the ultimate relation be a relation of equality. Thus the infinitesimal difference is the ghost of a difference between one Quantum and another, and, when it is thus imagined, the qualitative nature, according to which dx is related essentially as a determination of ratio not to x but to dy, is in the background. Relatively to dx, dx^2 is allowed to vanish; but dx vanishes still more relatively to x; and this, justly considered, means that it is related only to dy. —In this kind of demonstration, geometers are chiefly at pains to make intelligible the approximation of a magnitude to its limit, clinging to this aspect of the difference between Quantum and Quantum, where it is no difference and yet still is a difference. But in any case Approximation is a category which in itself means and makes intelligible nothing; dx has already passed through approximation, it is neither near nor is it nearer; and "infinitely near" itself means the negation of nearness and of approximation.

The process thus has been this, that the incrementa or infinitesimal differences have been considered only from the side of the Quantum which vanishes in them, and as the limit of this: they are, then, taken as unrelated moments. From this the inadmissible idea would follow, that it would be permissible to equate in the ultimate ratio abscissae and ordinates, or else sines and cosines, tangents, and versed sines,—anything, in fact.—This idea seems to be operating when a curve is treated as a tangent; for the curve too is incommensurable with the straight line, and its element of a different quality from the element of the straight line. And it seems even more irrational and less permissible than the confusion of abscissae and ordinates, versed sine and cosine, and so forth, when (*quadrata rotundis*) a part—though infinitely small—of a curve is taken for part of a tangent and thus is treated as a straight line.—However, this treatment differs essentially from the confusion we have just denounced; and this is its justification,—in the triangle, which has for sides the element of a curve and the elements of its abscissae and ordinates, the relation is the same as though this element of the curve were the element of a straight line—the tangent: the angles, which constitute the essential relation (that is, that which remains in these elements after abstraction made from the finite magnitudes belonging to them), are the same.—We can also express ourselves in this matter as follows:—straight lines, as being infinitely small, have passed over into curves, and the ratio which subsists between them in their infinity is a ratio of curves. The straight line according to its definition is the shortest distance between two points, and therefore its difference from the curve is based upon the determination of amount, upon the smaller number of distinguishable steps on this route,—and this is a quantitative determination. But when it is taken as intensive magnitude, or infinite moment, or element, this determination vanishes in it, and with it vanishes the difference from the curve, which is based solely on a difference in Quantum.—Taken as infinitesimal, therefore, straight line and curve have no quantitative relation, and hence (on the basis of the accepted definition) no qualitative difference relatively to each other: the latter now passes over into the former.

Different from the equation of heterogeneous determinations (although analogous) is the assumption, indeterminate in itself and quite indifferent, that infinitely small parts of the same whole are equal to one another; but when this is applied to an object heterogeneous within itself (that is, infected with essential non-uniformity of magnitudinal determination), it produces the peculiar inversion contained in that proposition of higher mechanics which asserts that infinitely small parts of a curve are passed through in equal (and infinitely small) periods of time at a uniform rate: and this is asserted of that kind of motion where, in equal finite (that is, existing) periods of time, finite (that is, existing) and unequal parts of a curve are passed through; the assertion is thus made of a movement which as existent is, and is assumed to be, non-uniform. This proposition is the verbal expression of the intended meaning of an analytic term which results from the development (quoted above) of the formula covering a motion which is non-uniform but, otherwise, conforms to some law. Older mathematicians attempted to express in words and sentences, and to exhibit in geometrical tables, the results of the newly invented infinitesimal calculus (which in any case had to do solely with concrete objects), chiefly in order to use them in theorems in accordance with the ordinary method of proof. The terms of a mathematical formula into which analytic treatment decomposed the magnitude of an object (for instance, motion) there received an objective meaning such as velocity, accelerated force, and so on; according to this meaning they were to produce correct propositions and physical laws; their objective connexions and relations, too, were to be determined by analytic means; for instance, it was said that in a uniformly accelerated motion, there existed a special velocity proportional to the periods of time, while besides an accretion was added uniformly from the force of gravity. In the modern analytical form of mechanics such propositions are regularly cited as results of the calculus, regardless of whether they had in themselves a real meaning—that is, a meaning having a corresponding existent—or whether such a meaning could be proved; and it is thought that the difficulty of making intelligible the connexion of such determinations, when they are taken in the express *real* meaning (like the transition from that plain uniform velocity to a uniform accelerated velocity), has been totally removed by the analytic treatment, since in it this connexion follows directly from the authority, now regarded as established, of the operation of the calculus. We are told that it is a triumph of science when supra-empirical laws—that is, existential propositions having no existence—are discovered by the pure calculus. But in the first and naïve stage of the infinitesimal calculus the task was to indicate a real meaning of these determinations and propositions, which were represented in geometrical diagrams: this meaning was to be made plausible, and with this meaning they were to be applied in the proof of the capital propositions which were in question. (See Newton's proof of his fundamental proposition of the theory of gravitation in *Princ. mathem. philosophiae naturalis lib. I., Sect. II., Prop. I.*, and cf. Schubert's[1] Astronomy—first Ed., Vol. III.,§ 20—, where it is admitted that things are not *exactly* as Newton assumes, that is, at that point which is the nerve of the proof.)

It will be impossible to deny that in this sphere much has been accepted as proof—chiefly veiled under the kindly mist of the infinitesimally small—on no other ground than that the result was always already known beforehand, and the proof, which was arranged in such a manner as to produce the result, at least effected the illusion of a framework of proof,—which illusion was preferred to mere belief or empirical knowledge. But I do not hesitate to regard this method as no better than demonstrational jugglery and counterfeiting; and I include even some of Newton's demonstrations, and especially such as belong to those just mentioned, for which Newton has been extolled to the skies and above Kepler, because what Kepler had discovered empirically *he* demonstrated mathematically.

The useless framework of such proofs was erected in order to prove physical laws. But mathematics is altogether unable to prove the magnitudinal determinations of physics in so far as they are laws based upon the qualitative nature of the moments, for this simple reason that this science is not philosophy and does not start from the concept, so that the qualitative element (in so far as it is not taken lemmatically from experience) lies outside its sphere. The assertion of the honour of mathematics, which demands the strict proof of all its propositions, often allowed it to forget its limit; thus it seemed against its honour to acknowledge simply experience as source and sole proof of empirical propositions. Thought at a later period achieved a more instructed view of this matter; and, until it clearly understands the difference between what is mathematically demonstrable and what can only come from another source, between the things which are terms of an analytical development and those which are physical existents, scientific method cannot achieve a strict and pure attitude.—But justice will no doubt be done to that framework of Newtonian proof as it was to another baseless Newtonian structure of optical experiments and conclusions connected with them. Applied mathematics still seethes with a similar brew of experience and reflection; but long ago one part after another of that optics began to be ignored by science in practice (with this inconsistency—that all the rest, though in contradiction, was allowed to stand); and, similarly, in practice some of these sophistical proofs have already been forgotten or replaced by others.

---

[1] Schubert, Friedrich Theodor von, 1758–1825, Director of the Observatory at S. Petersburg: *Lehrbuch der theoretischen Astronomie*, 1798; *Populäre Astronomie*, 3 vols., 1804–1810.

Observation 2

THE PURPOSE OF THE DIFFERENTIAL CALCULUS DEDUCED FROM ITS APPLICATION

In the previous Observation we considered, partly the conceptual determinateness of the infinitesimally small, which is used in the differential calculus, and partly the reason for its introduction into this calculus; both determinations are abstract and in themselves therefore easy. Their so-called application, however, offers greater difficulties but also a more interesting aspect: the elements of this *concrete* side are to be the object of this Observation.—The whole method of the differential calculus is complete in the proposition that $d(x^n) = n x^{n-1} dx$, or $\frac{f(x+i) - fx}{i} = P$, that is, that it is equal to the coefficient of the first term of the binomial $x + dx$, $x + i$, if the latter is developed according to the powers of dx or i. No need to learn anything further: the derivation of the next forms, the differential of a product, an exponential magnitude and so on, results mechanically; and in a short time, perhaps a mere half-hour—for the deduction of the differential also gives us the reverse, namely, the deduction of the original function, or integration, from these former—the whole theory may be had by heart. The only delay is due to the effort to understand and make it intelligible that that other part—the omission of the remaining terms of the series which arises, apart from the first terms—is valid, after the first part of the task, the deduction of the coefficient, was effected so easily by analytical, that is, by purely arithmetical means, through the development of the function of the variable magnitude, after an increment had given it the form of a binomial. And if it were the case that we needed only the coefficient, then, as was said, once it was determined the entire theoretical part would be done with in less than half an hour, and the omission of further terms of the series would cause no difficulty if only because they would not even come into question as terms of the series; being second, third, or other functions, they are determined with the determination of the first, and now are quite beside the issue.

And first we may observe that probably the method of the differential calculus shows on the face of it that it was neither invented nor constructed as an end in itself. Not only was it not founded for its own sake as a new kind of analytical process, but the arbitrary neglect of terms resulting from the development of a function is contrary to every mathematical principle;—arbitrary, because it is assumed that the whole of this development entirely belongs to the matter in hand—the matter being looked at as the difference of the developed function of a variable magnitude (after it has been given the form of a binomial) from the original function. The need for such a mode of procedure and its internal lack of justification immediately point to the fact that its origin and basis must be elsewhere. It happens in other sciences, too, that that which is placed first, as being elementary, and which is the source from which are ostensibly derived the propositions of the science, is not self-evident, and that it eventually appears that its reason and foundation lie rather in what follows. In the history of the differential calculus the course of things makes it plain that, especially in the various so-called tangential methods, the matter began in special artifices. The method of procedure, later becoming extended to further objects, reached consciousness and was cast into abstract formulae, which it was then attempted to raise to the rank of first principles.

We have shown that the *qualitative* quantity-determinateness of entities, which, primarily, are related to one another as Quanta, is the conceptual determinateness of the so-called infinitesimally small; from this there started the empirical investigation which attempted to demonstrate this conceptual determinateness in the descriptions or definitions of the infinitesimally small, in so far as it is taken as infinitesimal difference or something similar.—This was done only in the interest of abstract conceptual determinateness as such; it might further be asked, what is the nature of the transition thence to mathematical formation and application? To this end the theoretic part (the conceptual determinateness) must further be examined, and this will prove not wholly unfruitful in itself; next we must consider its relation to Application; and with both we must demonstrate, as far as here is possible, that the general conclusions are adequate to the end of the differential calculus and to the manner in which it brings that end about.

It must first be remembered that the mathematical form of the conceptual determinateness which is under discussion has already been mentioned in passing. The qualitative determinateness of the quantitative is first and generally stated in Quantitative Ratio, but already in the demonstration of the various so-called rules of arithmetic (see Observation on this subject) it was said that it was in the Ratio of Powers (to be considered later in its proper place) that Number is posited by the equation of Unit and Amount (the moments of its concept) as returned to itself, and thus acquires in itself the moment of Infinity or Being-for-Self, that is, of self-determinedness. Thus, the express qualitative magnitudinal determinateness essentially (as has also been mentioned already) refers to determinations of powers; and since it is the specific characteristic of the differential calculus that it operates with qualitative forms of magnitude, its peculiar object in mathematics must be the treatment of forms of powers; and all the problems, with their solutions, for which the differential calculus is used, show that all the interest lies only in the treatment of determinations of powers as such.

This foundation is important and immediately puts in the forefront something determinate instead of the merely formal categories of variable, continuous, or infinite magnitudes and the like, or of functions in general; but for all that it is still too general; other operations too are concerned here; the elevation to a power and the extraction of a root, the treatment of exponential magnitudes and of logarithms and series and the equations of higher orders, all are interested in and operate upon ratios only which are based upon powers. No doubt they must between them constitute a system of treatment of powers; which, however, of the various ratios wherein determinations of powers can be posited is the one which is the proper object and interest of the differential calculus, this can only be elicited from itself, that is, from its so-called applications. These in fact are the thing itself, the actual procedure in the mathematical solution of a certain group of problems; and this procedure was earlier than the theory or general part, and has been called application only with reference to the later created theory whose aim was to set up the general method of procedure and also to produce its first principles, that is, its justification. It has been shown in the previous Observation how idle an attempt it has been to find principles for the former manner of comprehending the procedure,—principles which really solved the contradiction which there was found, instead of excusing it or hiding it behind the insignificance of that which was requisite to the mathematical procedure (which here meant that which was to be omitted), or behind the possibility (which comes to the same thing) of infinite or arbitrary approximation, and the like. If the general part of the procedure were to be abstracted, in a manner different from that which has hitherto been followed, from that real part of mathematics which is called differential calculus, then these principles and all the trouble taken over them would prove superfluous, inasmuch as they reveal in themselves a distortion and an abiding contradiction.

If we search out this peculiarity by simply gathering what we find in this part of mathematics, we discover as its object:—

(α) Equations in which any number of magnitudes (we may, however, here confine ourselves to two) are combined into one determinate whole in such a manner that, firstly, they have their determinateness in empirical magnitudes which are their fixed limits, and, moreover, in the particular kind of union with them and with one another, which, indeed, is the case with equations generally; but, since there is only one equation for both magnitudes (or several equations for several magnitudes, but always fewer than the magnitudes), these equations belong to the class of indeterminate equations;—and, secondly, that one aspect of them (the determinateness of these magnitudes here being what it is) is that they are, or at least one of them is, present in the equation in a higher power than the first.

Certain observations may be made about this; and first, that according to the first determination mentioned these magnitudes are wholly of the character of such *variable* magnitudes as occur in the problems of *indeterminate* analysis. Their value is indeterminate, but is so in such a manner that when the one gets a perfectly determinate value (a numerical value) from without, then the other too is determined: one is a function of the other. The categories of variable magnitudes, functions, and the like, are, therefore, merely formal for that specific determinateness of magnitude with which we are dealing here, as was said above; for they are of a generality not yet containing that specific factor which is the one aim of the differential calculus; nor can that factor be explained from them analytically. In themselves they are simple, unimportant, easy determinations, which are made difficult only when what they do not properly contain is put into them in order to be then deduced, I mean the specific determination of the differential calculus.—With regard to the so-called *constant*, we may remark that it exists first as an indifferent empirical magnitude, determining the variable magnitudes only with respect to their empirical Quantum, as Limit of their minimum and maximum; while the nature of the connexion between the constants and the variable magnitudes is itself one of the moments of the nature of that special function which these magnitudes are. But, conversely, the constants also are functions; for instance, in so far as a straight line has the meaning that it is the parameter of a parabola, then its meaning is this, that it is the function $\frac{y^2}{x}$; and, generally, in the development of the binomial, the constant which is the coefficient of the first term of the development is the sum of the roots; the coefficient of the second is the sum of their products, in pairs, and so on: thus the constants here are just functions of the roots; where, in the integral calculus, the constant is determined from the given formula, it is in this respect treated as a function of the latter. We shall elsewhere consider these coefficients, in another determination, as functions, whose meaning in the concrete is the only matter of interest.

The peculiarity, however, by which the consideration of the variable magnitudes is distinguished, in the differential calculus, from its nature in the indeterminate problems, must be attributed to what has been mentioned, namely that at least one, and possibly every one, of these magnitudes is in a higher power than the first; and here again it is indifferent whether all of them are of the same higher power, or of unequal powers; the specific indeterminateness which they here have lies solely in the fact that they are functions of one another standing in such a ratio of powers. This gives a *qualitative* determination to the variation of the variable magnitudes: it is thus *continuous*, and this continuousness (which in itself is merely the formal category of an identity in general, a determinateness persisting unchanged in variation) here has its determinate meaning, and that only in the ratio of powers, which is supposed to have no Quantum for exponent and to constitute the non-quantitative and permanent determinateness of the ratio of variable magnitudes. On the other hand we may object to another formalism, that the first power is power only in relation to higher power: for itself, x is only any indeterminate Quantum. There is thus no meaning in differentiating in themselves the equations $y = ax + b$ (the equation of the straight line) or $s = ct$ (the equation of plain uniform velocity); if the equation $y = ax$, or $y = ax + b$, becomes $a = \frac{dy}{dx}$, or $s = ct$ becomes $\frac{ds}{dt} = c$, then, equally, $a = \frac{y}{x}$ is the determination of the tangents, or $\frac{s}{t} = c$ the determination of plain velocity. The latter, as $\frac{dy}{dx}$, is exhibited in connexion with what is asserted to be the development of uniformly accelerated motion; but it has been remarked above that it is an empty assumption, based upon the routine of method alone, that a moment of simple, merely uniform velocity (that is, velocity not determined by the higher power of one of the moments of motion), has a place in the system of such motion. The method proceeds from the idea of the increment to which the variable magnitude is subject; naturally, therefore, only a magnitude which is a function of a first power can be subject to an increment: when now, in order to discover the differential, the difference between the given equation and that which has thereby arisen, has to be found, the empty nature of this operation manifests itself; for, as we remarked, after as well as before it the equation is the same for the so-called increments as for the variable magnitudes themselves.

(β) What has been said determines the nature of the equation which is to be treated; we must now indicate upon what point of interest its treatment is directed. This consideration can furnish only known results, such (with respect to form) as are especially to be found in Lagrange's conception; but I have, of course, made this exposition wholly elementary in order to remove all heterogeneous determinations involved in it.—The basis of the treatment of an equation of this kind is seen to be this, that the power within itself is taken as a ratio or system of determinations of ratios. We stated above that Power is Number in so far as it has reached that stage where its variation is determined by itself, and its moments, Unit and Amount, are identical;—which, as was shown above, is found perfectly in the square, and more formally (which here makes no difference) in the higher powers. Now power is number (though the expression "magnitude" be preferred as more general, yet in itself it always is number), and thus is a multitude, and can be represented as a sum: it can, therefore, be divided within itself into any multitude of numbers, which relatively to one another and to their sum are without any determination except that together they are equal to the sum. But the power can also be divided into a sum of such differences as are determined by the form of the power. If the power is taken as sum, then its radical number or root too is taken as a sum, and, as regards multiplicity of division, is arbitrary,—which multiplicity, however, is the indifferent and empirical Quantitative. The sum, in which shape the root is supposed to exist, reduced to its simple determinateness, that is, its true universality, is the binomial; and any further multiplication of the terms is a pure repetition of the same determination, and, therefore, an empty process.[1] What matters is the determinateness of the terms (which thus is qualitative) which results from the raising of the root, taken as sum, to a power, and this determinateness lies entirely within that change which this process is. These terms, thus, are entirely functions of power and of raising to a power. Now this representation of number as sum of a multitude of such terms, which are functions of this process, and further the eagerness to find the form of such functions and also this sum from the multitude of such terms, this it is (the discovery being entirely dependent on that form) which constitutes, as is well known, the special doctrine of series. But here it is important to distinguish the further point of interest, namely, the relation of the basic magnitude itself (the determinateness of which, in so far as it is a complex, that is, in this instance, an equation, therefore includes a power) to the functions of its potentiation. This relation (apart altogether from the above-mentioned interest in the sum) will show itself to be the only truly scientific point of view by which the differential calculus is guided.

---

[1] It is only a part of the formalism of that universality to which analysis perforce lays claim, when $(a + b)^n$ is not taken for the development of the power, and $(a + b + c + d + \ldots)^n$ is substituted. This is often done elsewhere too; and this

But first another determination must be added to what has been said; or rather, a determination implied in it must be removed. It was said that the variable magnitude, in whose determination power is an element, is looked upon as a sum within itself, as being, in fact, a system of terms, in so far as these are functions of potentiation, and that thus the root too was considered as a sum, and, in its simply determinate form, as a binomial, namely

$$x^n = (y + z)^n = (y^n + ny^{n-1}z \pm \ldots).$$

This presentation started from Sum as such upon the process of developing Power, that is, upon the process of achieving its potentiational functions; here, however, the aim is not a sum as such, nor the series which arises from it: the only element which must be taken up from Sum is Relation. Relation of magnitudes as such is on the one hand all that remains after abstraction has been made from the *plus* of a sum as such, and is all that is needed on the other hand in

---

form, so to speak, should be taken for an affectation of the appearance of universality, for the matter is exhausted in the binomial; by the development of this the law is found, and it is the law which is the true universality, and not external and empty repetition of the law, which is all that this $a + b + c + d + \ldots$ effects.

order that the functions of development of Power may be found. But such a Relation is determined already by the fact that the object here is an equation ($y^m = a x^n$), that is, a complex of several (variable) magnitudes which contains their determination of powers. In this complex each of these magnitudes is posited as just being in relation with the other, with the meaning, as one might say, of a *plus* in itself,—as a function of the other magnitudes; their characteristic (that of being functions of one another) gives them this determination of a *plus*, which for that very reason is quite indeterminate and is not an addition, increment, or the like. This abstract point of view could, however, be neglected; we may simply remain at that point where, the variable magnitudes being given in the equation as functions of one another in such a manner that this determinateness contains a relation of powers, the functions of potentiation too are now compared one with the other;—which second functions have no determination whatever except that which comes from potentiation. So far we can assert that it is optional, or possible, to transpose an equation from the powers of its variable magnitudes to a relation of its functions of development; and the utilitarian quality of such a transformation must be indicated by some further purpose, advantage, or use; the transformation has been made only because of its usefulness. Above we started from the presentation of these determinations of potentiation, and experimented on a magnitude which was stated to be a sum and, therefore was to be assumed as being differentiated within itself; but this was only done, partly to indicate the nature of such functions, and partly because this implies the mode by which they are found.

We have thus reached the ordinary analytical development, which for the purposes of the differential calculus is taken in this manner, that an increment, *dx* or *i*, is given to the variable magnitude, and then the power of the binomial is explained by means of the series of terms belonging to it. The so-called increment, however, is intended to be not a Quantum but a form, whose only value is that it assists the development; what is wanted (and this is admitted, and most explicitly by Euler and Lagrange) in the above-mentioned idea of Limit, is the resulting determinations of powers of the variable magnitudes, the so-called coefficients of the increment (as is admitted) and its powers, according to which the series is ordered, and to which the different coefficients belong. To this we might also add that an increment (without Quantum) is only assumed for the sake of development, and that, therefore, it would be most convenient to take 1 (the One) for that purpose; for in the development this only occurs as factor; so that the factor One fulfils the purpose, which is, that the increment is not to imply the positing of any quantitative determinateness or change; dx, on the other hand, is infected with the false idea of a quantitative difference, and other symbols, like i, with the show, useless here, of universality, so that they always have the appearance and pretension of a Quantum and its powers; which pretension then involves the trouble that they must nevertheless be taken away and kept away. In order to preserve the form of a series developed on the principle of powers, the denominations of exponents (being *indices*) might equally well be placed behind a One. In any case abstraction must be made from the series and from the determination of coefficients according to their place in the series: the relation between all is the same; the second function is derived from the first in just the same manner as this from the original, and for the one which is counted second, the first, and derivative, function, is counted original. The essential point of interest is not, however, the series, but solely the determination of powers resulting from the development in relation to the magnitude which, to them, is immediate. They are not, therefore, determined as being coefficients of the first term of the development, since one term is designated as first in relation to others following it in the series, and such a power as the power of an increment, together with the series, is here out of place; therefore, the plain expression of derivative function of a power or, as was said above, function of potentiation of a magnitude, would be preferable,—the knowledge being presupposed of the manner in which the derivation is taken as a development included within a power.

The pure mathematical beginning in this part of the analysis is just the discovery of the function determined by the development of powers; it is a further question, what is to be done with the ratio thus obtained, where it is to be applied and used, or, in fact, for what purpose such functions are looked for. It is the process of discovering proportions in concrete objects which may be referred back to those abstract analytic proportions, that has given the differential calculus its great interest.

But as regards applicability, the immediate outcome of the nature of the matter—merely in virtue of the form which we have shown the moments of powers to possess, and no conclusion being yet drawn from any instances of application—is as follows. The development of powers whence result the functions of their potentiation contains (abstraction being made from closer determination) the reduction of magnitude to the next lowest power. Thus it is that this operation is applicable to such objects as also contain this distinction of determinations of powers. If now we consider spatial determinateness, we find that it contains the three dimensions which we may call the *concrete*, in order to distinguish them from the abstract distinctions of height, length, and breadth,—namely, line, surface, and solid space; and, when they are taken in their simplest forms and with reference to self-determination and, therefore, to analytic dimensions, we arrive at the straight line, the plane surface and surface taken as square, and the cube. The straight line has an empirical Quantum, but with the plane we reach the qualitative element—determination of power; we may neglect closer modifications,—the reflection, for instance, that this also happens to plane curves; for here we are only dealing with the distinction in general. Herewith also the need arises to pass from a higher to a lower determination of power, and conversely,—the attempt being for instance to derive linear determinations from given equations of plane and so on, or the other way about.—In motion, further, the magnitudinal proportion of the space passed through with its elapsed time must be considered, and motion manifests itself in various determinations, as simply uniform, as uniformly accelerated, and as alternately accelerated uniformly and retarded uniformly, returning upon itself; these different kinds of motion are expressed according to the magnitudinal proportion of their moments, space and time; and thus there result for it equations having different determinations of powers; and, in so far as it may be necessary to determine one kind of motion or of spatial magnitude to which one kind is tied, from another kind of these, the operation also involves the transition of one function of power to another, either higher or lower.—The examples of these two objects should suffice for the purpose for which they are cited.

The appearance of contingency which the differential calculus presents in its applications would certainly be simplified if the nature of the spheres where the application can take place, and the peculiar need for and condition of this application, were clearly understood. But now it is necessary further to know, within these spheres, between what *parts* of the objects of the mathematical problem such a relation takes place as is posited peculiarly by the differential calculus. And first it must be remarked that two kinds of relation are to be observed. The operation of the depotentiation of any equation (the equation considered according to the derivative functions of its variable magnitudes) gives a result which in itself really no longer is an equation but a proportion: this proportion is the object of the differential calculus proper. And precisely in this fact we are also presented with a second proportion, that between the higher determination of power (the original equation) itself and the lower (the derivative). This second proportion we must here provisionally neglect: it will prove to be the peculiar object of the integral calculus.

We will then consider the first relation, and, for the determination of the moment (which must be taken from the so-called application and contains the interesting part of the operation) we will take the simplest example, curves such as are determined by an equation of the second degree. The relation of the coordinates is, of course, given immediately through the equation in a determination of powers. From the fundamental determination there follow determinations of other straight lines connected with the coordinates—tangents, subtangents, normals, and so on. The equations, however, between these lines and the coordinates are *linear* equations; and the wholes, of which these lines are determined to be parts, are right-angled triangles of *straight* lines. Now the transition from the fundamental equation, which contains the determination of the powers, to these linear equations, contains the above transition from the original function (that is, the function which is an *equation*) to the derivative function (which is a *relation*—a relation subsisting between certain lines contained in the curve). What must now be discovered is the connexion between the *relation* of these lines and the *equation* of the curve.

It is not without interest to introduce this much of historicity, as to remark that the first discoverers can indicate their discovery only in a wholly empirical manner, without being able to render an account of the operation, which has remained quite external. In this regard I content myself with a reference to Barrow, who was Newton's master. In his *lect. Opt. et Geom.*, in which he treats problems of higher geometry according to the method of the indivisibles (which, in the first place, differs from the characteristic feature of the differential calculus), he also indicates his mode of determining the tangents, "because his friends had urged him" (*lect. X.*). The nature of this indication must be read in his own words if we would properly understand how this method is given quite as an external rule —in the same style in which formerly in arithmetical schoolbooks the "rule of three," or, better still, the so-called "test by casting out the nines," was enunciated. He enumerates the minute lines, which later were called the increments in the characteristic triangle of a curve, and then gives the instruction—which is a mere rule—to cast off as superfluous the terms which appear when the equations are developed as powers of these increments or products (*etenim isti termini nihilum valebunt*); adding that the terms which contain only magnitudes determined by the original equation must also be cast off (—which means that the original equation is subtracted afterwards from the equation formed with the increments), and that the ordinates themselves must be substituted for the increment of the ordinates, and the subtangents for the increment of the abscissae. The method could be set forth (if one may say so) in no more school-masterly manner;— the second substitution is the assumption of the proportionality of the increments of ordinates and abscissae to the ordinates and subtangents, which is the basis of the procedure for determining tangents in the ordinary differential method: in Barrow's rule this assumption appears in all its naïve nakedness. A simple way of determining subtangents had been found: Roberval's[1] and Fermat's methods come to the same issue;—the method for finding maximal and minimal values, from which the latter started, is based on the same foundation and the same procedure. It was a mathematical mania of that period to discover so-called methods, that is, rules of this kind, and to make a mystery of them; which was not only easy, but also, in one respect, necessary, and for the same reason for which it was easy,—namely that the inventors had found only an empirical and external rule and no method, that is, nothing derived from acknowledged principles. Such so-called methods Leibniz absorbed from his period, and Newton from it and, immediately, from his master: the generalization implied in their form and applicability opened new roads to the Sciences, but also brought the need of forcing the method out of the shape of merely external rules, and attempted to give it the justification which it demanded.

If now the method be more closely analysed, the true process is this. First the determinations of powers (powers, of course, of variable magnitudes) contained in the equation, are reduced to their first derivatives. But this changes the value of the members of the equation: hence no equation remains, and a proportion has arisen which subsists between the first derivative of one variable magnitude and the first derivative of the other; for $p x = y^2$ we have $p : 2 y$, or for $2 a x - x^2 = y^2$ we have $a - x : y$, which later used to be called the proportion $\frac{dy}{dx}$. The equation is an equation of the curve; this relation, on the other hand, wholly dependent on it and wholly derived from it (derived in the process described above in accordance with a bare rule), is linear, certain lines here being proportional one to the other; $p : 2 y$ or $a - x : y$ are themselves relations of straight lines of the curve, or of coordinates or parameters; but so far knowing this we know nothing. The aim is to know, with regard to other lines which occur in connexion with the curve, that a certain one of these relations holds between them,—that is, to identify two relations.—Thus, secondly, the question arises, which are the straight lines determined by the nature of the curve which are thus related?—But this was known already—namely, that a relation thus reached is the relation between ordinates and subtangents. The ancients had found this by an ingenious geometrical method; what modern inventors discovered was the empirical mode of arranging the equation of a curve in such a manner that the first relation should result, of which it is already known that it is equal to a relation which contains the line (here the subtangent) which it is desired to determine. Now, in part, this arrangement of the equation has been taken methodically and made methodically (Differentiation),—and, in part, the imaginary increments of the coordinates and the imaginary characteristic triangle (which is formed of these and of a similar increment of the tangents) have been invented,—simply in order that the proportionality between the relation found by the depotentiation of the equation and the relation between ordinates and subtangents should not be represented as taken up empirically from old acquaintance, but as a demonstrated truth. And yet old acquaintance proves itself, generally and most unmistakably in the form of rules (such as we quoted), as the only occasion and justification (where needed) of the assumption of the characteristic triangle and of that proportionality.

---

[1] Personne, Gilles, sieur de Roberval, 1602–1675.

It was Lagrange who rejected this pretence and followed the true scientific course: thanks to his method we know the real point at issue, for it consists in a separation of the two transitions which must be made in order that the problem may be solved, and in a separate treatment and proof of each of these sides.—In our more detailed explanation of the procedure we will retain as example the elementary problem of finding the subtangents.—Now one part of this solution, the theoretical or general part, which is the finding of the first function from the given equation of curves, is regulated by itself: this part gives us a linear relation, a relation, that is, of straight lines, which occur in the system of curve-determination. The other part of the solution now is the finding of those lines in the curve which are in this relation. This is done directly (*Théorie des Fonct. Anal. II. P. II. Chap.*), that is, without the characteristic triangle, which means that no assumption is made of infinitely small arcs, ordinates, or abscissae, nor are the determinations of dy and dx (that is, of being sides of this relation) attributed to them, which would mean immediately that it was equated with the ordinate and subtangent. A line (and also a point) has its determination in so far as it constitutes the side of a triangle, and the determination of a point too lies only in this. This (as may be mentioned in passing) is the fundamental proposition of analytical geometry, which introduces the coordinates as (what is the same thing) in mechanics it does the parallelogram of forces,—which for that very reason does not require all the efforts that are made to find a proof.—The subtangent is now made the third side of a triangle, the other sides of which are the ordinate and the relative tangent. The latter is a straight line, and, therefore, its equation is $p = aq$, (the addition of $+ b$ adds nothing to the determination and is made only for the sake of the fetish of universality);—the determination of the ratio $\frac{p}{q}$ falls within a, the coefficient of q, which in turn is the first derivative of the equation, but need be considered only as $a = \frac{p}{q}$,—being, as has been said, the essential determination of the straight line which is applied as tangent to the curve. Now, further, the first derivative of the curve-equation is taken, and, therefore, it is also the determination of a straight line; further, it is assumed that the coordinate p of the first straight line, and y, the ordinate of the curve, are identical, and, therefore, that the point where that first straight line (which is taken to be a tangent) touches the curve is also the beginning of the straight line which is determined by the first function of the curve: the task, therefore, is to show that this second straight line coincides with the first and, therefore, is a tangent; which may be thus algebraically expressed:—since $y = fx$ and $p = Fq$, and it is assumed that $y = p$ and hence that $fx = Fq$, therefore $f'x = F'q$. In order to prove that the straight line which is applied as tangent and the straight line of the equation which is determined as being its first function coincide, and, therefore, that the latter is a tangent, recourse is had to the increment i of the abscissa and to the increment of the ordinate which is determined during the development of the function. Thus here too that ill-famed increment is introduced; but its introduction for this purpose, and the development of the function under its guidance, must carefully be distinguished from the use (mentioned above) of the increment in the discovery of the differential equation and in the characteristic triangle. The use here made is justified and necessary; it falls within the scope of geometry, for it is part of the geometrical determination of a tangent as such that between it and the curve with which it has a point in common no other straight line can pass and pass also through that point. In this determination the quality of tangent or not-tangent is reduced to a magnitudinal difference: that line is the tangent of which simply *greater smallness* with respect to the essential determination is predicated. There is no empirical element whatever in this smallness, which appears to be only relative,—nothing, that is, which depends on a Quantum as such; it is posited as qualitative by the nature of the formula when the difference of the moment on which the magnitude which is to be compared depends, is a difference of powers; this difference is reduced to i and i^2, and i (which after all must signify a number) must then be imagined as a fraction, and thus i^2 *in itself* is smaller than i; so that the idea of an arbitrary magnitude as which i might be taken is here superfluous and even out of place. And for this very reason the demonstration of a greater smallness has nothing to do with an infinitesimally small quantity, the introduction of which is thus here by no means necessary.

I will now mention—if only for its beauty and for its fame, mostly forgotten now, but well deserved—the tangential method of Descartes; it moreover has a bearing on the nature of equations, on which an observation remains to be made. Descartes unfolds this independent method, where the linear determination required is discovered from the same derivative function, in his geometry, which has proved so fruitful in other respects too (*liv. II. p.* 357 *sqq. Oeuvres Compl. ed. Cousin Tom. V.*); for in it he teaches the great basis of the nature of equations and their geometrical construction, and of the Analysis of Geometry, the sense of which he had thereby so greatly extended. With him the form of the problem was this, to draw straight lines at right angles to given points of a curve, by which method the subtangent (and so forth) is determined; and it is easy to understand the satisfaction which he expresses there at his discovery, which concerned an object of general scientific interest at that period, and is purely geometrical and thereby stands high above the mere methods of rule (mentioned above) used by his rivals: "*J'ose dire que c'est ceci le problème le plus utile et le plus général, non seulement que je sache, mais même que j'aie jamais désiré de savoir en géometrie.*"—He bases the solution upon the analytic equation of the right-angled triangle which is formed by (1) the ordinate of the point of the curve to which the straight line demanded in the problem is to be perpendicular, by (2) this straight line itself (the normal), and by (3) that part of the axis which is cut off by the ordinate and the normal (the sub-normal). Now the equation of a curve is known, and from this equation the value of ordinate or abscissa is substituted into this equation of the triangle. Thus an equation of the second degree results; and Descartes shows also how curves whose equations contain higher degrees are reduced to this. In this equation only one of the variable magnitudes occurs, either as square or as first power;—a quadratic equation which at first appears as a so-called impure equation. On this Descartes reflects as follows:—If the point taken in the curve is imagined as the point of intersection of this and of a circle, then this circle will intersect the curve at another point, so that for the two unequal x's which will thus arise there will be two equations with the same constants and having the same form;—or else there will be only *one* equation with unequal values of x. But the equation is one only for the *one* triangle, in which the hypotenuse is perpendicular to the curve (or is normal),—which is imagined in this way, that the two points of intersection of the curve and the circle are allowed to coincide, so that the curve is allowed to touch the circle. But then also the fact that the x or y of the quadratic equation has unequal roots, disappears. But in a quadratic equation of two equal roots, the coefficient of the member contained by the unknown in the first power is twice the single root; and from this there results an equation by which the required determinations are discovered. This method must be considered the brilliant device of a true analytic mind, compared with which the arbitrarily assumed proportionality of subtangent and ordinate, together with the so-called increments of abscissa and ordinate (supposed to be infinitely small), is vastly inferior.

The final equation reached in this manner, in which the coefficient of the second member of the quadratic equation is equated with the double root or unknown, is the same as is found by the method of the differential calculus. When $x^2 - a\,x - b = 0$ is differentiated, there results the new equation $2\,x - a = 0$; or, again, $3\,x^2 - p = 0$ results from $x^3 - p\,x \mp q = 0$. And here we may observe that it is by no means self-evident that such a derivative equation is also correct. We have already considered the fact that in an equation with two variable magnitudes which never lose their quality of being unknown magnitudes just because they are variable, only a proportion results; and this for the simple reason indicated, that, when the functions of the potentiation are substituted for the powers themselves, the value of the two members of the equation is altered, and it remains as yet unknown whether an equation subsists between them with their values thus altered. The equation $\frac{dy}{dx} = P$ expresses nothing except that P is a proportion, and no other real meaning can be ascribed to $\frac{dy}{dx}$. And also it is still not known of this proportion $= P$, to what other proportion it is equal; it is only this equation, or proportionality, which gives value and meaning to the proportion.—It was mentioned that this meaning—and this was what was called application—was introduced empirically and from without; and, similarly, as the equations here under discussion are derived by differentiation, we must know from some other source whether they have equal roots, in order to know whether the equation reached remains correct. But this fact is not expressly brought to notice in the manuals; although it must be admitted that it is got out of the way when an equation with an unknown, reduced to nought, is straightway equated with y, whereupon of course only $\frac{dy}{dx}$ (a proportion) results from the differentiation. It is true that the functional calculus is supposed to deal with functions of potentiation, and the differential calculus with differentials; but it by no means follows immediately that magnitudes whose differentials or functions of potentiation are taken, are themselves *only* to be functions of *other* magnitudes. In any case, in the theoretic part, that is, where the instruction is given to derive the differential (or the functions of potentiation), no thought is yet given to the intention that the magnitudes, of which the treatment according to such a derivation is there taught, are themselves to be functions of other magnitudes.

With regard to the omission of the constant in the process of differentiation, this further observation may be made, that differentiation here means that the constant is indifferent for the determination of the roots when they are equal, the determination being exhausted by the coefficient of the second member of the equation. In the example quoted from Descartes the constant is itself the square of the roots, so that it can be determined from the constant as well as from the coefficients; for, generally, like the coefficients, it is a function of the roots of the equation. In the ordinary exposition the omission of the so-called constants (which are connected with the other members only by plus or minus) is brought about by the mere mechanism of the method,—when, in order that the differential of a composite expression may be found, only the variable magnitudes receive an increment, and the expression thus formulated is subtracted from the original expression. The meaning of the constants and of their omission, the question how far they are themselves functions and serve or do not serve in this determination, finds no expression here.

In connexion with the omission of constants, an observation may be made about the names of Differentiation and Integration, similar to the observation which was made above about the expressions Finite and Infinite,—namely, that their determination contains the opposite of what is denoted by the terms. To differentiate denotes that differences are posited, whereas differentiation in fact reduces an equation to lesser dimensions, and the omission of the constant removes one moment of the determinateness; for, as we remarked, the roots of the variable magnitude were placed upon an equality, and thus the difference between them was cancelled. And in integration the constant is supposed to be reintroduced; and though by this process the equation is integrated, it is so in this sense, that the difference of the roots, which had just been cancelled, is reconstructed, so that the equalization is differentiated once more.—The ordinary expression adds its share in obscuring the essential nature of the matter and in setting everything in a point of view which is subordinate and even alien to the main issue: I mean the point of view of the infinitely small difference, the increment, and the like, and also of the bare difference generally between the given and the derivative function, no designation being made of the specific, namely the qualitative, difference.

Mechanics is another important field where the differential calculus is applied; mention has already been made incidentally of the different functions of powers that result from the elementary equations of its object, which is motion, and of their significance. I will admit these directly here. The equation (that is, the mathematical expression) for motion which is simply uniform, $c = \frac{s}{t}$ or $s = c\,t$, where the spaces passed through are proportionate to the times elapsed according to an empirical unit c (the magnitude of velocity), offers no meaning for differentiation: the coefficient c is already fully determined and known, and no further development of powers can take place.—The analysis of $s = a\,t^2$, the equation of the motion of a falling body, has already been recorded;—the first member of the analysis $\frac{d\,s}{d\,t} = 2\,a\,t$ is translated into language (or into fact, as the case may be) when it is postulated that a member of a sum (an idea which we banished long ago) must be one part of this motion; and that, further, this part must be added to the force of inertia (a merely uniform velocity) in such a manner that the motion is uniform in infinitesimally small parts of time, and not uniform in finite parts of time, that is, in those which actually exist. It is true that $f\,s = 2\,a\,t$; and the meaning of a and t is known already, together with the fact that this suffices to posit the determination of the uniform velocity of a motion; for since $a = \frac{s}{t^2}$, then $2\,a\,t = \frac{2\,s}{t}$ universally; but having this we are no wiser at all, and only the deceitful assumption that $2\,a\,t$ is part of the motion regarded as a sum gives the deceitful appearance of a physical proposition. The factor itself (a, the empirical unit, a Quantum as such) is ascribed to gravity; but if the category of force of gravity is employed at all, it should rather be said that the whole $s = a\,t^2$ is the effect, or, better, the law, of gravity.—Similar to this is the proposition derived from $\frac{d\,s}{d\,t} = 2\,a\,t$, which enunciates that if gravity ceased to act, the body, moving at the velocity reached at the end of its fall, would describe twice the space it has already passed through in a time equal to that occupied by its fall.—This contains a metaphysic which in itself is unsound: the end of the fall, or the end of a period of time during which the body has been falling, still is itself a period of time; if it were not, a state of rest, which excludes velocity, would be assumed; velocity can be introduced only according to the space passed through in a *period* of time and not at the *end* of a period.—When the differential calculus is actually applied in other spheres of physics where there is no motion at all, for instance in the behaviour of light (apart from its so-called propagation in space) or in the magnitudinal determinations of colours, the first derivative of a quadratic function being here too called velocity, then we must regard this as a still more illegitimate example of the formalism which feigns real existence.—

The motion (says Lagrange) which is represented by the equation $s = a\,t^2$ is empirically given in falling bodies; the next simplest motion after this equation would be that whose equation was $s = c\,t^3$, but nature knows no such motion; and we do not know what meaning the coefficient c could have. This is true; but there is a motion whose equation is $s^3 = a\,t^2$, and this is Kepler's law of the motion of the bodies of the solar system;—and indeed it would appear an interesting task to show the intended meaning here of the first derivative function $\frac{2\,a\,t}{3\,s^2}$, and so on, to treat this equation further and directly by means of differentiation, and to develop the laws and determinations of that absolute motion from this starting point,—a task in which analysis might display a brilliance most worthy of itself.

Thus the application of the differential calculus to the elementary equations of motion offers in itself no real interest: formal interest is derived from the general mechanism of the calculus. But another significance is gained by the analysis of motion with respect to the determination of its trajectory: if this is a curve and its equation contains higher powers, then the transition is necessary of rectilinear functions (as functions of potentiation) to the powers themselves; the former must be extracted from the original equation of motion, which contains the factor of time, and time must be eliminated; and so at the same time this factor must be reduced to lower functions of development, from which these equations of linear determinations can be worked out. This aspect leads us to the interesting element of the other part of the differential calculus.

What has been said so far was said with the purpose of emphasizing and fixing the simple and specific determination of the differential calculus, and of demonstrating it in a few elementary examples. It was seen that this determination consisted in the following process: the coefficient of the member of development (the so-called first derivative) is found from an equation of functions of powers; this function is a proportion which is demonstrated in moments of the concrete object; an equation results which determines these moments themselves as between the two proportions. And also we must briefly consider the principle of the integral calculus, and what results from its application for its specific concrete determination. The consideration of this calculus has been simplified and more correctly determined through the fact that it is no longer taken as a method of summation, as it was called in opposition to differentiation; the increment was there considered the essential ingredient, and with this it appeared in essential connexion with the serial form.—The task of this calculus, as of the differential, is, first, theoretical or rather formal, but, as is well known, it is the converse of the differential. Here a beginning is made from a function which is considered as derivative and as the coefficient of the next member, which originates from the development of an equation as yet unknown; and from it the original function of power is to be calculated. The function which in the natural order of development must be regarded as primary is here considered derivative, and that which before was considered derivative here is considered as given and in fact as beginning to exist. But it appears that the formal part of this operation has already been performed by the differential calculus; since in it the transition and relation in general between original function and function of development is established. In order to apply the function from which we must start, and also in order to effect the transition from it to the original function, it is necessary in many cases to have recourse to the serial form; but it must be remembered that this form as such has nothing to do directly with the peculiar principle of integration.

It appears next that the other part of the task of the calculus, with respect to its formal operation, is the application of the latter. This is now itself the task, namely, to know the *meaning* (in the sense indicated above), as a separate object, of the original function of the given function, which is considered the first derivative. It might appear that this doctrine was quite done with in the differential calculus; but a further circumstance enters into play which does not allow the matter to be so simple. For it resulted from this calculus that the proportion—which is linear—was obtained from the first derivative of the equation of a curve; and, therefore, knowing this we also know that the integration of this proportion gives us the equation of the curve in the proportion of abscissa and ordinate; or, if the equation were given for the plane of a curve, then it would be the case that the differential calculus *ought* already to have taught, with respect to the meaning of the first derivative of such an equation, that this function exhibited the ordinate as function of the abscissa, and, therefore, the equation of a curve.

But the question is, which of the determining moments of the object is itself given in the equation; for the analytic treatment can proceed only from what is given, and pass thence to the remaining determinations of the object. Thus what is given is not the equation of an area of the curve, nor of the body arising from its revolution, nor of an arc of the curve; but only the proportion of the abscissae and ordinates in the equation of the curve itself is given. Therefore, the transitions from those determinations to this equation itself cannot already be treated in the differential calculus: it is reserved for the integral calculus to find these proportions.

But further it has been shown that the differentiation of an equation of more than one variable magnitude gives the power of development or differential coefficient, not as an equation, but only as a proportion: the task, then, is to indicate in the moments of the object a second proportion that shall be equal to this first proportion, which is the derivative function. In the integral calculus, on the other hand, the object is the proportion itself of the original function to the derivative (which is here supposed to be given); and the task is, to indicate the meaning of the original function (which is to be discovered) in the object of the given first derivative, or rather, since this meaning has already been declared to be the problem (the meaning is, for instance, the plane of a curve, or the curve, imagined as rectilinear, which remains to be rectified; and so on), to demonstrate that such a determination is found by the original function, and to show which is the moment of the object that must be taken for this purpose as initial function of the derivative function.

Now the ordinary method, which uses the idea of the difference as equivalent to the infinitely small, makes its task simple: thus, for the quadrature of curves, an infinitely small rectangle, a product of ordinate and element, that is, the infinitely small part of the abscissa, is taken for the trapezium which is supposed to have for one of its sides the infinitesimally small arc opposite to that infinitesimally small part of the abscissa. The product is integrated in this sense, that the integral gives the sum of the infinitely great number of trapezia, the plane whose determination is required, that is, the finite magnitude of this element of the plane. And, similarly, it forms a right-angled triangle out of the infinitesimally small elements of the arc and the ordinates and abscissae belonging to them; in this the square of that arc is supposed to be equal to the sum of the squares of the other two infinitesimally small elements, the integration of which presents us with this arc as a finite arc.

This procedure is based upon the general discovery which is the foundation of this part of Analysis; and here is based upon it in that the quadrated curve, the rectified arc, and so on, stand to a certain function which is given by the equation of the curve, in the relation of so-called original to derivative function. The question now is this: when a certain part of a mathematical object (for instance, of a curve) is assumed to be the derivative function, what other part of it is expressed by the corresponding original function? We know that, when the function of the ordinate given by the equation of the curve is taken to be the derivative function, then the (relatively) original function expresses the magnitude of the area of the curve which is cut off by this ordinate; and, when a certain tangential determination is regarded as derivative function, then its original function expresses the magnitude of the arc belonging to this tangential determination, and so on: but the method which uses the infinitesimally small and operates with it does not take the trouble to recognize and to demonstrate that these ratios (firstly that which subsists between original and derivative function, and secondly that between the magnitudes of two parts, or conditions, of the mathematical object) together form one proportion. It is the peculiar merit of intellectual insight to have discovered from results already known from without, that certain and specified sides of a mathematical object stand in the relation one to the other of original and derivative function.

In this calculus, of these two the derivative function, or (as it has been determined) the function of potentiation, is the one which is *given* relatively to the original function; the latter still remains to be discovered from the former by integration. But it is not given immediately, nor is it given for itself which part or determination of the mathematical object is to be looked at as derivative function, in order to find, through tracing it back to the original function, the other part or determination, whose magnitude the problem requires. The ordinary method, as has been stated, immediately represents certain parts of the object as infinitesimally small, in the form of derivative functions which can be determined from the original equation of the object by differentiation in general (—thus for the rectification of a curve it takes the infinitesimal abscissae and ordinates); and this method takes instead such as can be brought into a connexion with the object of the problem (in the example, with the arc, which also is imagined as infinitesimal) which is fixed and determined by elementary mathematics, by which procedure, these parts being known, that part also of which the magnitude was to be found, is determined. Thus, for rectification, the three infinitesimals we mentioned are connected in the equation of the right-angled triangle, while for quadrature, the ordinate and the infinitesimal abscissa are connected in a product, a plane being taken, in the general arithmetical manner, as a product of straight lines. The transition from such a so-called element of plane or arc, and so forth, to the *magnitude* of plane or arc itself now only counts as ascent from the infinitesimal to the finite expression or to the *sum* of an infinite number of elements, of which the required magnitude is supposed to consist.

We can, therefore, make only the superficial observation that the integral calculus is just the problem of the differential calculus, but inverted, and, in general, more difficult; while the real interest of the integral calculus concerns itself entirely with the relation to each other of the original and the derivative function in concrete objects.

Lagrange nowhere banished the difficulty of any problem in the facile manner of these direct assumptions, nor did he consent to do so in this part of the calculus. It will help the elucidation of the nature of the matter if here too we indicate the detail of his procedure in a few examples. For the task of his method is to demonstrate in itself that a relation of original to derivative function subsists between separate determinations of a mathematical whole, for instance a curve. But in this sphere, because of the nature of the relation itself, this cannot be done in a direct manner, since in the mathematical object the relation connects curved with straight lines, linear dimensions and their functions with plane dimensions and their function, and so on,—connects, that is, terms which are qualitatively different; the determination thus can only be taken as the mean between a greater and a less. Thus here again there enters spontaneously the form of increment with its *plus* and *minus*, and that vigorous "*Développons*" is here in place; but we have already discussed that purely arithmetical and finite meaning which here is all that belongs to the increments. When we develop the condition that the magnitude which is to be determined must be greater than one and less than another limit (itself easily determinable), we derive such facts as that the function of the ordinate stands in the relation of first derivative function to the function of the area.

Lagrange's exposition of the rectification of curves starts from Archimedes' principle, and is interesting, therefore, as affording an insight into the translation of the Archimedean method into the principle of modern analysis, which allows us to look upon the inner and true meaning of an affair which by the other mode is pursued but mechanically. The procedure is necessarily analogous to that which has just been indicated: no direct equation results from the principle of Archimedes, which is that the arc of a curve is greater than its chord and less than the sum of two tangents drawn between the ends of the arc and their point of intersection. This fundamental determination of Archimedes is translated into the modern analytic form when an expression is found which is in itself a simple fundamental equation, whereas the form of Archimedes only *postulates* that there must be an infinite progress between two elements, respectively too great and too small, which each time determine themselves,—this progress ever giving but a new pair of great and small, with an ever narrower limit of inaccuracy. The formalism of the infinitesimally small immediately gives us the equation $dz^2 = dx^2 + dy^2$. But Lagrange's demonstration, starting from the foundation which has been indicated, shows that the magnitude of the arc stands in the relation of original to a derivative function, in which the characteristic member itself is a function which comes from the relation of a derivative to the original function of the ordinate.

The method of Archimedes, and, at a later date, Kepler's treatment of stereometrical objects, employ the idea of the infinitesimally small,—a fact which has often been quoted as authorizing the use made of this idea in the differential calculus, while its peculiarity and distinguishing quality were not sufficiently emphasized. The infinitesimally small means, first, the negation of Quantum as such, that is, of a so-called finite expression, or of that perfect determinateness which belongs to Quantum as such. And, similarly, the fundamental determination in the subsequent famous methods of Valerius,[1] Cavalieri, and others, which are based on the consideration of the relations of geometrical objects, is, that the Quantum (as such) of determinations which so far are being considered merely as related terms, is to be neglected for this purpose, and consequently they are to be taken as non-magnitudinal.

---

[1] Valerius, Lucas, died 1618 at Rome, called by Galileo the Archimedes of his time: *de quadratura parabolae per simplex falsum.*

But here the general affirmative element which is latent in the merely negative determination remains unrecognized and unnoticed—an element which, abstractly, proved to be qualitative magnitudinal determinateness based, more precisely, upon the ratio of powers;—and also, since this relation itself includes a number of more closely determinate relations like powers and the functions of their development, attempts have been made to base these too on the general and negative determination of the same infinitesimally small, and to derive them thence. In Lagrange's exposition which we have just examined, the determinate affirmative which is contained in Archimedes' manner of developing the problem has been discovered, and thus the procedure, which was infected with an unlimited overpassing, has been given its correct limit. The importance of the modern invention in itself, and its capacity to solve problems hitherto intractable, and to treat in a simple manner those which were not insoluble before, is due solely to the discovery of the relation of the original to the so-called derivative, and of the parts which, in a mathematical whole, stand in this relation.

What has been said may serve to make clear what is characteristic in that proportion of magnitudes which is the object of the particular kind of calculus under discussion. Our exposition was able to confine itself to simple problems and the methods of solving them; and it would neither have been suitable for the determination of the concept (which here alone was our object), nor would it have been in the power of the author, to examine the whole scope of the so-called application of the differential and integral calculus, and to complete the induction that they are based upon the principle which we discovered, by tracing back to it all their problems and their solutions. But our contribution has sufficiently shown that as every other calculus has for object a separate determinateness or relation of magnitude, and addition, multiplication, the raising to powers and extraction of roots, operations with logarithms and with series, and so on, constitute such objects, so too the differential and integral calculus; and the name of relation of a function of powers and of the function of its development or potentiation, is perhaps fittest for whatever is akin to this calculus, for this name places us nearest to an insight into the nature of the matter. Only, operations according to other magnitudinal relations (addition and so on) are also generally used in this calculus, and so also logarithmic, circular, and serial relations are applied, especially in order to make more tractable expressions for the purpose of the necessary operations of the derivation of original functions from functions of development. The differential and integral calculus has this point of interest in common with the form of the series, that it determines the functions of development which, in series, are called coefficients of the terms; but the interest of this calculus is directed only upon the relation of the original function to the next coefficient of its development, and thus the series tries to represent a sum in the multitude of terms arranged according to powers which have these coefficients. The infinite of the infinite series—that indeterminate expression of the negative element of Quantum in general—has nothing in common with the affirmative determination which is contained in the infinite of this calculus. And also the infinitesimally small (which appears under the shape of increment), which gives to development the form of series, is a merely external means to this end, and the only meaning of its so-called infinity is to have no meaning except as this methodological device: the series, which in fact is not what is wanted, produces an excess the removal of which causes the unnecessary trouble. Lagrange's method too—he took up the serial form again by preference—is hampered by this trouble; although it is in this method that the true peculiarity stands revealed in what is called the application, for, without forcing the forms of dx, dy, and so on, into the objects, that part is directly indicated to which the determinateness of the derived function (function of development) belongs in them: so that it is clear that here the real matter in hand is not the form of the series.[1]

---

[1] In the above-mentioned criticism (*Jahrb. für wissensch. Krit.*, II Bd. 1827, Nr. 115, 6 sqq.), interesting views of a sound scholar in this science, Mr. Spehr,* may be found; they are quoted from his *Neue Prinzipien des Fluentenkalkuls*, Brunswick, 1826, relating to a fact which he asserts materially to contribute to the obscurities and unscientific parts of the differential calculus, and they agree

---

* Spehr, Friedrich Wilhelm, 1799–1833, a mathematician of Brunswick: *Vollständiger Lehrbegriff der reinen Kombinationslehre*, 1824.

Observation 3

FURTHER FORMS CONNECTED WITH THE QUALITATIVE DETERMINATENESS OF MAGNITUDE

In the differential calculus the infinitesimally small appears in its affirmative meaning as qualitative magnitudinal determinateness; and of this it has further been shown that it is present in this calculus not only as determinateness of powers in general, but more especially as the determinateness of the ratio of a function of powers to the power of development. The qualitative determinateness is also present in a wider and, so to say, weaker form, which, together with the connected employment of the infinitesimally small and its meaning in this employment, will be considered in this Observation.

We begin from what precedes, and must first remark in this regard that the various determinations of powers, in the analytic aspect, manifest themselves as purely formal and quite homogeneous, since they denote numerical magnitudes, which, as such, do not possess this mutual relation of qualitative difference. But when applied to objects of space, the analytic relation shows itself in its qualitative determinateness as the transition from linear to plane determinations, from determinations of straight lines to determinations of curves, and so

with what has been said about the general nature of the theory of this calculus. "Purely arithmetical investigations," he says, "(admittedly related more closely than any others to the differential calculus) were not distinguished from the differential calculus proper, and have ever been confused (as by Lagrange) with the matter itself, which latter was regarded as a mere application of them. These arithmetical investigations include the rules of differentiation, the derivation of Taylor's theorem, etc., and even the various methods of integration. But the reverse is the case, and these applications are the object of the real differential calculus, which, from the analysis, presupposes all these arithmetical developments and operations."—We have shown how with Lagrange the separation of the so-called application from the procedure of the general part, which starts from series, serves only to emphasize the peculiar nature of the differential calculus for itself. And it is strange that the author, having this interesting understanding that the so-called applications are just what constitutes the object of the differential calculus proper, should enter upon the formal metaphysics (there quoted) of Continuous Magnitude, Becoming, Flow, and so on, and should even desire to add new ballast to this old. These determinations are *formal* because they are only general categories, which do not indicate what is specific in the matter which was to be learned and abstracted from the concrete doctrines, the applications.

on. This application further involves that spatial objects which from their very nature are given in the form of continuous magnitudes, are taken as discrete, the plane as a multitude of lines, the line as a multitude of points, and so on. This solution is interesting only in one respect, which is that it itself determines the points into which the line is analysed and the lines into which the plane, and so forth, in order that from such a determination it may proceed analytically, which really means arithmetically; the starting-points for the magnitudinal determinations which are to be found are those elements whence are to be derived the function and equation for the concrete, that is, the continuous magnitude. In those problems which are interesting chiefly because they employ this procedure, something which is *determinate in itself* is demanded, in the element, for the starting-point,—in opposition to the method which is indirect, because it can begin on the contrary only with limits between which is supposed to lie that entity, determinate for itself, which is its objective. In both methods the result is the same if only it is possible to discover the law of further determination and impossible to reach the perfect (that is, so-called finite) determination which is demanded. To Kepler is ascribed the honour of first having thought of this reversal of the progress, and of making the discrete its starting-point. He expresses this simply when he explains how he understands the first theorem in Archimedes' cyclometry. Archimedes' first theorem is, of course, that a circle is equal to a right-angled triangle having one catheter equal to the radius and one equal to the circumference of the circle. Kepler takes the meaning of the theorem to be this, that the circumference of the circle has as many parts as it has points, that is, infinitely many, each of which may be considered as the base of an isosceles triangle, and so on: he thus gives expression to the dissolution of the continuous into the form of discreteness. The expression "infinite" which here occurs is still far distant from the determination which it is destined to have in the differential calculus.—A determinateness or function having now been found for such discreta, they must next be united, and exist essentially as elements of the continuous. But a sum of points produces no line and a sum of lines no plane: the points are, therefore, immediately taken as linear and the lines as plane in nature. But also these linear entities must not yet be lines (which they would be if they were taken as quantum): they are, therefore, imagined as infinitesimally small. But what is discrete can be united only externally, the moments retaining their meaning of discrete Ones; the analytic transition from them is made only to their sum, and is not also the geometric transition from point into line, line into plane, and so on; and, therefore, the element which has its determination as point or as line is given the quality of line with the former, and the quality of plane with the latter determination, so that the sum, being a sum of little lines, may become a line, and being a sum of little planes, a plane.

The need of acquiring this moment of qualitative transition and of recurring to the infinitesimally small to this end must be regarded as the source of all the ideas which, intended to overcome this difficulty, are the greatest difficulty themselves. In order that one might dispense with this expedient, it would have to be possible to show that the analytic procedure itself, which appears as a mere summation, in fact already contains multiplication. But in this regard a new assumption enters, which constitutes the basis of this application of arithmetical relations to geometrical figurations. This assumption is, that arithmetical multiplication is a transition to a higher dimension for the geometrical determination too, and that the arithmetical multiplication of magnitudes which, according to their spatial determinations, are lines, also extracts a plane from the linear determination. Three times four linear feet are twelve linear feet, but three linear feet times four linear feet are twelve plane feet (square feet), for the unit in both, since each is a discrete magnitude, is the same. The multiplication of lines by lines at first appears meaningless, since multiplication only deals with numbers; that is, it is a change of entities which are perfectly homogeneous with that into which they pass over (the product), and change only their magnitude. On the other hand, a process of multiplying a line as such by a line—which has been called *ductus lineae in lineam*, like *plani in planum*, and is also *ductus puncti in lineam*—is a change not only of the magnitude but also of the line as qualitative determination of spatiality, as a dimension; the transition of line into plane must be taken as its self-externalization, which for the point is a line, and for the plane, a volume. It is this process which is imagined when it is said that the movement of a point is a line, and so forth; but movement includes the determination of time, and in this idea, therefore, appears rather as a contingent and external variation of the condition; whereas it is the conceptual determinateness (which was expressed as self-externalization) which must be taken,—that qualitative change, which in arithmetic is the multiplication, of Unit (the point and so on) into Amount (the line and so on).—We may here make this further remark, that in the self-externalization of the plane (which would manifest itself as a multiplication of plane into plane) the appearance of a difference between arithmetical and geometrical products results when the self-externalization of the plane, as *ductus plani in planum*, would in arithmetic produce the multiplication of two determinations of the second degree, that is, a product of four dimensions, which in the geometrical determination is however reduced to three. Although on the one hand Number, because it has One for principle, provides the fixed determination for the external quantity, yet equally its productive power is formal. Taken as a numerical determination, 3 × 3, when it reproduces itself, is 3 × 3 × 3 × 3; but this same magnitude as plane determination is not allowed, when it reproduces itself, to proceed beyond 3 × 3 × 3, because space, imagined as a progress from the point, or the merely abstract limit, has its true limit as concrete determinateness after line in the third dimension. This difference might prove powerful with regard to free movement, wherein one (spatial) side is governed by geometrical determination (in Kepler's law $s^3 : t^2$), and the other (temporal) side by arithmetical determination.

It will now be evident without further remark how the qualitative here considered differs from the subject of the previous Observation. There the qualitative element consisted in the determinateness of powers: here, like the infinitesimally small, it stands in the mere arithmetical relation of factor to product, or as point to line, line to plane, and so on. And the qualitative transition which must be made to the continuous from the discrete (into which continuous magnitude is imagined as dissolved) is effected by a process of summation.

However, this would-be mere summation contains in fact a multiplication and, therefore, a transition from linear to plane determination; and this appears most simply in the manner in which (for instance) it is shown that the area of a trapezium is equal to the product of the sum of the two opposite parallel lines and half the height. This height is imagined simply as the *amount* of a multitude of discrete magnitudes which must be summed up. These magnitudes are lines which lie parallel between those two limiting parallels; their number is infinite, for they must constitute the surface, and also they are lines which, therefore, in order to be of plane nature, must be posited together with negation. In order to escape the difficulty that a sum of lines is to produce a plane, lines are immediately assumed to be planes, but *infinitely narrow*, for their only determination lies in the linear element of the parallel limits of the trapezium. Being parallel, and being limited by the other pair of rectilinear sides of the trapezium, they can be imagined as terms of an arithmetical progression, having a uniform difference which, however, need not be determined, and having those two parallels for first and last terms; the sum of this series is of course the product of the parallels and half the amount of the terms. This latter quantum is called Amount only relatively to the idea of the infinitely many lines: it is the magnitudinal determinateness of something continuous, namely height. Clearly that which is called sum is also a *ductus lineae in lineam*, multiplication of linear by linear element, and, therefore, the production (according to the above determination) of something of plane nature. In the simplest case of any rectangle *A B*, each of the two factors is a simple magnitude; already in the next example (itself elementary) of the trapezium, only one factor is *once* half the height, while the other is determined by a progression: it too is of linear nature, but the determinateness of its magnitude is more complex: and since it can be expressed only by a series, the endeavour to sum it up is called analytical, that is, arithmetical; while multiplication here is the geometrical moment, the qualitative part of the transition from the dimension of line into plane: one factor is taken, discretely, only for the arithmetical determination of the other, and, like the other, is itself the magnitude of a linear something.

The method in which planes are imagined as sums of lines is also frequently used when multiplication as such is not used in order to find the result. This happens where the object is not to indicate the magnitude in the equation as Quantum, but as a proportion. For instance, it is commonly proved that the area of a circle is to the area of an ellipse, of which the major axis is equal to the diameter of the circle, as the major axis is to the minor axis, by assuming that each of these areas is the sum of the relative ordinates. Each ordinate of the ellipse is to the corresponding ordinate of the circle as the minor is to the major axis: it is concluded that, therefore, the sums of the ordinates too (that is, the areas) are in the same proportion. Those who here wish to avoid the idea that an area is a sum of lines have recourse to the common and quite gratuitous makeshift of making the ordinates into trapezia of infinitely small breadth: the equation is only a proportion, and therefore only one of the two linear elements of a plane enters into the comparison. The other element (the axis of the abscissa) is assumed to be equal in ellipse and in circle, and therefore as = 1 in so far as it is a factor of arithmetical magnitudinal determination: the proportion therefore depends solely upon the relation of the one determining moment. The two dimensions are essential to the *idea* of area: but the magnitudinal determination, as it is required to be indicated in this proportion, concerns itself only with one moment; and when the idea of sum is added to this one moment (which is a surrender to, or attempted propping of, the idea), then this means that the real point demanded by mathematical determinateness is here missed.

This exposition also contains the criterion of Cavalieri's method of the indivisible (which has been mentioned above): this, too, is therefore justified and need not have recourse to the infinitesimally small. These indivisibilia are lines when he is considering a plane, and squares or plane circles when he is considering a pyramid, cone, and so on. The basic line or plane (which he takes as determinate) is called the *regula*; it is the constant, and with reference to a series it is its first or last term; and to it these indivisibilia are considered parallel, that is, as having the same determination with regard to the figure. Now Cavalieri's general fundamental proposition is (*Exerc. Geometr. VI.*, in the later work *Exerc. I.*, p. 6), "that all figures, both plane and solid, are proportionate to all their indivisibilia, and compare these collectively or, when there is a common proportion in them, distributively."—For this purpose he compares, in figures of the same base and height, the proportions between lines drawn parallel to the former and at equal distance from it; all such lines in a figure have one and the same determination and constitute its whole content. In this manner, too, Cavalieri proves, for instance, the elementary proposition that parallelograms of equal height are proportionate each to its base: any two lines drawn at an equal distance from the base and parallel to it in the two figures are in the same proportion to the bases; and so, therefore, are the whole figures. In fact, the lines do not constitute the content of the figure in so far as it is *continuous*, but only in so far as it is to be determined arithmetically; the linear factor is its element, and through it alone its determinateness must be seized.

And here we are led to reflect upon the difference which exists with respect to that element in which the determinateness of a figure consists; it is either of the same nature as is here the height of the figure, or it is external limit. In so far as it is determinateness as being external limit, it is admitted that the continuity of the figure follows, so to speak, upon the equality or the proportion of the limit; thus the equality of figures which coincide with one another follows from the fact that the limiting lines coincide with one another. But when parallelograms are of equal height and base, only the latter determinateness is external limit; height, and not parallelism (and on this the second capital determination of figures, that is their ratio, depends), introduces a second principle which determines external limits. Euclid's proof that parallelograms of equal height and base are equal, reduces them to triangles, that is, to continua limited externally: in Cavalieri's proof, and first in his proof of the proportionality of parallelograms, the limit is magnitudinal determinateness as such in general, which is explained to be taken as applied to each pair of lines drawn at an equal distance in the two figures. These lines, which are either equal or in an equal ratio with the base, being taken collectively, produce the figures which also are in the same ratio. The idea of an aggregate of lines is incompatible with the continuity of a figure; the mere consideration of lines quite exhausts the essential determinateness. Cavalieri frequently answers the objection that the idea of the indivisible necessarily involves the comparison of lines or planes infinite in amount (*Geom. Lib. II. Prop. I. Schol.*); and makes the just distinction, that it is not their amount, which we do not know (—it is, as has been observed, an empty auxiliary idea—), but only magnitude, that is quantitative determinateness as such, equal to the space enclosed by these lines, which he compares. This space is contained in limits, and, therefore, its magnitude too is contained within these limits: the *continuous is nothing other than the indivisibilia themselves*, says Cavalieri: if it were outside them it would be incommensurable: but it would be absurd to say that limited continua were incommensurable.

Clearly, thus, Cavalieri attempts to distinguish what belongs to the external existence of the continuous from that which constitutes its determinateness and needs to be emphasized only for comparison and for theorems dealing with it. It is true that the categories which he uses for this purpose (the fact that the continuous is *composed* or *consists* of the indivisibilia, and the like) are inadequate, since they also involve the intuition, or, as was said before, the external existence, of the continuous; instead of saying that "the continuous is nothing but the indivisibilia themselves," it would be more correct and therefore also immediately more clear to say that the magnitudinal determinateness of the continuous is the same as that of the indivisibilia themselves.—Cavalieri does not care for the faulty conclusion that there are greater and less infinites, which (he says) is drawn by the schools from the idea that the indivisibilia constitute the continuous; and, further, (*Geom. Lib. VII. Præf.*) he expresses the more definite knowledge that his method of proof by no means forces him into the idea of the composition of the continuous from indivisibilia: the continua only follow the proportion of the indivisibilia. He claims to have taken the aggregates of indivisibilia, not in that manner in which, for the sake of an infinite multitude of lines or planes, they appear to fall into the determination of infinity, but in so far as they contain a determinate condition and nature of limitedness. But in order finally to remove this stumbling-block, he does not spare himself the pains of proving the capital propositions of his geometry (in the seventh book, specially added for this purpose) in a manner designed to be uninfected with infinity.—This manner reduces the proof to the ordinary form (mentioned before) of the coincidence of figures, that is, as was observed, to the idea of determinateness as external spatial limit.

We may further remark about this form of coinciding, that altogether it is what may be called a childish aid for sensuous intuition. In the elementary theorems about triangles, two triangles are imagined side by side; of their six component parts, three are assumed equal to the corresponding three in the other triangle, and then it is demonstrated that such triangles are equal in all respects; that is, that each has the remaining three parts also equal to those of the other, because it follows from the equality of the first three that they coincide. If the matter be taken more abstractly, it is just because of this equality of each pair of the parts corresponding to each other in both, that there is only one triangle, in which three parts are taken as already determinate, whence follows the determinateness of the three remaining parts. In this manner determinateness is demonstrated as complete in three parts; and, for determinateness as such, the three remaining parts represent a superfluity, the superfluity of sensuous existence, that is, of the intuition of continuity. Expressed in this form, qualitative determinateness manifests itself as distinct from the object of intuition, which is the whole as continuous in itself; and coincidence does not allow this distinction to be perceived.

With parallel lines and parallelograms there comes into play (as has been observed) a new circumstance, namely, partly the equality of the angles only and partly the height of the figures, from which latter their external limits (the sides of the parallelograms) are distinct. Here a doubtful question arises, that is, how far in these figures—apart from the determinateness of one side, the base, which is an external limit—we are to take for the other determinateness (*a*) the other external limit, namely, the other side of the parallelogram, or (*b*) the height. Where there are two such figures having the same base and height, the one being rectangular and the other having very acute angles (the opposite pair being therefore very obtuse), the latter could easily appear greater to intuition than the former, in so far as it might take its long side as determinant and (according to Cavalieri's method of imagining the matter) might compare the areas according to the multitude of parallel lines which can pass through them: the longer side might be considered as affording a potentiality of more lines than the vertical side of the rectangle affords. Such an idea, however, provides no objection to Cavalieri's method; for the multitude of parallel lines imagined for purposes of comparison in the two parallelograms presupposes their equidistance from one another or from the base: it follows, therefore, that height and not the other side of the parallelogram is the other determining moment. But this changes further when two parallelograms are compared with each other which have the same height and base but are not in the same plane, and are at different angles with a third plane: here the parallel sections which arise when the third plane is imagined as passing through them and moving parallel to itself, are no longer equidistant from one another, and the two planes are unequal. Cavalieri is very careful to draw attention to this distinction, which he defines as the distinction between a *transitus rectus* and a *transitus obliquus* of the indivisibilia (already in *Exercit. I. n. XII. sqq.* and also in *Geometr. I. II.*), and prevents a superficial misunderstanding which might arise in this direction. Barrow in his above-cited work (*Lect. Geom. II.* p. 21) also uses the method of indivisibilia, although he adulterates it with the assumption (which from him passed on to his pupil, Newton, and to other mathematical contemporaries, among them Leibniz) that a curvilinear triangle, like the so-called characteristic triangle, may be equated with a rectilinear triangle in so far as both are infinitely, that is, *very* small; and I remember that he quotes an objection of Tacquet,[1] an ingenious geometer who worked upon what then were new methods, which points to that same end. The difficulty which he raises refers to the question, which line, in the calculation of conical and spherical surfaces, should be taken as fundamental moment of determination for the consideration based upon application of the discrete. He says that Tacquet's objection to the method of indivisibilia is this, that in the calculation of the surface of a right-angled cone the triangle of the cone is imagined, in this atomistic method, as composed of straight lines parallel to the base and at right angles to the axis, which also are the radii of the circles of which the surface of the cone consists. And if this surface is determined from the sum of the circumferences, and this sum from the number of their radii, that is, from the magnitude of the axis, or the height of the cone, then (he proceeds) such a result is in contradiction with the truth elsewhere taught and demonstrated by Archimedes. And Barrow shows that, to determine the surface, it is not the axis but the side of the triangle of the cone which must be taken as the line the revolution of which produces the surface; this line, therefore, and not the axis, must be taken as the magnitudinal determinateness for the multitude of circumferences.

---

[1] Tacquet, Andr., 1611–1660, Professor in the Jesuit College at Antwerp: *Cylindricorum et annularium libri V*, 1651–9.

Such objections and hesitancies have their sole origin in the indeterminate idea which they use of the infinite multitude of points of which a line, or of lines of which a plane, and so forth, is regarded as consisting: the essential magnitudinal determinateness of lines and planes is obscured by this idea.—It has been the intention of these Observations to make evident the affirmative determinations which so to speak remain in the background in the varied use which is made of the infinitesimally small in mathematics; and to raise them from the obscurity in which they are wrapped by this purely negative category. In the infinite series (as in Archimedes' cyclometry) the Infinite means nothing more than that the law of further determination is known, but that the so-called finite (that is, arithmetical) expression, is not given, so that it is impossible to effect the reduction of the curve to the straight line: this incommensurability constitutes their qualitative difference. The qualitative difference between discrete and continuous in general also contains a negative determination, which causes them to appear incommensurable and introduces the infinite in this sense, that the continuous (which is to be taken as discrete) must now have no Quantum according to its continuous determinateness. The continuous (which, arithmetically, is to be taken as product) is thus posited discretely in itself, as being analysed into the elements which are its factors: in these lies its magnitudinal determinateness; and just because they are these elements or factors, they are of a lower dimension, and, in so far as there is any determinateness of power, of a lower power than the magnitude of which they are elements or factors. Arithmetically this difference appears as merely quantitative, as a difference of root or power or any other determinateness of power; but when the expression concerns only the quantitative as such (for instance $a : a^2$ or $d. a^2 = 2a : a^2 = 2 : a$, or, for the law of gravitation, $t : a t^2$), the ratios which result are meaningless ($1 : a$, $2 : a$, $1 : a t$); the sides ought to be held apart against their merely quantitative determination by a difference in qualitative meaning, as $s : a t^2$, where the magnitude as a quality is expressed as function of the magnitude of another quality. Here then it is only quantitative determinateness which is present to consciousness; with it, it is easy to operate according to its manner, and no objection can be offered if the magnitude of one line be multiplied by the magnitude of another; but from the multiplication of these same magnitudes there also results the qualitative change of the transition of line into plane: and in so far a negative determination comes into play. It is this that causes the difficulty which is solved by an understanding of its peculiarity and of the simple nature of the matter, but by the help of the infinite, which is intended to remove it, is suspended quite unsolved and in confusion.

CHAPTER III

THE QUANTITATIVE RATIO

The Infinity of Quantum has proved itself to be the negative Beyond of Quantum; but it belongs to Quantum. This Beyond is the Qualitative in general. Infinite Quantum, as the union of the two moments (quantitative and qualitative determinateness), is Ratio.

In Ratio the determinateness of Quantum no longer is indifferent: it is qualitatively determined as absolutely related to its Beyond. It continues itself into its Beyond; and this, for the present, is simply *any* other Quantum. But essentially they are related to one another not as external Quanta; each has its determinateness in this relation to the other. In this their otherness they have thus returned upon themselves; what each is, it is in the other; and the other constitutes the determinateness of each.—Thus the passing of Quantum beyond itself now has this meaning, that it did not merely change into an Other, nor into its abstract Other (its negative Beyond), but that it there reached its determinateness: it finds *itself* in its Beyond, which is another Quantum. The quality, or conceptual determinateness, of Quantum is its externality in general. Now in Ratio it is posited as having its determinateness in its externality and in another Quantum, and as finding its true nature in its Beyond.

The entities that have the relation which we have found, are Quanta. This relation is itself a magnitude; Quantum is posited, not only as being in a Ratio, but as being itself a Ratio; it is a Quantum in general which contains this qualitative determinateness within itself. Taken thus as Ratio, it expresses its existence as self-contained totality and its indifference to limit, through the fact that it has within itself the externality of its determinateness and in this externality is related only to itself, that is, is in itself infinite.

Ratio is divided into:—

(1) Direct Ratio. Here the qualitative element does not show forth for itself as such: it is in a purely quantitative

manner that Quantum is posited as having its determinateness in its externality.—In itself Quantitative Ratio is the contradiction of externality and of self-relation, of the existence of Quanta and of their negation;—it transcends itself when next

(2) in Indirect Ratio there is posited the negation as such of one Quantum together with the alteration of the other, and the alterability of Direct Ratio itself;

(3) in Ratio of Powers, however, unity, which in its difference is self-related, asserts itself as simple self-production of Quantum; this qualitative element being finally posited in simple determination and as identical with Quantum, becomes Measure.

—In the preceding Observations, which deal with the infinite of Quantity (that is, with the qualitative moment in Quantity), much that is true of the nature of the following Ratios has been anticipated. All that remains, therefore, is to explain the abstract concept of these Ratios.

A

THE DIRECT RATIO

1. In that Ratio which is immediate and, therefore, direct, the determinateness of one Quantum is contained reciprocally in the determinateness of the other. There is only one determinateness or limit of the two, which is also itself a Quantum, and this is the *exponent* of the Ratio.

2. The exponent may be any Quantum; but it is a qualitatively determinate Quantum self-related in its externality only in so far as it has in itself its difference, its Beyond and otherness. But this difference of the Quantum in itself is the difference of Unit and Amount: Unit is self-determinateness; Amount, the indifferent oscillation about determinateness, the external indifference of Quantum. At first Unit and Amount were moments of Quantum; now, in Ratio (which to this extent is Quantum realized) each of its moments appears as an independent Quantum, and as determinations of its existence, as limitations of its otherwise purely external and indifferent magnitudinal determinateness.

The exponent is this difference as simple determinateness, that is, it immediately contains in itself the meaning of both determinations. It is, first, Quantum, and thus is Amount; if one side of the Ratio, which is taken as Unit, is expressed as numerical One—and as such only it is counted,—then the other, Amount, is the Quantum of the exponent itself. Secondly, it is simple determinateness as the qualitative element of the sides of the Ratio: if the Quantum of one is determined, the Quantum of the other too is determined by the exponent, and it is quite indifferent how the first is determined: as Quantum determinate for itself it no longer has a meaning, but can equally well be any other without changing the determinateness of the Ratio, which rests wholly upon the exponent. The one, which is taken as Unit, always remains Unit, however great it becomes; and the other, however great it likewise becomes in the process, must always remain the same Amount of that Unit.

3. Thus really both only constitute one Quantum; one relatively to the other has the value only of Unit and not of an Amount, and the other has only that of an Amount; and thus, according to their conceptual determinateness, they are not themselves complete Quanta. But this incompleteness is a negation in them,—not a negation which follows from their general alterability according to which one (and each is one of the two) can assume any magnitude, but according to the determination that when one is altered the other increases or decreases proportionally; this means (as has been shown) that only one (Unit) changes as Quantum; the other side (Amount) remains the same Quantum *of units*, and even the former has *meaning* only as Unit, however much it may change as Quantum. Thus each of the two sides is only one of the two moments of Quantum, and that independence which is its peculiarity, is negated in itself: in this qualitative connexion they must be posited as negative to each other.

The exponent is supposed to be the complete Quantum, for in it the determination of the two sides coincides; but in fact, as quotient, it only has the value of the Amount, or of the Unit. There is here no determination showing which of the sides of the ratio must be taken as Unit and which as Amount; if one (the Quantum *B*) be measured against the Quantum *A* as Unit, then the quotient C is the Amount of such Units; but if A itself be taken as Amount, then the quotient C is the Unit which the Amount A demands for the Quantum B; thus this quotient, as exponent, is not posited as what it is supposed to be, namely, the determinant of the ratio or its qualitative unity. It is posited as such only in so far as it has the meaning of being the union of both moments, of Unit and Amount. These sides are present as Quanta (such as they are supposed to be in explicit Quantum, which is Ratio), but also only in the value which they are supposed to have as being sides of the Ratio,—which means that they are incomplete Quanta and count only as one of these qualitative moments; and, therefore, they must be posited as having this their negation. Thus there arises a Ratio more real and better corresponding to its determination; the exponent here has the meaning of being their product; Ratio according to this determinateness is Inverse Ratio.

B

THE INVERSE RATIO

1. The Ratio which has now been reached is *transcended* Direct Ratio; the first stage was immediate and, therefore, not truly determinate Ratio; here determinateness has been added, in such a manner that the exponent counts as product, as the unity of Unit and Amount. With regard to immediacy, it could indifferently be taken as Unit and as Amount, as was shown above, and so it also existed as Quantum only in general, and hence, by preference, as Amount; one side had to be taken as Unit and as One, to which the other stood in the relation of fixed Amount, which also was the exponent; its quality, therefore, was only this, that this Quantum is taken as fixed, or rather that what is fixed has the meaning only of Quantum.

Now in Inverse Ratio, too, the exponent, as Quantum, is taken as immediate; and any Quantum can be taken as fixed. But this Quantum is not related as fixed Amount to the One of the other Quantum in the ratio; this ratio, which before was fixed, is now posited as variable; if another Quantum is taken as the One of the one side, then the other no longer remains the same amount of units of the first. In Direct Ratio this unit is only the common part of both sides; as such it continues itself into the other side, which is Amount; and Amount itself for itself, or the exponent, is indifferent to the Unit.

But under the present determinateness of Ratio, Amount as such is varied relatively to the One to which it is related as the other side of the Ratio. It becomes another when another One is taken as Quantum. Consequently, though the exponent is only immediate and a Quantum arbitrarily taken as fixed, yet it does not preserve itself as such in the side of the Ratio: it, and with it the direct ratio of the sides, is variable. Thus in the present Ratio the exponent as determining Quantum is posited as negating itself as Quantum of the Ratio: hence it is posited as qualitative, as Limit, so that the qualitative element emphasizes itself in opposition to the quantitative.—In Direct Ratio the variation of both sides is the single alteration only of the Quantum which the unit (the common element) is supposed to be; one side is increased or decreased in the same proportion as is the other; the Ratio itself is indifferent to this variation, which remains external. But in Inverse Ratio the variation, although likewise arbitrary according to the indifferent quantitative moment, is *contained within* the Ratio; and even this arbitrary quantitative overpassing is circumscribed by a limit, namely, the negative determinateness of the exponent.

2. This qualitative nature of Indirect Ratio must be further considered—in its realization; and the complication of the affirmative with the negative which it contains must be analysed.—Quantum is posited as qualitatively determining Quantum, that is, itself, as representing itself in itself as its own limit. Thus, first, it is an immediate magnitude as simple determinateness; it is the whole as existent and affirmative Quantum. Secondly, however, this immediate determinateness is also limit; and hence it is differentiated into two Quanta, which stand primarily in the relation of Other to each other; —but, since it is their qualitative and their complete determinateness, it is the unity of Unit and Amount; it is product, and they are its factors. Thus partly the exponent of their ratio is identical with itself in them and is their affirmative, and thus they are Quanta; and partly, as negation posited in them, it is the element of unity in them, according to which

each is an immediate and limited Quantum, and limited in such a way that it is identical with its Other only in itself. Thirdly, as simple determinateness, it is the negative unity of this its differentiation into two Quanta and the limit of their reciprocal limitation.

According to these determinations, the two moments limit each other in the exponent; and since the exponent is their determinate unity, each is the negative of the other; one increases in the same ratio in which the other decreases; each has its magnitude in so far as it is in contact with the magnitude of the other and is what the other lacks. Thus each negatively continues itself into the other; it cancels its own Amount in the other, and is what it is only by virtue of the negation or limit which the other posits in it. Thus each contains and has its measure in the other, for each is to be only that Quantum which the other is not; the magnitude of the other is indispensable to the value of each, and therefore inseparable from it.

This continuity of one in the other constitutes the moment of unity by which they form a Ratio,—or the one determinateness and simple limit which is the exponent. This unity, the whole, constitutes the Being-in-Self of each, which is distinct from their magnitude as it is found: according to the latter, each exists only in so far as it withdraws from the other part of their common Being-in-Self or whole. But it can withdraw from the other only as much as makes it equal to this Being-in-Self; it has its maximum in the exponent, which, according to the second determination which we indicated, is the limit of their reciprocal limitation. And since each is moment of the Ratio only in so far as it limits the other and therefore is limited by it, it loses this its determination by equating itself with its Being-in-Self; in it not only the *other* magnitude becomes nought, but it vanishes *itself*, since it is not mere Quantum, but is to be what it is as such only as being such a moment of the Ratio. Thus each side is the contradiction of the determination as their Being-in-Self, that is, of the unity of the whole (which is the exponent), and of the determination as moment of the Ratio: and this contradiction is once more infinity in a new and peculiar form.

The exponent is *limit* of the sides of its Ratio, within which they increase and decrease relatively to each other; and according to the affirmative determinateness which the exponent (as Quantum) is, neither can become equal to it. As limit thus of their reciprocal limitation, it is (α) their Beyond, to which they can approximate infinitely; but they cannot reach it. This infinity in which they approximate to it is the bad infinity of the infinite progress; this itself is finite, and has its barrier in its opposite, in the finitude of the two sides and of the exponent; it, therefore, is only approximation. But (β) bad infinity is here also posited as what in truth it is, namely as the merely negative moment in general, according to which the exponent is simple limit as Being-in-Self in relation to the distinct Quanta of the Ratio; their finitude is referred to this as to a pure variable, although it remains quite distinct from them as their negation. This infinite, to which they can but approximate, further is found and is present as affirmative Hither: this is the simple Quantum of the exponent. Here the Beyond is reached with which the sides of the Ratio are tainted: it is in itself the unity of both and, therefore, in itself the other side of each; for each only counts for as much as the other does not, and thus the whole determinateness of each resides in the other; and this their Being-in-Self is, as affirmative infinity, simply the exponent.

3. But now the result is that Inverse Ratio has passed over into a determination different from that which it had at first. The determination was, that a Quantum (as immediate) is related also to another Quantum in such a manner that it is greater in the proportion in which the other is smaller, that it is what it is by virtue of its negative attitude to the other; and similarly a third magnitude is the common barrier of this their increase. Here this variation (as opposed to the Qualitative as fixed limit) is their peculiarity: they have the determination of variable magnitudes, having this fixed limit as an infinite Beyond.

We have found and must coordinate the determinations that this infinite Beyond exists as a present and finite (but optional) Quantum; and, further, that its fixity (by virtue of which it is thus related to the quantitative as infinite Beyond) which is the qualitative element of Being only as abstract self-relation, has developed itself with itself as mediation of itself in its Other, namely the finite moments of the Ratio. The universal part of this is contained in the fact that the whole as exponent is the limit of the reciprocal limitation of the two members; thus negation of negation and, therefore, infinity (an affirmative attitude to itself) is posited. A closer determination is, that in itself the exponent is product and, therefore, unity of Unit and Amount, whereas each of the two members is only one of these moments; it, therefore, includes them, and in them (as containing them) relates itself to itself. But in Inverse Ratio the difference develops into the externality of quantitative Being; and the Qualitative is not merely what is fixed or immediately includes the moments, but is found coalescing with itself in self-external otherness. It is this determination which manifests itself as result in the moments which we discovered. For the exponent is found to be Being-in-Self, the moments of which are realized in Quanta and in their variability in general; the indifference of their magnitudes in their variation manifests itself as infinite progress, and this is based upon the fact that in their indifference it is their determinateness that they have their value in the value of an other. Thus (α) in the affirmative aspect of their Quantum they are, in themselves, the whole of the exponent. Equally, (β) their negative moment, for their reciprocal limitation, is the magnitude of the exponent; their limit is that of the exponent. The infinite progress of their determinate existence and of their limitation, and the negation of any particular value, implies that they have no other immanent limit or fixed immediacy. This negation, therefore, is the negation of the self-externality of the exponent which is represented in them; and the exponent (taken as Quantum in general and as analysed into Quanta) is posited as that which preserves itself amid the negation of their indifferent persistence and collapses into itself, and, therefore, as the determinant of any such transition beyond itself.

Hence Ratio has come to be determined as the Ratio of Powers.

C

THE RATIO OF POWERS

1. Quantum, which in its other-being identifies itself with itself and determines its self-overpassing, has reached Being-for-Self. It is thus qualitative totality, and, positing itself as

developed, it has for moments the conceptual determinations of number—Unit and Amount. Even in Inverse Ratio the latter is a multitude determined, not by the former as such, but from without by a third element: now it is posited as determined only by the former. This happens in the Ratio of Powers, where Unit, which in itself is Amount, is also Amount relatively to itself as Unit. Other-being, the Amount of Units, is Unit itself. Power is a multitude of Units each of which itself is this multitude. Quantum as indifferent determinateness varies; but where this variation means raising to a power, this its other-being is limited purely by itself.—Thus in Power Quantum is posited as having returned upon itself: it is immediately itself and also its other-being.

The exponent of this Ratio no longer is an immediate Quantum as in Direct and also in Inverse Ratio. In the Ratio of Powers it is of a wholly qualitative nature; it is the simple determinateness that Amount here is Unit and that Quantum in its other-being is self-identical. This also contains the aspect of its quantitative nature, which is, that limit or negation is not posited as existing immediately, but that Determinate Being is posited as continued into its otherness; for the true nature of quality is this, that it is quantity (or immediate determinateness in so far as it is transcended).

2. Ratio of Powers first appears as an external variation applied to any Quantum; but it has this closer connexion with the concept of Quantum, that Quantum, in that Determinate Existence into which it has developed in this Ratio, reaches and completely realizes this concept: this Ratio represents what Quantum is in itself, and expresses its determinateness or quality by which it is distinct from anything else. Quantum is determinateness indifferent and posited as transcended; that is, determinateness as limit which also is no limit, and continues itself into its otherness, and consequently remains self-identical. It is thus posited in the Ratio of Powers, and its other-being or self-overpassing into another Quantum is determined by itself.

If we compare the progress of this realization in the Ratios with which we have dealt hitherto, then the quality of Quantum to be its own posited self-difference, is just this, that it is Ratio. As Direct Ratio it is, as such posited difference, only general or immediate, so that its self-relation, which, as exponent, it possesses in relation to its differences, counts only as the fixity of an Amount of Units. In Inverse Ratio, Quantum in its negative determination is an attitude of itself to itself,—to itself as its own negation, wherein, however, it has its value; as affirmative self-relation it is an exponent, which, as Quantum, is the determinant of its moments only in itself. But in the Ratio of Powers it is present in the difference, because the difference is a self-difference. Externality of determinateness is the quality of Quantum; and according to its concept this externality is posited as its self-determination, as its self-relation and its Quality.

3. But Quantum, being now posited as it is in accordance with its concept, has passed over into another determination. This might also be expressed thus—that its determination is its determinateness, and its Being-in-Self its Determinate Being. It is Quantum in so far as the externality or indifference of determinateness (which means that it is that, as the phrase goes, which can be increased or diminished) counts and is posited only simply or immediately; and it has become its Other (namely Quality) in so far as this externality is now posited as mediated by Quantum and as being such a moment as refers itself in it to itself and is Being as Quality.

Thus first Quantity as such appears as opposed to Quality. But Quantity is itself a quality; it is self-relating determinateness in general, distinct from that determinateness which is other to it, namely Quality as such. But it is not only *a* quality: the truth of Quality itself is Quantity, and the former has manifested itself as passing over into the latter. On the other hand, Quantity in its truth is externality which has returned to itself and is not indifferent. And thus it is Quality itself, and, outside this determination, Quality as such would be nothing.—In order that the totality may be posited, the twofold transition is required,—not only the transition of one determinateness into its other, but also the transition of the latter into the former, its regress into the first. The first transition gives us only the identity in itself of both;—Quality is contained in Quantity, but so far Quantity is a one-sided determinateness. It is the result of the second transition that, conversely, Quantity is contained in Quality, and that it too exists only as having been transcended. This is the regress into the first. This observation upon the necessity of the twofold transition is of great importance for the whole scientific method.

Now Quantum no longer is indifferent or external determination, but, as such, is transcended, and is Quality, and is that by virtue of which something is what it is: this is the truth of Quantum, and this is Measure.

Observation

It has been explained above, in the Observations upon the Quantitative Infinite, that this and the difficulties which result have their origin in the qualitative moment which manifests itself in the quantitative; and further, how especially the qualitative moment of the Ratio of Powers runs into manifold developments and complexities. It was shown that the fundamental flaw which prevented the concept from being apprehended was, that a halt was made at the negative determination of the infinite (where it is negation of Quantum), and that no progress was made to the simple affirmative determination which states that this is the qualitative.—Here it only remains to remark upon the intrusion, in philosophy, of quantitative forms into the pure qualitative forms of thought. The Ratio of Powers has especially been applied in recent times to conceptual determinations. The concept in its immediacy was called *first power*, in its other-being or difference (the Determinate Being of its moments) it was called *second power*, and in its return upon itself or totality, *third power*.—Here it is immediately evident that Power thus employed is a category essentially belonging to Quantum: this Power is not meant as *potentia*, the δύναμις of Aristotle. The Ratio of Powers thus expresses determinateness as reaching its truth in Difference, as Difference exists in the special notion of Quantum, but not as it exists in the Notion as such. Quantum does not yet by any means possess that negativity which belongs to the nature of the Notion, as posited in its peculiar determination: differences which are proper to Quantum are superficial determinations for the Notion itself, and are far from being determinate as they are determinate in the Notion. It was in the childhood of philosophic thought that numbers were used (as by

Pythagoras) to designate universal and essential distinctions; and here first, second, or other Powers are in no way better than numbers. This was a rudimentary form of pure thinking comprehension; the determinations of thought themselves were not discovered—that is, brought to consciousness for themselves—till after Pythagoras. To return from these to numerical determinations is the part of thought which knows its impotence and, in opposition to what our philosophic culture—no stranger to determinations of thought—had reached, commits the further folly of trying to vindicate this weakness as something new, superior, and in the line of progress.

In so far as Powers are used only as a symbolical expression, they are unobjectionable, just as much as are numbers and other symbols of concepts,—but also they are as objectionable as all symbolism whatever which attempts to represent pure conceptual or philosophic determinations. Philosophy needs no such help either from the sensible world or from active imagination or from subordinate fields of philosophical speculation, the determinations of which, therefore, are unfitted for higher spheres and for the whole. The latter happens whenever categories of the finite are applied to the infinite; the common determinations of force, or substantiality, cause and effect, and others, are themselves too only symbols used to express other relations, like vital and spiritual relations; that is, they are untrue determinations of those relations, and still more so are Powers of Quantum and numbered powers, both for such and for speculative relations generally.—If numbers, powers, the mathematical infinite, and the like are to be used not as symbols but as forms for philosophic determinations and hence themselves as philosophic forms, then first of all their philosophic meaning, that is, their conceptual determinateness, must be demonstrated. If this is done, they are superfluous designations: the conceptual determinateness designates itself, and its own is the only correct and fitting designation. The use of these forms is, therefore, nothing but a convenient means of escaping the trouble of seizing, proclaiming, and justifying the conceptual determinations.

SECTION THREE

MEASURE

Abstractly the statement may be made that in Measure Quality and Quantity are united. Being as such is the immediate self-identity of determinateness. This immediacy of determinateness has transcended itself. Quantity is Being which has returned upon itself in such a manner that it is simple self-identity as indifference to determinateness. But this indifference is merely externality—possessing its determinateness not in itself but in an Other. Thirdly, there is self-relating externality: as self-relation it is also transcended externality and has in itself its own difference from itself. This difference as externality is the quantitative moment, and, as having returned to itself, the qualitative moment.

Modality is enumerated among the categories of transcendental idealism after Quantity and Quality, and where Relation is inserted; so that it may be mentioned here. The category there means that it is the relation of the object to thought. In this idealism thought is essentially external to the thing-in-itself. In so far as other categories have only the transcendental determination of belonging to consciousness as its objective element, modality, which is the category of relation to the subject, to this extent relatively contains the determination of intro-reflection; that is, objectivity, which is stated to be a quality of the other categories, is lacking in the categories of modality; and these (in Kant's expression) do not in the least add to the concept as determination of the object, but only express the relation to the possibility of cognition (*Critique of Pure Reason*, 2nd Ed., see pp. 99, 266).—The categories which Kant groups under Modality—Possibility, Actuality, and Necessity—will occur later in their place; that infinitely important triple form was not applied by Kant—for with him it appeared only as a formal flash of light—to the *genera* of his categories (Quantity, Quality, etc.); and this *name* too he applied only to their species. It was impossible for him, therefore, to find the third member for Quality and Quantity.

With Spinoza, too, the *Mode* is third after Substance and Attribute: he declares that the Mode is the *affections* of Substance, or is that element in an Other by virtue of which it is comprehended. According to this concept this third element is only externality as such; and indeed it has been mentioned elsewhere that in Spinoza in general rigid substantiality lacks the return into itself.

The observation here made extends more generally to those systems of Pantheism upon which thought has done its elaborating work. Being, the One, Substance, the Infinite, or Essence, is the first; in opposition to this abstract element the second, namely every kind of determinateness, can equally abstractly be grouped as that which is merely finite, accidental, perishable, unessential, and non-essential; and this is the next and ordinary step in purely formal thought. But the connexion of the second with the first makes itself so evident that both must needs be taken as one unity; and thus with Spinoza the Attribute is the whole Substance, that is, as taken by Understanding—itself a limitation or Mode: and Mode (which is the non-substantial in general which can be understood only from an Other) is thus the other extreme for Substance, the third in general. Indian Pantheism in all its monstrous imagination has also, taken abstractly, received this elaboration; this is the tempering thread which leads through its riot to this point of moderate interest, that Brahma, the One of abstract thought, passes through the shape of Vishṇu (especially in the form of Kṛishṇa) to the third form, Śiva. The determination of this third is Mode, change, arising and passing away, the field of externality in general. If this Indian triad has led to a comparison with the Christian, it must be recognized that they have a common element of conceptual determination; but it is essential that the difference be more definitely brought to consciousness: the difference is not only infinite, but true infinity constitutes the difference itself. According to its determination this third principle is the explosion of substantial unity into its opposite, and not its return to itself,—the non-spiritual, not Spirit. In the true triad not only unity is found, but harmony—the consummation of a pregnant and real unity, which in its wholly concrete determination is Spirit. The principle of Mode and change does not indeed exclude unity: thus with Spinoza the Mode as such is the false, and Substance alone is the true, and everything must be reduced to it; which is a jettison of all content into the void, into a unity merely formal and without content; and similarly, Śiva once more is the great whole, not distinct from Brahma: it is Brahma itself; that is, the difference and the determinateness vanish again, but are neither preserved nor transcended; unity is not led back to concrete unity, nor dissension to reconciliation. The highest goal for man transplanted into the sphere of arising and passing away, of modality in general, is submersion into unconsciousness, unity with Brahma, annihilation, which is the same as the Buddhist Nirvāna, Nibbāna, and so forth.

The Mode is abstract externality in general and indifference to qualitative and to quantitative determinations; and in essence what is external and unessential should not matter; but on the other hand it is often admitted that all depends upon the how and why. The Mode is thus declared essentially to belong to the substantial part of a thing. This very indefinite relation contains at least this, that this external part is the external not quite so abstractly.—

The Mode here has the definite meaning of Measure. Spinoza's Mode, like the Indian principle of Change, is the measureless. The Greek idea, though indeterminate as yet, that everything has a Measure (which led Parmenides to introduce, after abstract Leing, Necessity as the ancient Limit imposed on all things), is the beginning of a much higher concept than that contained in Substance and the difference between Mode and Substance.

When more fully developed and reflected, Measure becomes Necessity; Fate and Nemesis were generally limited to the determinateness of Measure, which meant that what is presumptuous, or grows too high and great, is reduced to the other extreme by being brought down to annihilation, and that thus the mean of Measure, mediocrity, is restored.—"The Absolute, or God, is the Measure of all things" is a definition not more strongly pantheistic but infinitely more true than "the Absolute, or God, is Being."—Measure is indeed an external way or manner, a more or less, but it is also reflected into itself, and is a determinateness not merely indifferent and external, but existing in itself. It is thus *the concrete truth of Being*; and, therefore, mankind has revered in Measure something inviolable and holy.

The idea of Essence is already contained in Measure, namely that it is identical with itself in the immediacy of determinateness, so that this self-identity reduces the immediacy to a mediate; and also this mediate is mediated only through this externality, but is self-mediation; it is the reflection whose determinations *are*, but, thus being, exist only as moments of their negative unity. The qualitative is quantitative in Measure: determinateness or difference is indifferent, and, therefore, the difference is no difference, it is transcended: and this quantitativity, as return upon self, where it exists as the qualitative, constitutes that Being-in-and-for-Self which is Essence. But Measure is Essence only in itself or in the concept; and this concept of Measure has not yet been posited. Measure, while it is Measure, is the existent unity of the qualitative and the quantitative: its moments are as a Determinate Being, a Quality and its Quanta, which are inseparable only in themselves at first, but have not yet the meaning of this reflected determination. The development of Measure contains the differentiation of these moments, but also their relation, so that the identity which they are in themselves becomes their mutual relation, that is, is posited as such. The meaning of this development is the realization of Measure, where it posits itself as being related to itself and hence as a moment. This mediation determines it as being transcended; its immediacy and the immediacy of its moments disappear, and they exist as being reflected; and Measure, which thus has manifested itself as being what it is according to its concept, has passed over into Essence.

Measure now is immediate unity of the Qualitative and the Quantitative. Thus:—

first, it is a Quantum having qualitative meaning, and, therefore, existing as Measure. The process of further determination is, that the difference of its moments (qualitative and quantitative determinateness) appears in this self-determined entity. These moments further determine themselves as each a whole Measure, which thus is independent; and, these being essentially related to one another, Measure becomes,

secondly, the relation of Specific Quanta as independent Measures. But at the same time their independence essentially is based on quantitative ratio and magnitudinal difference; thus their independence becomes a reciprocal transition. Consequently Measure perishes in the measureless.—But this Beyond of Measure is its negativity only in itself; hence,

thirdly, the indifference of determinations of Measure is posited; and Measure (as real by virtue of the negativity which it contains) is posited as the Inverse Ratio of Measures, which, as independent Qualities, are essentially based only upon their Quantity and their negative relation to one another; they thus turn out to be no more than moments of their truly independent unity, which is their intro-reflection and its positing, that is, Essence.

The development of Measure which has been attempted in what follows is exceedingly difficult; it starts from immediate, external Measure, and should, therefore, on the one hand, proceed to the abstract further determination of the Quantitative (a natural mathematics), and, on the other, demonstrate the connexion between this Measure-determination and the Qualities of natural objects, at least in a general manner; for the exact demonstration of the connexion between Qualitative and Quantitative which arises from the concept of the concrete object belongs to the special science of the concrete,—examples of which may be looked up in the *Encyclopaedia of the Philosophical Sciences*, 3rd Ed. §§ 267 and 270, Observations on the Law of Gravity and that of free celestial motion. We may here observe generally that the various forms in which Measure realizes itself belong also to different spheres of natural reality. The complete and abstract indifference of developed Measure, that is, of its laws, can occur only in the sphere of Mechanism where the concrete corporeal is only matter, abstract itself; its qualitative differences essentially have the Quantitative for their determinateness; Space and Time are themselves pure externalities, and the multitude of materials or masses, the intensity of weight, are also external determinations having their peculiar determinateness in the Quantitative. However, such magnitudinal determinateness of the abstractly material is marred by plurality, and hence by a conflict of qualities, in the physical and even more in the organic sphere. And here not merely the conflict of qualities as such operates, but Measure is subordinated to higher relations, and the immanent development of Measure is reduced to the simple form of Immediate Measure. The members of the animal organism have a Measure which, as a simple Quantum, is in a ratio to other Quanta of the other members: the proportions of the human body are fixed ratios of such Quanta; and natural science has much to discover of the connexion of such magnitudes with the organic functions, on which they wholly depend. But Motion is the nearest example of the reduction of an immanent Measure to a magnitude merely externally determined. In the celestial bodies it is free motion determined only by the concept, and, therefore, its magnitudes also depend on it alone (see above); but it is reduced from organic to arbitrary or mechanically regular motion, that is, to an altogether abstract, formal motion.

And in the realm of Spirit a peculiar and free development of Measure takes place even less. It is evident, for instance, that a republican constitution, like that of Athens, or an aristocratic constitution tempered by democracy, can exist only where the State has a certain magnitude, and that in developed civic society aggregates of individuals belonging to different trades are in a certain relation to one another; but this yields neither laws of Measure nor peculiar forms of it. In the realm of Spirit as such differences occur of intensity of character, strength of imagination, of feelings, of conceptions, and so on; but determination does not pass beyond this indeterminate concept of strength or weakness. And it is realized how poor and wholly void, in the end, are these so-called laws which are set up about the relation of strength and weakness of sensations, images, and so on, when the psychologies are examined which labour on these matters.

CHAPTER I

THE SPECIFIC QUANTITY

Qualitative Quantity is, firstly, an immediate specific Quantum; this,

secondly, is in a relation to another, and therefore becomes a quantitative specifying, a transcendence of indifferent Quantum. This Measure thus is a Rule, and contains, unsynthetized, the two moments of Measure, namely, self-existent quantitative determinateness and external Quantum. But being thus held apart the two sides become Qualities, and the Rule becomes a relation between them; hence Measure manifests itself,

thirdly, as a Relation of Qualities, which have first One Measure; later on, this specifies itself as an internal difference of Measure.

A

THE SPECIFIC QUANTUM

1. Measure is the simple self-relation of Quantum, its own determinateness determined by itself: thus Quantum exists qualitatively. And first, as immediate Measure, it is an immediate Quantum, determined accordingly as being just any Quantum; and the Quality belonging to it is similarly immediate and is determined similarly.—Thus Quantum, as being this no longer indifferent limit, but a self-relating externality, is itself Quality; and being differentiated from Quality it does not pass beyond it, nor does Quality pass beyond Quantum. It is thus determinateness which has returned into simple self-identity; it is one with Determinate Being as Determinate Being is one with its Quantum.

If the determination which has been reached is to be formulated, it may be expressed in the proposition that *everything which exists has a Measure.* All Determinate Being has a magnitude, and this magnitude belongs to the nature of Something itself; it constitutes its determinate nature and Being-in-Self.

Something is not indifferent to this magnitude, nor does it remain unchanged when the latter changes: a change in the magnitude would change its quality. As Measure, Quantum has ceased to be a limit which is no limit; it now is a determination of the Thing in such a manner that an increase or decrease in this Quantum would destroy it.—

A Measure as standard in the ordinary sense is a Quantum which is arbitrarily taken as Unit, determinate by itself, against an external Amount. Such a unit can, of course, also be a unit determinate by itself in fact, like a foot and other original measures; but, in so far as it is also used as standard for other things, it is for them an external and not their original Measure.—In this way the diameter of the earth or the length of the pendulum may be taken as specific Quantum for itself. But what fraction of the diameter of the earth or length of the pendulum is taken, and the degree of latitude under which the latter is taken when it is to be used as standard,—this is arbitrary. And for other things such a standard is still more something external. These have specified the general specific Quantum in yet another particular manner, and have thereby become particular things. It is, therefore, foolish to speak of a natural standard of things. Moreover, a general standard is designed to serve only for external comparison; and in this most superficial meaning, where it is taken as General Measure, it is quite indifferent what is used as Measure. It is not meant to be a fundamental Measure, which would mean that in it the natural Measures of particular things would be represented and would hence, according to a Rule, be recognized as specifications of a universal Measure, the Measure of their universal body. But without this meaning an absolute standard is interesting and significant only as being common to all; and such a common element is universal not in itself, but only by convention.

This immediate Measure is a simple magnitudinal determination, like the magnitude (for example) of organic beings, of their members, etc. But everything that exists has a magnitude which makes it what it is and allows it to have Determinate Being.—As Quantum it is an indifferent magnitude, open to external determination and capable of oscillation along the scale of more and less. But, as Measure, it is also distinct from itself as Quantum (which is such an indifferent determination), and is a limitation of this indifferent oscillation along a limit.

In Determinate Being quantitative determinateness thus is double; first, Quality is bound to it, but, secondly, oscillation may take place along it and Quality yet survive. It thus comes about that whatever has a Measure may perish when its Quantum is varied. In a manner this destruction appears unexpected in so far as changes can be made in Quantum while Measure and Quality are unchanged; and yet it can be made quite intelligible—for it is gradual. Recourse is so readily made to this category in order to render intelligible to the eye or to the mind the disappearance of a Quality or of something; for thus the illusion is created that one can almost be eye-witness of disappearance; for, Quantum being posited as limit external and variable by its very nature, change (as a change of Quantum only) needs no explanation. But in fact nothing is thereby explained; the change is also essentially the transition of one Quality into another, or (a more abstract transition) of one existence into a non-existence; and this contains a determination different from that of "gradual," which is only a decrease or increase and a one-sided retention of magnitude.

2. But already the ancients were aware of the connexion by which a change appearing as merely quantitative turns into one which is qualitative, and they illustrated the confusions which arise from ignorance of this connexion by popular examples; the well-known *ἐλεγχοί* called those of the Bald and of the Heap belong here; these (according to Aristotle) are devices to compel the assertion of the contradictory of a first assertion. The question was, whether (for example) the removal of one hair from a head, or from a horse's tail, produced baldness; or whether a heap ceased to be a heap after the removal of one grain. This could be denied without hesitation, for such a removal constitutes a wholly negligible quantitative difference; and thus one hair or grain is removed, and this is repeated, one only being removed after each acquiescence: there appears at last the qualitative change,—the head (or tail) is bare, the heap has vanished. At each acquiescence not only the repetition was forgotten, but also the fact that the

quantities negligible in themselves (like disbursements negligible in themselves from a capital) add up, and that the sum forms the qualitative whole, which, therefore, at the end has vanished: the head is bald, and the purse empty.

The embarrassment and contradiction which result are no sophism in the ordinary meaning of the term: this contradiction is not vicious nor illusory. The error is committed by the Other which was assumed, namely by ordinary consciousness, when it takes a quantity as a merely indifferent limit, that is, precisely, in the definite meaning of a Quantity. This assumption is upset by the truth to which it is led, namely, that it is a moment of Measure and is connected with Quality: what is refuted is the one-sided clinging to abstract determinateness of Quantum.—And therefore these divagations are no idle and pedantic joke; they are in themselves correct and the product of a consciousness which takes an interest in the phenomena which occur in thought.

Quantum when it is taken as indifferent limit is that side from which a Determinate Being can unsuspectedly be attacked and destroyed. It is the *cunning of the Notion* to seize it from this side, where its Quality does not appear to come into play;—and this so much so that the aggrandizement of a State or of a property, and so on, which leads in the end to disaster for the State or the owner of the property, may at first actually appear as their good fortune.

3. Measure in its immediacy is an ordinary Quality of a determinate magnitude belonging to it. Now that side according to which Quantum is indifferent limit along which oscillation can take place while the Quality remains unchanged, is also different from its other side, according to which it is qualitative or specific. Both are magnitudinal determinations of one and the same thing; but, further, this difference must be taken as immediate in accordance with the immediacy in which Measure first is found: hence the two sides have different existences. The existence of Measure, then, which is magnitude determinate in itself, is, in its attitude to the existence of the variable and external side, a transcendence of its indifference; it is a *specifying* of Measure.

B

SPECIFYING MEASURE

This is
firstly, a Rule, a Measure external to mere Quantum;
secondly, Specific Quantity, determining external Quantum;
thirdly, both sides, as Qualities of specific quantitative determinateness, are related to one another as One Measure.

(*a*) The Rule

Rule, or Standard (which has already been spoken of), is first a magnitude determinate in itself; it is Unit with reference to a Quantum, which is a particular existence, existing in a Something other than the Something of the Rule, and measured by the Rule, that is, determined as Amount of that Unit which is the Rule. This comparison is an external activity, and the Unit is an arbitrary magnitude, which in turn can be posited as Amount (the foot as an Amount of inches). But Measure is not only external Rule, but being specific it must also in itself be related to an Other which is a Quantum.

(*b*) The Specifying Measure

Measure is a specific determining of external magnitude, that is, of the indifferent magnitude which is posited by some other existence in general in the Something of Measure; Measure is itself a Quantum, but, as distinguished from this, it is the qualitative element determining merely indifferent and external Quantum. Something has in it that side of Being-for-Other to which indifferent increase and decrease is proper. This immanent mensor is a Quality of Something, to which the same Quality in another Something is opposed; but in the latter the Quantum is relatively measureless as opposed to the former Quality which is determined as mensor.

In so far as Something is a Measure in itself, a change in the magnitude of its Quality is external, and does not make of it an arithmetical plurality. But its Measure reacts against it, is in the relation of an intensive to this plurality, and assimilates it in a peculiar manner; it changes the change which is externally posited, makes this Quantum another, and, by means of this Specification, manifests itself in this externality as Being-for-Self.—The plurality which is specifically assimilated is itself a Quantum, dependent upon the other plurality, which also remains merely external to it. The specified plurality is consequently also variable; but it is not, therefore, a Quantum as such, but is external Quantum specified in a constant manner. Thus Measure has its determinate being as Ratio, and its specific element is in general the exponent of this Ratio.

It was seen, when these determinations were being considered, that in Intensive and Extensive Quantum it is one and the same Quantum which we find, in one case in the form of intensity, and in the other in the form of extensity. In this difference the basic Quantum undergoes no change: the difference is only an external form. On the other hand, in Specifying Measure, Quantum in the first case exists in its immediate magnitude, but in the second is taken (by virtue of the exponent of the Ratio) in another Amount.

The exponent, which constitutes the specific element, might at first appear to be a fixed Quantum, as being the quotient of the ratio between the external and the qualitatively determinate term. But then it would be nothing but an external Quantum; by exponent nothing must here be understood but the qualitative moment itself which specifies the Quantum as such. The properly immanent qualitative element of Quantum is (as was seen above) only the Determination of Power. This it must be which constitutes the ratio, and which here, as the self-existent determination, has opposed Quantum as being external modification. The principle of Quantum is the numerical One, which constitutes its determinateness-in-itself; and the relation of the numerical One is external; and the change (which is determined only by the nature of immediate Quantum as such) consists in itself in the addition of such a numerical One, of yet another, and so on. Thus external Quantum changes in arithmetical progression; and thus the specifying reaction of the qualitative nature of Measure produces another series, which is related to the first and increases and decreases with it, but in a ratio which is not determined by a numerical exponent, but is incommensurable with a number, according to a determination of powers.

Observation

Temperature—to cite an example—is a Quality where these two sides, of being external and specified Quantum, are distinguished. As Quantum it is external temperature (even that of a body as the general medium), of which it is assumed that its change moves along the scale of arithmetical progression, and that it increases or decreases uniformly; whereas it is absorbed in a different manner by the different individual bodies which it includes, since these, by virtue of their immanent Measure, determine the temperature which they receive from without, and the change in the temperature of any one does not correspond directly with that of the medium or of any other. Different bodies, compared under the same temperature, give relative numbers of their specific heats, of their capacities for heat. But these capacities of bodies vary under different temperatures, and thus a change in the specific configuration is introduced. Hence a particular specification manifests itself in the increase or decrease of temperature. The relation of the temperature which is imagined as external, to the temperature of a given body, which also depends upon the former, has no fixed exponent of ratio; the increase or decrease of this heat does not progress uniformly with the increase or decrease of external heat.—A temperature is here assumed which is altogether external, and changes quite externally or purely quantitatively. But the temperature is itself the temperature of air or some other specific temperature. More closely considered, therefore, the ratio would really have to be taken not as the relation between a merely quantitative and a qualifying Quantum, but between two specific Quanta. And indeed Specifying Ratio will immediately proceed to determine itself in such a manner that the moments of Measure do not consist only of two sides of one and the same Quality, a quantitative side and a side qualifying the Quantum, but of the relation between two Qualities which in themselves are Measures.

(*c*) Relation of Both Sides as Qualities

1. The qualitative and self-determinate side of Quantum exists only as relation to the externally quantitative; as its

specification it is the transcendence of its externality, by virtue of which Quantum exists as such; thus it logically depends on, and begins from, Quantum. But Quantum differs from Quality itself also in a qualitative manner; and this difference between the two must be posited in the *immediacy* of Being in general, which as yet is still the sphere of Measure: the two sides are thus qualitatively related to each other, and each for itself is such a Determinate Being; and the one Quantum (as yet formal and not determinate in itself) is the Quantum of a Something and of its Quality, and also is the specific magnitude of these qualities, since it is the case that their mutual relation has proceeded to the determination of Measure in general. These Qualities are related to each other according to the determination of Measure, and this determination is their exponent; but they are so related in themselves already in the Being-for-Self of Measure: in its double existence Quantum is both external and specific, so that each of the different Quantities has this double determination in itself and also is absolutely interlocked with the other: it is just in this alone that the Qualities are determinate. Thus they are not only determinate existences existing for each other, but they are posited as inseparable, and the magnitudinal determinateness connected with them is a qualitative unity—*one* determination of Measure in which, according to their concept, they cohere. Measure is thus the *immanent* quantitative mutual attitude of *two* Qualities.

2. In Measure the essential determination of *variable magnitude* appears, for Measure is Quantum as transcended, that is, no longer as that which it is supposed to be in order to be Quantum, but as Quantum and also something else; this Other is the Qualitative, and, as was determined, nothing else than its Ratio of Powers. This change is not yet posited in immediate Measure: any Quantum (and in fact one individual Quantum) is there connected with the one Quality. But in the Specifying of Measure (the preceding determination) as a changing of merely external Quantum by means of the Qualitative, a difference is posited between the two magnitudinal determinatenesses, and thus a plurality of Measures in a common external Quantum; and Quantum shows itself as existing Measure only here where it is differentiated from

itself, for here it is manifested both as one and the same (*e.g.* the same temperature of the medium) and also as different (quantitative) Determinate Being (in the different temperatures of the bodies contained in the medium). This differentiation of Quantum in the different Qualities (the different bodies) gives another form of Measure, namely the one where both sides are mutually related as Quanta qualitatively determined: this might be called *Realized Measure*.

Magnitude as magnitude in general is variable, for its determinateness is a limit which also is no limit; variation then affects only a particular Quantum, for which another is substituted. But the true change is the change of Quantum as such; this leads to the determination, interesting if taken in this way, of the variable magnitude of higher mathematics: here no halt must be made at the mere form of variability in general, nor must any other determination be introduced than the simple determination of the concept, according to which the Other of Quantum is only the Qualitative. Thus the true determination of real variable magnitude is that it is magnitude qualitatively determined, that is (as has sufficiently been shown) determined by a Ratio of Powers: in this variable magnitude it is posited that Quantum is counted not as such, but according to its other, or qualitative, determination.

The two sides between which this attitude subsists are, in the abstract, Qualities in general, and as such have some particular meaning, such as Space and Time. Taken in their general relation of Measure as magnitudinal determinations they are, the one Amount oscillating in external arithmetical progression, and the other an Amount which is specifically determined by the first, which for it is Unit. If each were only any particular quality in general, they would contain no difference that could determine which should, in view of its magnitudinal determination, be taken as merely externally quantitative, and which as varying in quantitative specification. If, for instance, they are related as root and square, it is indifferent in which the increase or decrease is considered to be merely external and moving in arithmetical progression, and which on the other hand as specifically determining itself in this Quantum.

But in fact the Qualities are not indeterminately different from each other, for being moments of Measure they are to contain its qualification. The next determinateness of the Qualities themselves is, for the one, that it is the Extensive, or externality, in itself, and, for the other, that it is the Intensive, that which is in itself and is negative to the former. Thus of the quantitative moments Amount belongs to the former and Unit to the latter; in simple direct ratio the former must be taken as dividend and the latter as divisor, in specifying ratio the former as the power or other-becoming and the latter as root. In so far as counting here still takes place, and external Quantum (which thus is a magnitudinal determinateness wholly contingent and empirically hit on) is reflected upon, and consequently change too is taken as proceeding in external arithmetical progression, this falls within the side of Unit or intensive Quality; while the external or extensive side must be represented as varying in the specified series. Direct relation, however (like velocity in general, $\frac{s}{t}$), is here reduced to a formal and not existing determination, belonging only to abstracting reflection; even in the relation of root and square (as in $s = a\,t^2$) the root must be taken as empirical Quantum and proceeding in arithmetical progression, while the other side must be taken as specified: but the higher realization of the qualification of the quantitative, more truly corresponding with the concept, is this, that both sides are related in higher determinations of powers (as is the case in $s^3 = a\,t^2$).

Observation

What has here been discussed with respect to the connexion of the qualitative nature of a Determinate Being with its quantitative determination in Measure, has an application in the example (already indicated) of movement: here, in velocity as the direct ratio of space passed through and time elapsed, the magnitude of time is taken as denominator, and the magnitude of space as numerator. If velocity is altogether only a ratio of the space and time of a movement, it is indifferent which of the two moments is to be considered as Amount or as Unit. But space (like weight in specific gravity) is external and real Whole as such, and hence Amount: time, on the other hand (like volume), is that which is of ideal nature, the negative, the side of Unit.—Essentially, too, this is the proper place of a more important ratio, that of free movement;—movement first conditional, as in gravitation (where, of temporal and spatial Quantity, the former is root and the latter square), or next absolutely free, as of the celestial bodies, where, of the period of revolution and the distance, the former is by one power lower than the latter, since the two are related to each other as square and cube. Such fundamental ratios are based upon the nature of the related qualities of space and time, and upon the class of relation in which they stand, which is either mechanical movement—that is, not free and not determined by the concept of the moments,—or gravitational—that is, movement conditionally free,—or, thirdly, absolutely free, as celestial movement. These classes of movement and their laws are based upon the development of the concept of their moments, space and time; for these Qualities prove as such and in themselves, that is, in their concept, to be inseparable, and their quantitative ratio is the Being-for-Self of Measure, it is only one determination of Measure.

With respect to the absolute relations of Measure, it may be proper to recall that the Mathematics of Nature, if it would be worthy of the name of Science, must essentially be a science of measures,—a science for which much has been done empirically, but little in a truly scientific, that is philosophic, manner. Mathematical principles of Natural Philosophy—as Newton called his work—if they were to fulfil this determination in a philosophic and scientific meaning deeper than that which was reached by Newton and the whole Baconian generation, must contain quite other things in order to bring light into these regions, dark as yet, but most worthy of contemplation.—It is a great merit to become acquainted with the empirical numbers of nature (as the distances from one another of the planets), but an infinitely greater merit to cause the empirical Quanta to disappear and to raise them into a universal form of quantitative determinations, so that they become moments of a law or Measure;—immortal merits, acquired (for instance) by Galileo with regard to gravity and by Kepler with regard to the movements of the celestial bodies.

They demonstrated the laws which they discovered by showing that with them the totality of details corresponds with observation. But a higher proof of these laws must be demanded, nothing less than that their quantitative determinations be known from the qualities or determinate concepts which (like time and space) are related. There is no trace of this kind of demonstration either in those mathematical principles of natural philosophy nor in further works of this kind. We observed above (in connexion with the show of a mathematical demonstration of natural relations based on an abuse of the infinitesimally small) that the attempt to conduct such demonstrations in a properly mathematical manner, that is, with neither experience nor the concept for starting-point, was a contradictory undertaking. Such proofs presuppose their theorems (namely precisely these laws) out of experience: their work is to reduce them to abstract expressions and convenient formulae. No doubt but that the whole real merit which is ascribed to Newton over Kepler in relation to the same objects will be reduced, with full knowledge (the false fabric of proofs being banished) and after clearer reflection upon what mathematics can do and has done, to this transformation of the expression,[1] and to the analytic treatment introduced in a rudimentary form.

C

BEING-FOR-SELF IN MEASURE

1. In the form of specified Measure which we have just considered, the quantitative element of both sides is qualitatively determined (both in the Ratio of Powers): thus they are moments of one determinateness of Measure of qualitative nature. But the qualities are thus posited so far only as immediate and *merely* different; they are not themselves in the relation in which is the magnitudinal determinateness of each, that is, having neither meaning nor existence *outside* this ratio; for this is contained in the determinateness of powers

---

[1] See Encyclopaedia of Philosophical Sciences, Observation on § 270 on the transformation of Kepler's $\frac{s^3}{t^2}$ into $\frac{s^2 \times s}{t^2}$ (which is Newton's), the part $\frac{s}{t^2}$ being called the force of gravity.

of magnitude. Thus the qualitative veils itself, as though specifying not itself, but magnitudinal determinateness: it is posited only as applied to this, while for itself it is immediate quality as such, which, beyond the fact that magnitude is differentiated from it, and beyond its relation to its other, has a Determinate Being which subsists for itself. Thus space and time, apart from that specification which their magnitudinal determinateness contains in the movement of gravitation, or in absolutely free movement, count as space or as time in general, space persisting permanently for itself beyond and without time, and time flowing for itself independently of space.

This immediacy of the qualitative as against its specific relation of Measure is, however, equally bound up with a quantitative immediacy and with the indifference of a quantitative element in itself as opposed to this its relation; immediate Quality has a Quantum which also is only immediate. Consequently specific Measure has also a side of—at this point—external variation; it does not interfere with its progress, which is merely arithmetical and contains external and therefore only empirical magnitudinal determinateness. When Quality and Quantum thus appear outside of specific Measure, they are also related to it; immediacy is a moment of such things as themselves belong to Measure. Thus the immediate Qualities also belong to Measure, they are also related, and, apart from magnitudinal determinateness, they are in a relation which in being outside the specified relation (determination of powers) is itself only direct relation and immediate Measure. This conclusion and its corollaries must be expounded in greater detail.

2. Immediately determinate Quantum as such is, in itself, and as moment of Measure, based upon a conceptual complex; nevertheless it is given as external in its relation to specific Measure. But the immediacy which is hereby posited is the negation of qualitative determination of Measure: this was demonstrated above of the sides of this determination of Measure, which therefore appeared as independent Qualities. This negation and the return to immediate quantitative determinateness are contained in qualitatively determinate ratio in so far as the relation of differentiated somethings in general contains their relation as *one* determinateness, which, accordingly, here in the quantitative, and distinct from the

determination of ratio, is a Quantum. As negation of the differentiated and qualitatively determined sides, this exponent is a Being-for-Self, it is absolute determinedness; but it is such Being-for-Self only *in itself*: as Determinate Being it is a simple, immediate Quantum, it is quotient or exponent as of a ratio of the sides of Measure (this ratio being taken as direct), but, generally, the empirically appearing unit in the quantitative element of Measure.—In the fall of bodies the spaces passed through vary as the squares of the times elapsed ($s = a\,t^2$);—this is a ratio of powers, specifically determined, of time and space; the other or direct ratio would belong to space and time as Qualities indifferent to each other: it is supposed to be the ratio of space to the *first* moment of time, and in all the following points of time the same coefficient a remains the Unit as an ordinary Quantum for the Amount which, for the rest, is determined by Specifying Measure. At the same time it counts as the exponent of that direct ratio which belongs to imaginary and absolute velocity, that is velocity which is formal and not specifically determined by the concept. Such velocity does not exist here any more than that mentioned above which was supposed to belong to a body at the end of a moment of time. The former velocity is attributed to the *first* moment of time occupied by the fall, but this so-called moment of time is itself only an assumed unit, and thus, as an atomic point, has no Determinate Being; the beginning of the movement—and the smallness which is attributed to the beginning could make no difference—is forthwith a magnitude, and a magnitude specified by the law of gravitation. This empirical Quantum is ascribed to the force of gravity, in such a manner that this force itself is to have no relation to the given specification (the determinateness of powers) and to what is peculiar in the determination of Measure. The *immediate* moment —namely, that in the movement of falling one unit of time (one second—that which is called the first) goes with an amount of some fifteen spatial units (which are taken as feet)—this moment is an *immediate* Measure, like the measure-magnitude of human limbs, the distances and diameters of the planets, and so on. The determination of such a measure does not belong to the sphere of qualitative determination of Measure (which here is the law of gravitation itself); and the concrete sciences have not yet given us any clue to that upon which depend such numbers, being as they are that element of a Measure which appears only immediately, that is, empirically. Here we are concerned only with this conceptual determinateness, which is, that that empirical coefficient constitutes *Being-for-Self* in the determination of Measure, but constitutes the moment of Being-for-Self only in so far as it exists in itself and therefore immediately. The rest is the developed aspect of this Being-for-Self, the specific measure-determinateness of the sides.—In the relation of gravitation (a movement which indeed is half conditioned and half free), gravity according to this second moment must be looked upon as a force of nature, its ratio being determined by the nature of time and of space, so that this specification (the ratio of powers) falls within gravity: the former simple and direct ratio expresses only a mechanical behaviour of time and space, formal velocity externally produced and determined.

3. Measure has reached the determination that it is a specified ratio of magnitudes, which as qualitative has applied to it ordinary external Quantum; but this is not a Quantum in general, but essentially is the determining moment of Ratio as such; thus it is exponent, and, as immediate determinateness, it is now an invariable exponent, and exponent therefore of the direct ratio (already mentioned) of those same Qualities by which their magnitudinal ratio to each other is specifically determined. In the example which we used (the Measure of the movement of gravitation) this direct ratio is anticipated, so to speak, and assumed to be given; but, as was observed, it does not yet in fact exist in this movement.—But it constitutes the further determination that Measure is now realized in this manner, that its two sides are Measures (distinguished as an immediate and external Measure and a Measure specified within itself), and that it is their unity. As such a unity Measure contains the relation in which the magnitudes are determined by the nature of the Qualities and are posited as different: its determinateness is therefore quite immanent and independent and, simultaneously, has collapsed into the Being-for-Self of immediate Quantum, the exponent of a direct ratio. And here its self-determination is negated, for it has its last and self-existent determinateness in this its Other; and,

conversely, immediate Measure, which in itself is supposed to be qualitative, in truth has qualitative determinateness only in the former. This negative unity is *real Being-for-Self*, the category of a Something as unity of Qualities related as Measures,—a complete *independence*. Immediately, the two (which have shown themselves to be two different ratios) yield a twofold Determinate Being; or rather, such an independent whole, since it is a Being-for-Self in general, is also a schism into different *stable somethings* whose qualitative nature and subsistence (materiality) lies in their mensural determinateness.

CHAPTER II

REAL MEASURE

MEASURE is determined as a Relation of Measures which constitute the Quality of different stable Somethings, or, more familiarly, Things. The Measure-Relations which we have just considered belong to abstract Qualities like space and time: specific gravity, and further the chemical properties, are examples (which are determinations of *material* existences) of those which still remain to be considered. Space and time are also moments of such Measures; but now, subordinated to further determinations, they are no longer related only in accordance with their peculiar conceptual determination. In sound, for instance, the time in which a certain number of vibrations occurs, and the spatial elements of length and thickness of the vibrating body, are among the determining moments; but the magnitudes of these ideal moments are determined externally: they manifest themselves no longer in a ratio of powers, but in ordinary direct ratio; and harmony reduces itself to the wholly external simplicity of numbers the ratios of which are most easily apprehended, and therefore afford a satisfaction which belongs entirely to sensation, since the spirit finds no image, phantasy, thought, nor anything of this kind, to fill it. The sides which now constitute the Measure-Relation are Measures themselves, but are also real Somethings; and thus their Measures are, first, immediate Measures and, as Relations, direct Relations. The relation to each other of such relations must now be considered in its progressive determination.

Measure is now real Measure, and thus

first, it is the stable Measure of a corporeality which is in a relation to others, and in this attitude specifies them and also, thereby, specifies stable materiality. This specification, as an external relating to many others in general, is the production of other Relations and therefore of other Measures; and specific stability does not continue to consist in *one* direct

Relation, but passes over into specific determinateness, which is a Series of Measures.

Secondly, the direct Relations which thus result are Measures determinate and exclusive in themselves (Elective Affinities); but their difference is also only quantitative, and thus there is a progress of Relations, which partly is merely externally quantitative, but also is interrupted by qualitative Relations, and forms a *nodal line* of specific stable entities.

Thirdly, however, in this progress measurelessness in general and, more determinately, the infinity of Measure, enter into play for Measure: here the mutually exclusive stabilities are One with one another, and the stable entity enters into a negative relation to itself.

A

THE RELATION OF STABLE MEASURES

Measures must now be called not merely immediate, but stable, since they in themselves now become Relations of Measures which are specified; they are thus first, in this Being-for-Self, physical somethings, material things. Now the whole which is a Relation of such Measures is

(*a*) first, itself immediate; thus the two sides, which are determined as being such Stable Measures, severally subsist in separate things; their connexion is effected from without.

(*b*) The stable materialities are, however, what they are qualitatively, only by virtue of the quantitative determination which they have as Measures; that is, they are determined by virtue of a relation to others which is itself quantitative, and are determined as themselves different from these (this is so-called Affinity) and as members of a series of such quantitative attitude.

(*c*) This indifferent manifold attitude also rounds itself off into *exclusive* Being-for-Self,—into so-called Elective Affinity.

(*a*) UNION OF TWO MEASURES

In itself, Something is determined as a Measure-relation of Quanta, which, further, have Qualities; and the Something is the relation of these Qualities. One of them is its Being-in-Self, which makes it a Being-for-Self—a material (like weight, taken intensively; or extensively, multitude, that is of material parts). The other is the *externality* of this Being-in-Self (the abstract or ideal, or space). These Qualities are determined quantitatively, and their relation constitutes the qualitative nature of the material Something;—the relation between weight and volume, determinate specific gravity. The volume (that which is of ideal nature) must be taken as Unit, and that which is intensive (which appears in quantitative determinateness and, in comparison with the former, as extensive magnitude, a multitude of Ones of which each is for self) as Amount.—The purely qualitative attitude between one magnitudinal determinateness and the other according to a ratio of powers has vanished in the fact that, in the stability of Being-for-Self (or material Being), immediacy has returned; and here the magnitudinal determinateness is determined as a Quantum as such, and the ratio of such a Quantum to the other side is also determined in the ordinary exponent of a direct ratio.

This exponent is the specific Quantum of a Something, but it is immediate Quantum; and this (and therefore the specific nature of such a Something) is determined only in *comparison* with other exponents of such ratios. It constitutes the *specific* determinateness-in-self, the inner peculiar Measure of something. But this its Measure is based upon Quantum, and therefore it is only external and indifferent determinateness; and hence such a Something is variable, in spite of the internal measure-determination. The Other towards which it can be variable is not a material multitude, a Quantum in general (for its specific determinateness-in-self can withstand this), but a Quantum which is also exponent of such a specific ratio. Two things having different internal Measures are related and enter into union,—like two metals of different specific gravities;—and what kind of homogeneity of nature (since it is not the case that the union of a metal, for instance, with water, is here in question) is further required to make possible such a union need not here be considered.—On the one hand now each of the two Measures preserves itself in the variation, which might reach it through the externality of Quantum, because it is Measure; on the other hand this self-preservation itself is a negative attitude to this Quantum, a specification of it; and, since the Quantum is exponent of the Measure-relation, it is a change in Measure itself—a reciprocal specification.

According to the merely quantitative determination, Union is a mere summation of the two magnitudes of the one Quality and of the two magnitudes of the other; for instance, the sum of the two weights and the two volumes in the union of two bits of matter of different specific gravity, with the condition that not only the weight of the compound remains equal to that sum, but also the space which it occupies equal to the sum of those spaces. But in fact only the weight is equal to the sum of the weights which were found before the union: that side is summed up which, because it is for itself, has become fixed Determinate Being and therefore has permanent immediate Quantum, and this is the weight of matter, or (which is the same from the point of view of quantitative determinateness) the multitude of material parts. It is into the exponents that the change falls, for as Measure-relations they are the expression of qualitative determinateness, of Being-for-Self, which also manifests itself as negative towards this externality, since Quantum as such undergoes the contingent and external change by accretion which is summed up. This immanent determining of the quantitative cannot appear in weight (as has been shown), and therefore manifests itself in the other Quality, which is that side of the relation which is of ideal nature. It may be remarkable to sensuous perception that, after the mixture of two specifically different bits of matter, a change,—and usually a diminution,—of the volume which is summed up should appear: space itself constitutes the persistence of material things existing externally to one another. But this persistence as against negativity (which contains Being-for-Self) is that which is not in itself, or is the variable; in this manner space is posited as that which in truth it is—that which is of ideal nature.

But thus not only is one of the qualitative sides posited as variable, but Measure itself and the qualitative determinateness of the Something, which is based on it, has shown itself not to be in itself fixed, but to have its determinateness (like Quantum in general) in other Measure-relations.

(*b*) Measure as a Series of Measure-Relations

1. If something which enters into union with another, and this other too, were determined only by simple Quality and were what it is only for this reason, then they would only cancel each other in this union. But something which is a Measure-relation in itself is stable, and therefore it can enter into union with another such; and being cancelled in this union it preserves itself by virtue of its indifferent and quantitative persistence, and at the same time functions as specifying moment in a new Measure-relation. Its Quality is enveloped in the quantitative; thus it is also indifferent to the other Measure, and continues itself into it and into the newly-formed Measure; the exponent of the new Measure is itself only an indifferent Quantum, external determinateness, and manifests its indifference when the specifically determinate Something enters upon similar neutralizations of pairs of Measure-relations with similar Measures; its specific peculiarity fails to express itself when it and another form *One* only.

2. This union with a plurality of others which also are Measures in themselves yields different ratios, which accordingly have different exponents. What is stable has the exponent of its determinateness-in-self only in the comparison with others: now the neutrality with others constitutes its real comparison with them; it is its comparison with them through itself. But the exponents of these ratios are different, and thus it exhibits its qualitative exponent as the *series* of these different Amounts to which it is related as Unit,—or as a series of specific attitudes to others. The qualitative exponent as *one* immediate Quantum expresses one single relation. In truth the differentia of what is stable is the peculiar series of exponents which, if taken as Unit, are formed by it together with other such stable entities, one Other of them being similarly brought into relation with these and (taken as Unit) forming another series.—Now the relation of such a series within itself constitutes the qualitative element of that which is stable.

Now such a stable entity forms a series of exponents with a series of stable entities, so that at first it appears to be differentiated from an Other (outside this series) with which it is compared, by the fact that this latter forms another series of exponents with the same opposed entities. In this way, however, these two stable entities would be incommensurable, for here each is regarded as Unit to its exponents, and the two series which arise from this relation are other, but indeterminate else. The two entities which are to be compared as stable are so far different from each other only as Quanta; and in order that their relation may be determined, a common Unit which is for itself is required. And this determinate Unit must be looked for there only where the entities which are to be compared have the specific Determinate Being (as has been shown) of their Measure: that is, in the ratio which subsists between the ratio-exponents of the series. But this ratio of the exponents themselves is a Unit which is in fact determinate and for itself only in so far as the terms of the series have it to both as a constant relation among themselves; and then it can be their common Unit. Here alone then we have the commensurability of the two stable entities which are assumed to be indifferent to, and not neutralizing, each other. Each severally and apart from comparison is the Unit of the relations with the terms which stand opposed: the terms are Amounts as opposed to this Unit, and thus represent the series of exponents. But conversely this series is Unit for those two which, when compared with each other, are Quanta each to the other; as such they are themselves different Amounts of their lately demonstrated Unit.

But further those entities which, together with the pair (or rather the many in general) opposed to and compared with one another, produce the series of the exponents of their behaviour, are, in themselves, stable too, each being a specific Something of a Measure-relation which in itself belongs to it. And so each can in this regard be taken as Unit; so that they have a series of exponents in the first-mentioned and merely internally compared pair or rather indeterminate plurality; which exponents are the numbers resulting from comparison among themselves of the entities just mentioned; and conversely, if the entities are now severally taken as stable, then their numbers of comparison, among themselves, are the series of exponents for the terms of the first series. Thus both sides are series; in them, each number, firstly, is Unit in general to the series which opposes it, and in this latter it has its determinateness-in-self as a series of exponents; secondly, it is itself one of the exponents for each term of the opposite series; and thirdly, it is number of comparison for the other numbers of its series, and as being such an Amount (which belongs to it also as exponent) it has its Unit which is determinate in itself, in the opposite series.

3. In this attitude the manner has recurred in which Quantum as existing for self, that is as Degree, is posited as simple, but also as having magnitudinal determinateness in a Quantum which exists outside it and is a circle of Quanta. But in Measure, this external field is not merely a Quantum and a circle of Quanta but a series of ratio-numbers; and the determinateness-for-self of Measure lies in their totality. It is here as in the Being-for-Self of Quantum as Degree: the nature of Stable Measure has converted itself into this externality of itself. Its self-relation is, so far, immediate relation; and hereby, immediately, its indifference to an Other consists only in the Quantum. Its qualitative side therefore falls within this externality, and its attitude to Other becomes what constitutes the specific determination of this stable entity. Thus it consists, absolutely, in the quantitative character of this attitude, and this character is determined by the Other as much as by itself, and this Other is a series of Quanta, and itself is reciprocally a Quantum. But this relation, where two specific entities specify themselves to something, to some third element (the exponent), contains this too, that one has not here passed over into the other, so that not *one* negation in general has been posited, but *both* are posited here as negative; and, each preserving itself indifferent, its negation itself is negated. This their qualitative unity is thus self-existent *exclusive* unity. The exponents, which at first are numbers of comparison among themselves, have their true specific determinateness relatively to one another only in the moment of exclusion, and thus at the same time their difference becomes qualitative. But it is based upon the quantitative element: the stable element is in a relation to a plurality of its qualitatively different side, firstly, only because it is also indifferent in this attitude; secondly, this neutral relation, by virtue of the quantitativity which it contains, is not only change, but is posited as negation

of negation and as exclusive unity. And so the *Affinity* of a stable entity to the plurality of the other side is no longer an indifferent relation, but an *Elective Affinity*.

(*c*) ELECTIVE AFFINITY

The expression *Elective Affinity* has here been used as above "Neutrality" and "Affinity" were used,—expressions which refer to the chemical relation. For in the chemical sphere the material essentially has its specific determinateness in the relation to its Other: as this difference only does it exist. Further, this specific relation is tied to Quantity, and at the same time is not only the relation to one single Other, but to a series of such entities, different from and opposed to it; its combinations with this series are based upon a so-called Affinity with each of its members; but, while indifferent, each is also exclusive of others,—which relation of opposite determinations remains still to be considered.—But it is not only in the chemical sphere that the specific manifests itself in a circle of combinations: a single note too has meaning only in the attitude to and combination with another and with a series of others; the harmony or disharmony in such a circle of combinations constitutes its qualitative nature, which at the same time is based upon quantitative ratios that form a series of exponents and are the ratios between the two specific ratios which each of the notes that are combined is in itself. A single note is the key-note of a system; but also it is a single member in the system of any other key-note. The harmonies are exclusive Elective Affinities; but their qualitative peculiarity equally resolves itself again into the externality of merely quantitative progress.—As for the question as to where the principle of a Measure resides for these Affinities, which (be they chemical or musical or other) are Elective Affinities among and to one another, an observation remains to be made with regard to chemical affinity in the sequel; but this higher question is connected in the closest manner with the Specific of the properly Qualitative, and belongs to the separate parts of concrete Natural Science.

Inasmuch as the member of a series has its qualitative unity in its attitude to the totality of an opposed series, the members of which are differentiated from one another only by the Quantum according to which they neutralize themselves with the former, in so far the more special determinateness in this multiple affinity too is only quantitative. In Elective Affinity —as exclusive and qualitative relation—the attitude avoids this quantitative difference. The next determination which presents itself is this, that upon the distinction of multitude (that is, of *extensive* magnitude) which subsists between the members of the one side for the neutralization of a member of the other side should depend the Elective Affinity of this member to the members of the other series (to all of which it has affinity). Exclusion—as a *firmer* coherence against other possible combinations—which would be justified by this, would appear, when thus transformed, in a proportionately greater *intensity* according to the identity of the forms of intensive and extensive magnitude (which was proved above), since in these two forms magnitudinal determinateness is one and the same. But this transformation of the one-sided form of extensive magnitude into its other, the intensive form, makes no difference in the nature of the fundamental determination, which is one and the same Quantum: in fact, then, no exclusion would be posited here, but indifferently either only one combination, or, equally, one combination of an indeterminate number of members, could take place, if only the portions of these which came into play corresponded with the Quantum asked for in accordance with their relations to one another.

However, the combination which we have also called neutralization, is not only the form of intensity: the exponent essentially is measure-determination, and hence is exclusive; and the numbers, in this side of exclusive attitude, have lost their continuity and fluid power of mingling; the "more" or "less" acquires a negative character, and the advantage which one exponent has over others is not confined to magnitudinal determinateness. But equally there is present this other side, in which it is indifferent to a moment if it receive the neutralizing Quantum from several opposed moments, and from each according to its specific determinateness against the other; the exclusive negative attitude at the same time undergoes this infringement from the quantitative side.—Thus here there is posited a transformation of indifferent and merely quantitative attitude into qualitative, and conversely a transition of specific determinateness into merely external relation,—a series of relations which sometimes are of merely quantitative nature, and sometimes are Measures, and specific.

Observation

Chemical substances are the properest examples of such Measures as, being Measure-moments, have that which constitutes their determination only in their attitude to others. Acids and alkalis or bases appear as things immediately determinate in themselves, but still more as incomplete elements of bodies, as constituent parts which do not really exist for themselves, but have this existence only, that they transcend their isolated persistence and combine with an other. Further, the distinctness which makes them stable does not consist in this immediate quality, but in the quantitative character of the attitude. For this is not confined to the chemical opposition of acid and alkali or base in general, but is specified to be a measure of saturation, and consists in the specific determinateness of the quantity of the substances which neutralize one another. This quantity-determination in respect of saturation constitutes the qualitative nature of a substance: it makes it what it is for itself, and the number which expresses this is essentially one of several exponents for an opposed unit.—Such a substance has a so-called Affinity for another; in so far as this relation were purely qualitative, the one determinateness —like the relation of the magnetic poles or of the two kinds of electricity—would be merely related as negative to the other, and the two sides would not also manifest themselves as indifferent to each other. But the relation is also quantitative, and therefore each of these substances is capable of neutralizing itself with more than one; it is not confined to the one only to which it is opposed. It is not only the acid and the alkali or base which are in relation, but acids and alkalis or bases are related to one another. First they are characterized as against one another according as one acid requires (for example) more of an alkali than another in order to become saturated by it. But the stability[1] (which is for itself) manifests itself in the fact that the Affinities remain exclusive of one another, and one remains superior to the others, since one acid can, by itself, combine with all the alkalis, and conversely. Thus the capital difference as between two acids consists in the closer affinity of one to a base,—that is, in its so-called Elective Affinity.

---

[1] *Selbständigkeit.*

A law has been discovered with regard to the chemical affinities of acids and alkalis, which states that, if two neutral solutions are mixed, so that there results a separation and hence two new combinations, these products likewise are neutral. It follows that the amounts of two alkaline bases required to saturate an acid must be in the same ratio in order to saturate another acid; and generally, if for one alkali (which is taken as unit) the series of ratio-numbers has been determined in which the various acids saturate it, then the series is the same for any other alkali; only the different alkalis must be taken in different amounts relatively to one another,—amounts which again on their part form a similar fixed series of exponents for any of the opposite acids, for they are related to each acid in the same ratio as to any other.—Fischer[1] first extracted these series in their simplicity from the works of Richter:[2] see his observations on the translation of Berthollet's[3] treatise on the *Law of Affinity in Chemistry*, p. 232, and Berthollet, *Statique Chimique*, I. Part. p. 134 sqq.—Our knowledge of the ratio-numbers of mixtures of chemical elements has grown more perfect in every respect since that was written: to take it into consideration here would be a digression, because (among other reasons) this extension, which is empirical and partly only hypothetical, remains confined within the same conceptual determinations. But we may add a few observations on the categories used, and further on views about chemical Elective Affinity and its relation to the quantitative, and likewise on the attempt to base this upon determinate physical qualities.

It is well known that Berthollet modified the common idea of Elective Affinity by the concept of the activity of a *chemical*

---

[1] Fischer, Ernst Gottfried, Professor of Physics at Berlin, Member of the Academy, 1754–1831.

[2] Richter, Jeremiah Benjamin, 1762–1807, *Bergassessor* in Berlin.

[3] Berthollet, Claude Louis, Count, 1748–1822, Professor at the Polytechnic School in Paris.

mass. It must be clearly distinguished that this modification has no influence upon the quantitative ratios of the laws of chemical saturation themselves; but the qualitative moment of exclusive Elective Affinity as such is not merely weakened but transcended. If two acids act upon an alkali, and that acid, of which it is said that it has the closer affinity to the alkali, is present also in that Quantum which is sufficient to saturate the Quantum of the base, then, according to the earlier idea of Elective Affinity, only this saturation takes place: the other acid remains quite inactive and is excluded from the neutral combination. But according to this other concept of the activity of a chemical mass, each of the two is active in a ratio which is composed of the amount in which they are present and of their saturation-capacity or so-called Affinity. Berthollet's investigations have indicated in greater detail the circumstances in which the activity of the chemical mass is suspended, and one acid (the more powerfully allied) appears to cast out the other (the weaker) and exclude its activity, that is, to be active within the meaning of the earlier Elective Affinity. He has shown that the circumstances in which this exclusion takes place are such as cohesive force or the insolubility in water of the salts formed, and not the qualitative nature of the agents as such;—circumstances which can be suspended in their activity by other circumstances in turn—by temperature, for example. These obstacles removed, the chemical mass enters into activity uncrippled, and that which appeared as a purely qualitative exclusion, as Elective Affinity, shows itself as consisting only in external modifications.

Berzelius[1] is the next writer who best deserves to be heard on this matter. But in his *Manual of Chemistry* he brings forward nothing peculiar or more definite on the subject. Berthollet's views have been incorporated and repeated word for word, tricked out only with the peculiar metaphysics of an uncritical reflection; the categories of this alone therefore offer themselves for further consideration. The theory goes beyond experience, and partly invents sensuous images of a kind which experience does not offer of itself, partly it employs determinations of thought: in each way it makes itself the object of logical criticism. We will therefore scrutinize what we are told about the theory in the Manual itself (Vol. III., Part I., transl. by Wöhler, pp. 82 sqq.). Now here we read that we must imagine that in a fluid uniformly mixed each atom of the dissolved body is surrounded by an equal number of atoms of the solvent: "when several substances are dissolved together, they must share among themselves the interstices between the atoms of the solvent, so that, when the fluid is mixed uniformly, a symmetry arises in the arrangement of the atoms such that all the atoms of the individual bodies are in a uniform position relatively to the atoms of the other bodies; one could therefore say that the solution is characterized by *symmetry* in the position of the atoms, and the *combination* by the fixed *proportions*."—Berzelius proceeds to exemplify this by means of the combinations which result when sulphuric acid is added to a solution of copper chloride: unfortunately the example demonstrates neither that atoms exist, nor that a number of atoms of the bodies which are dissolved surround atoms of the fluid, nor that free atoms of the two acids arrange themselves around those which remain combined (with the copper oxide), nor that there *is* any symmetry in position and arrangement, nor that there *are* interstices between the atoms,—and least of all that the dissolved substances share among themselves the interstices between the atoms of the solvent. This would mean that the atoms of the dissolved substance take up their position where the solvent is *not*,—for the interstices between its atoms are spaces *empty* of it,—and hence that the dissolved substances are not *in* the solvent but *outside* it (although they surround and are ranged around it, or conversely), and therefore certainly are not dissolved by it. Consequently it is not clear why we must form such images, which are not exhibited in experience, essentially contradict themselves immediately, and are not corroborated in any other manner. This could be effected only by a consideration of these images themselves, that is, through Metaphysics, which is Logic; but this confirms them as little as experience—quite the contrary.—For the rest, Berzelius admits (what also was said above) that Berthollet's propositions are not opposed to the theory of determinate proportions; although he adds that they also do not oppose the views of the corpuscular philosophy, that is, the images, cited above, of the atoms and of a solvent fluid which fills the interstices with the atoms of the undissolved bodies, and so on;—but this latter and baseless metaphysic has essentially nothing to do with the proportions of saturation itself.

---

[1] Berzelius, Johann Jakob, Baron von, 1779–1848, from 1807 Professor of Chemistry at Stockholm: *Manual of Chemistry*, 3 volumes, 1808 to 1828.

Thus that specific nature which is expressed in the laws of saturation applies only to the multitude of units of a body which are themselves quantitative (and not atoms), with which multitude the quantitative unit (which equally is no atom) of another body chemically different from the first, neutralizes itself; the difference consists solely of these different proportions. And if Berzelius, in spite of the fact that his theory of proportions is wholly a determination of multitudes, speaks (for instance, p. 86) also of *degrees* of affinity, when he explains Berthollet's chemical mass, as the sum of the degree of affinity, from the given quantity of the active body, then he himself falls here into the form of *intensive* magnitude. Berthollet here more consistently uses "*capacité* de saturation." But this form constitutes the peculiar element of the so-called dynamic philosophy which above (p. 29, *loc. cit.*) he calls "the speculative philosophy of certain German schools," and expressly rejects in favour of the excellent "Corpuscular Philosophy." He there states of this dynamic philosophy that it assumes that the elements *interpenetrate* one another in their chemical combination, and that neutralization consists in this mutual interpenetration: this merely means that the chemically different particles, which are in the relation one to another of multitudes, collapse into the simplicity of an intensive magnitude,—a fact which also manifests itself as a diminution of volume. In the corpuscular theory, on the other hand, the atoms, even when chemically combined, are supposed to remain outside one another in the interstices (juxtaposition); and in such an attitude, as being a merely extensive magnitude and persisting multitude, degree of affinity has no meaning. If it is stated in the same place that the appearances of determinate proportions came quite unexpectedly for the dynamic view, then this would be a merely external historical circumstance, apart from the fact that Richter's stoechiometric series (in Fischer's arrangement) were already known to Berthollet and are quoted in the first edition of this Logic, which demonstrates the invalidity of the categories upon which are based the corpuscular theories—the old and that which strives to be new. But Berzelius's judgment is wrong when he says that, under the dominion of "the dynamic view," the appearances of determinate proportions would have been "for ever" unknown,—meaning that this view is incompatible with the determinateness of proportions. In any case this determinateness is only a magnitudinal determinateness, and it is indifferent whether it is in extensive or intensive form; so that even Berzelius, however much he is attached to the first form (multitude), himself uses the idea of degrees of affinity.

Affinity hereby is traced back to the quantitative difference, and therefore is transcended as Elective Affinity; while the *exclusive* element which occurs in it is traced back to circumstances, that is to determinations which appear as something external to the affinity—to cohesion, insolubility of the combinations reached, and so forth. With this idea, in a measure, the procedure adopted in considering the effect of gravity may be compared: what in itself belongs to gravity (the fact that the moving pendulum, through its operation, necessarily passes into a state of rest) is there taken only as a concomitant fact due to the external resistance of air, of the thread, and so on, and is attributed to friction alone instead of to gravity.—Here it makes no difference to the nature of the qualitative which is contained in Elective Affinity, whether it appears in the form of these circumstances (these being taken as its conditions) and is so conceived. With the qualitative as such a new order begins, of which the specification no longer is quantitative difference only.

Accordingly the difference of chemical affinity is exactly fixed in a series of quantitative ratios as against Elective Affinity: it is fixed as a difference of qualitative determinateness which begins to function, the behaviour of which by no means coincides with that order; but this difference is wholly confused by the manner in which electricity has recently been coupled with chemical behaviour; and the hope to reach a clue about the most important principle, namely, Measure-relation, from this, which professes greater profundity, is entirely disappointed. In this theory the phenomena of electricity and of chemism are completely identified in so far as it refers to physical properties and not to Measure-relations only;

here the theory need not further be considered, and need be noticed only in so far as it contributes to confuse the differentiation of Measure-determinations. It may be called shallow in itself, for shallowness is the identification of things which differ, the differentia being omitted. As for affinity, chemical processes being thus identified with electrical, as well as with phenomena of light and fire, this is reduced "to neutralization of opposite electricities." It is almost funny to see the identification of electricity and chemism put into the following words (p. 63, *loc. cit.*): "it is true that electrical phenomena explain the action of bodies at a greater or less distance, their attraction before they are united (that is, behaviour which is not yet chemical), and the fire (?) caused by this union; but they offer us no explanation for the combination of the bodies which powerfully persists even after the opposite electrical condition has been destroyed;" that is, the theory tells us that electricity is the cause of chemical behaviour, but electricity cannot tell us what is chemical in the chemical process.—By the fact that chemical difference in general is traced back to the opposition of positive and negative electricity, the difference in affinity of the agents which belong to either side is determined between them as the order of two series of electro-positive and electro-negative bodies. When electricity and chemism are identified according to their general determination, it is overlooked already that the former in itself and in its neutralization is transient and remains *external* to the Quality of bodies, while chemism in its action, and especially in neutralization, occupies and alters the *whole* qualitative nature of the bodies. And within electricity the opposition of positive and negative is equally transient; it is so unstable that it depends upon the smallest external circumstances, and cannot even be compared with the determinateness and fixity of the opposition of acids (for instance) and metals, and so on. The variability which can have place in this chemical behaviour through exceedingly violent activity (for instance of a raised temperature and the like) cannot be compared with the superficiality of the electrical opposition. Finally the further difference, within the series of each of the two sides, between more or less positively electrical or more or less negatively electrical nature is entirely uncertain and undemonstrated. But from these series of bodies (Berzelius, *loc. cit.*, p. 84 sq.) "the electrochemical system is to be evolved according to their electrical dispositions,—a system fittest of all to give an idea of chemistry": these series are next indicated; and of their actual nature, this is added (p. 67): "this, *roughly*, is the order of these bodies; but this matter has been so little investigated that nothing quite certain can yet be determined of this relative order."—The ratio-numbers of these series of affinity (first made by Richter), as well as the exceedingly interesting reduction of the combinations of two bodies into the simplicity of a few quantitative ratios (first made by Berzelius), are wholly independent of this would-be electrochemical brew. It may be that in these proportions and in their universal extension, made since Richter, the experimental method was the true guiding star; if so, the mixture of these great discoveries with the so-called corpuscular theory, a wilderness lying apart from the road of experience, contrasts with it all the more; and only this beginning (by deserting the principle of experience) could cause that idea once more to be taken up after it had already been commenced upon earlier (especially by Ritter[1]), namely, to set up fixed orders of electro-positive and electro-negative bodies which were also to have chemical significance.

The opposition between electro-positive and electro-negative bodies is not in fact as true as it is assumed to be; and when this is made the basis of chemical affinity, then the vanity of this basis soon shows itself even in experiment, and leads to further inconsistencies. It is admitted (p. 73, *loc. cit.*) that two so-called electro-negative bodies, like sulphur and oxygen, combine in a far more intimate manner than, for example, oxygen and copper, although the latter is electro-positive. Thus the basis of affinity founded upon the general opposition of positive and negative electricity must come second to a mere more or less within one and the same series of electrical determinateness. From this it is concluded that the degree of affinity between bodies does not only depend upon their specific unipolarity (it is here irrelevant with what hypothesis this determination is connected: here it means only the alternative determinations of positive and negative), and that the degree of affinity must chiefly be derived from the intensity of their polarity in general. Thus here the consideration of affinity passes over to the relation of Elective Affinity, which is our chief concern; and we must consider what now results for this subject. It is forthwith admitted (*ibid.* p. 73) that the degree of this polarity, unless it exists merely in our imagination, seems to be no constant quality, but greatly depends on temperature; so that after all this the result announced is not only that every chemical effect is ultimately an electrical phenomenon, but also that what appears to be effect of so-called Elective Affinity is really brought about only by an electrical polarity which is stronger in some bodies than in others. At the end of our meanderings in hypothetical images we remain at the category of *greater intensity*, which is the same formality as Elective Affinity in general; and the latter, after it has been founded on greater intensity of electrical polarity, is no nearer to being on a physical basis than before. But even that which is here supposed to be determined as greater specific intensity is later traced back only to the modifications demonstrated by Berthollet, which have already been cited.

---

[1] Ritter, Johann Wilhelm, 1776–1810, Member of the Academy of Munich: *Das elektrische System der Körper*, 3 volumes, 1805–1806.

The merit and fame which were won for Berzelius by his doctrine of proportions, which has been extended to all chemical relations, are in themselves no reason why the deficiencies of this theory should not be set forth; and there is a further reason in the fact that such merit in one branch of Science (as with Newton) tends to become the authority for a baseless fabric of invalid categories which is connected with it, and that it is just this kind of metaphysic which is proclaimed and echoed with the greatest pretension.

Apart from the forms of Measure-relation which refer to chemical and to Elective Affinity, others too may be considered with respect to quantities which qualify themselves into a system. Chemical bodies form a system of relations in regard to saturation: saturation itself depends on the determinate proportion in which combine the two aggregates which are opposed to each other as having separate material existence. But there are also Measure-relations of which the moments are inseparable and cannot be represented in a private and independent existence. These are what before were called *immediate Stable Measures*: they are represented in the specific gravity of bodies.—Within the bodies they are a ratio of weight to volume; the ratio-exponent which expresses the determinateness of a specific weight and thus differentiates it from others, is a determinate Quantum of comparison only, a ratio external to them in an external reflection, not based upon its own qualitative attitude to an opposite existence. The task would be this,—from a rule, to understand the ratio-exponents of the series of specific gravities to be a system,—the rule specifying a merely arithmetical plurality to be a series of harmonic knots.—And the same demand would be applied to the understanding of the chemical affinity-series which have been mentioned. But Science has far to go before it will reach this point,—as far as it must go before it can comprehend in one Measure-system the numbers relating to the distances between the planets of the solar system.

Although at first specific gravities appear to be in no qualitative relation, they too do become qualitatively related. When bodies are chemically combined, or even only amalgamated or synsomatized, a neutralization of specific gravities also appears. The phenomenon has been mentioned above, that the volume, even of a mixture of substances which really remain chemically indifferent to one another, is not equal to the sum of their volumes before they were mixed. In the mixture they reciprocally modify the Quantum of the determinateness with which they enter into the relation; and in this manner they show that the relation subsisting between them is qualitative. Here the Quantum of specific gravity manifests itself not merely as a fixed number of comparison, but as a ratio-number, which is variable; and the exponents of the mixtures furnish series of Measures whose progress is determined by a principle other than the ratio-numbers of the specific gravities which are combined. The exponents of these ratios are not exclusive Measure-determinations: their progress is continuous, but contains a specifying law that is different from the ratios (which progress formally) in which the amounts are combined. The two progresses thus are made incommensurable.

B

NODAL LINE OF MEASURE RELATIONS

The last determination of Measure-Relation was that, being specific, it is *exclusive*: exclusiveness is proper to neutrality as *negative* unity of the differentiated moments. This unity is *for self*; it is Elective Affinity: with regard to its relation to the other neutralities no further principle of specification has shown itself; the latter resides only in the quantitative determination of Affinity in general, according to which the amounts which neutralize one another, and thereby stand opposed to other relative Elective Affinities of their moments, are determinate. But further, by virtue of its fundamentally quantitative determination, *exclusive* Elective Affinity also *continues* itself into neutralities other to it; and this continuity is not only external relation of the various relations of neutrality as a comparison, but, besides, neutrality as such contains a *separability*, since the entities whose unity produced it enter into relation, as stable Somethings and each as indifferent as to whether it combines with one or with other of the opposing series, although they do so in different and specifically determinate amounts. Thus this Measure, being in itself based upon such a relation, is in itself infected with indifference: it is external in itself, and in its relation to itself is variable.

The relation to itself of the Measure-Relation is a different matter from its externality and variability as its quantitative side: as relation to itself as against these it is an existent and qualitative foundation,—an enduring and material substratum which ought to contain in its Quality this principle of the specification of this externality together with the continuity of Measure in its externality with itself.

Exclusive Measure in this closer determination is external to itself in its Being-for-Self; it repels itself from itself and posits itself as another—both as quantitatively other, and also as another such relation as also is another Measure; it is determined as unity specifying in itself and producing Measure-Relations as against it. Such relations are different from the previous kind of affinities, in which one stable entity is related to stable entities of a different Quality, and to a series of such: they take place against one and the same substratum within the same moments of neutrality; Measure determines itself as repelling itself towards other relations which are only quantitatively different and also form Affinities and Measures, alternating with such as remain merely quantitative differences. They form in this way a Nodal Line of Measures along a scale of more and less.

We have here a Measure-Relation, an independent reality, qualitatively different from others. Such a Being-for-Self is also essentially a relation of Quanta, and therefore open to externality and variation of Quantum: it has some play, within which it remains indifferent to this change and does not alter its Quality. But there comes a point in this quantitative change at which Quality changes and Quantum shows itself as specifying; so that the altered quantitative relation is turned under our hands into a Measure and thereby into a new Quality and a new Something. The relation which has taken the place of the first is determined by this, partly according to the qualitative identity of the moments which have affinity to one another, and partly according to quantitative continuity. But, since the difference falls within this quantitative part, the new Something is indifferent to its predecessor: their difference is one of Quantum, and hence external. It has therefore appeared, not out of the former, but, immediately, out of itself, that is, out of internal specifying unity which has not yet entered into existence.—The new Quality or the new Something is subject to the same progress of change, and so on to infinity.

In so far as the progress of one Quality is in steady continuity of Quantity, the relations which approach to one qualifying point are differentiated, quantitatively considered, only by more and less. In this regard change is gradual. But this gradual nature affects only the external part of change, not its qualitative part; the preceding quantitative relation, which is infinitely near to that which follows, is still a different qualitative existence. Hence the merely quantitative progress of gradualness, which is no limit in itself, is broken short absolutely on its qualitative side; the new emerging Quality in its merely quantitative relation is, relatively to the disappearing one, an indeterminate other, is indifferent, and so far the transition is *per saltum*: both are posited as wholly external to each other.—Men like to try to make a change conceivable by means of the gradualness of transition; but rather gradualness is precisely the merely indifferent change, the opposite of qualitative change. Rather, in gradualness the connexion of the two realities—whether taken as states or as independent things—is suspended; it is posited that neither is limit of the other, but that one is just external to the other; and hereby, precisely, that which is needful in order that change may be understood is eliminated, however little may be required to this end.

Observation

Already the natural numerical system exemplifies such a Nodal Line of qualitative moments, which manifest themselves in the merely external progression. There is, partly, a merely quantitative oscillation, a persistent addition or subtraction, so that each number has the same arithmetical relation to its predecessor and successor as either of these had to its predecessor and successor, and so on. But the numbers which thus result have also a *specific* relation to other numbers that precede or follow, which makes them either such a multiple of one of them as is expressed by a whole number, or else power and root.—In musical relations a harmonic relation in the scale of quantitative progression is introduced by a Quantum, while yet this Quantum for itself has no other relation in the scale to its predecessor and successor than these in turn to their predecessors and successors. While succeeding notes appear progressively to move further from the key-note, and numbers through arithmetical progression to become more and more other, suddenly a *return* or surprising concord emerges, which was not qualitatively led up to by what immediately preceded, but appears as an *actio in distans*, as a relation to a distant entity. The progression along merely indifferent relations which do not change the preceding specific reality (or do not even form one) suddenly interrupts itself, and, while it continues in the same manner quantitatively, a specific relation breaks in *per saltum.*

Among chemical combinations, when mixture-proportions are progressively altered, certain qualitative nodes and jumps occur, such that two materials form products at particular points of the scale of mixture, which then show particular qualities. These products are differentiated not only by more and less, nor are they already given (in a weaker degree, for instance) in the relations which are akin to these nodal relations, but are actually bound to such points. The combinations of oxygen and nitrogen, for instance, produce the various oxides of nitrogen and nitric acids, which appear only in definite quantity-relations of the mixture and have essentially different qualities, so that in intermediate mixture-relations no combinations of specific existences take place.—Metal oxides (for instance, lead oxides) are formed at certain quantitative points of oxidation, and differ in colour and in other qualities. They do not gradually merge in one another; the relations which lie between those nodes produce no neutral and no specific Determinate Being. A specific combination appears which has not passed through these intermediate steps, but is based on a Measure-Relation and has qualities of its own.—Again, water, when it changes its temperature, not merely becomes less hot, but passes through conditions of hardness, of drop-forming liquidity, and of elastic liquidity; these different conditions do not appear gradually; but the merely gradual progress of change of temperature is suddenly interrupted and checked by these points, and the appearance of such a new condition is a jump.—All birth and death, instead of being a continued graduality, are rather an interruption of this and are the jump from quantitative into qualitative change.

It is said, *natura non facit saltum*; and ordinary imagination, when it has to conceive an arising or passing away, thinks it has conceived them (as was mentioned) when it imagines them as a *gradual* emergence or disappearance. But we saw that the changes of Being were in general not only a transition of one magnitude into another, but a transition from the qualitative into the quantitative, and conversely: a process of becoming other which breaks off graduality and is qualitatively other as against the preceding Determinate Being. Water on being cooled does not little by little become hard, gradually reaching the consistency of ice after having passed through the consistency of a paste, but is suddenly hard; when it already has quite attained freezing-point it may (if it stands still) be wholly liquid, and a slight shake brings it into the condition of hardness.

The gradualness of arising is based upon the ideas that that which arises is already, sensibly or otherwise, *actually there*, and is imperceptible only on account of its smallness; and the gradualness of vanishing on the idea that Not-being or the Other which is assuming its place equally is *there*, only is not yet noticeable;—there, not in the sense that the Other is contained in the Other which is there *in itself*, but that it is there as Determinate Being, only unnoticeable. This altogether cancels arising and passing away; or the In-itself, that inner somewhat in which something is before it attains Determinate Being, is transmuted into a smallness of external Determinate Being, and the essential or conceptual distinction into a difference external and merely magnitudinal.—The procedure which makes arising and passing away conceivable from the gradualness of change is boring in the manner peculiar to tautology; that which arises or passes away is prepared beforehand, and the change is turned into the mere changing of an external distinction; and now it is indeed a mere tautology. The difficulty for such Understanding which attempts to conceive consists in the qualitative transition of something into its Other in general and into its opposite; Understanding prefers to fancy identity and change to be of that indifferent and external kind which applies to the quantitative.

In Ethics, in so far as it is considered in the sphere of Being, the same transition from quantitative to qualitative takes place, and different qualities appear to base themselves on differences in magnitude. A more or less suffices to transgress the limit of levity, where something quite different, namely, crime, appears; and thus right passes over into wrong, and virtue into vice.—Thus too do States—other things equal—derive a different qualitative character from magnitudinal difference. Laws and constitution become something different when the extent of the State and the number of citizens increases. The State has a certain measure of its magnitude, and if forced beyond this it collapses helplessly under that very same constitution which was its blessing and its strength for as long as its extent alone was different.

C

THE MEASURELESS

In its realized Being-for-Self exclusive Measure still remains infected with the moment of quantitative Determinate Being, and therefore remains capable of oscillation along the scale of Quantum, on which the alteration of relations takes place. A Something or a Quality which rests on such a relation is forced beyond itself into the *Measureless*, and perishes by the mere alteration of its magnitude. Magnitude is that characteristic wherein, while no harm appears to be intended, a Determinate Being can be seized, and whereby it can be destroyed.

The abstract Measureless is Quantum in general as lacking determination in itself and as merely indifferent determinateness by which the Measure is not changed. In the Nodal Line of Measures determinateness is also posited as specifying; the abstract Measureless transcends itself into qualitative determinateness; the first Measure-Relation passes over into another which is new, and is a Measureless in regard to it, while in itself it is just as much a Quality which is for itself; thus then is posited the alternation of specific existences with one another and also with relations which remain merely quantitative,—and so to infinity. This transition then contains the negation of specific relations as well as the negation of the quantitative progress,—that is, infinity which is for itself.—*Qualitative* infinity (as in Determinate Being) was the eruption of the infinite in the finite, as *immediate transition* and disappearance of the Hither in its Beyond. But *quantitative* infinity is according to its determinateness the *continuity* of Quantum—a continuity of it beyond itself. The qualitatively finite becomes infinite: the quantitatively finite is its Beyond in itself and points beyond itself. But this infinity of the specification of Measure posits the qualitative as well as the quantitative, each *cancelling* the other, and thereby also posits their first and immediate *unity* (which is Measure in general) as having returned upon itself, and therefore as itself posited. The qualitative—a specific existence—passes over into another in such a manner that nothing takes place except a change in the magnitudinal determinateness of a relation: thereby the change of qualitative itself into qualitative is posited as external and indifferent, as a coincidence with itself; and apart from this the quantitative transcends itself as turning into qualitative, as being determinate in itself. This unity which thus continues itself into itself in its alternation of Measures is the stable Matter or Thing which in truth persists.

We have, then, (α) one and the same Thing which is posited as the substratum of its differentiations, and as persisting. This severance of Being from its determinateness begins already in Quantum in general; something has magnitude as indifferent to its determinateness as Being. In Measure the Thing is already in itself the unity of the qualitative and the quantitative—the two moments which constitute the distinction in the general sphere of Being, each being the Beyond of the other; in this manner the persisting substratum, so far, has in itself the determination of infinity which is. (β) This undifferentiatedness of the substratum is posited in the fact that the qualitative stabilities into which the measure-determining unity is broken up consist only in quantitative differences, so that the substratum continues itself into this its differentiating process; (γ) in the infinite process of the nodal series the continuation of the Qualitative into the quantitative progress, as into an indifferent variation, is posited; and in this is posited the *negation* of the Qualitative and therefore of merely quantitative externality. The quantitative reference beyond itself to an Other, as being another Quantitative, perishes in the emergence of a Relation-measure or Quality, and the qualitative transition transcends itself precisely in the fact that the new Quality itself is only a quantitative relation. This transition of Qualitative and Quantitative into each other takes place upon the foundation of their unity, and the only meaning of this process is the Determinate Being, the demonstration or positing that it is based on such a substratum, which is their unity.

In the series of Stable Measure-Relations, the one-sided members of these series are immediate qualitative Somethings (specific gravities, or chemical substances, basic or alkaline, for instance acids); and next their neutralizations (—by which here must be understood also combinations of substances of different specific gravity—) are stable and even exclusive Measure-Relations; relatively to one another they are indifferent totalities of Determinate Being which is for itself. Now such relations are determined as being only nodes of one and the same substratum. This reduces the Measures and their implied stabilities to *states*. The change is change only of a state, and that which passes over is posited as remaining the same in the process.

We now survey the progress in determination which Measure has covered. The moments of this progress are grouped in such a manner that Measure first is the unity (itself immediate) of Quality and Quantity, since it is an ordinary Quantum, which however is also specific. Here it is a quantitative determinateness referring not to other but to itself; it is, essentially, Relation. And further, therefore, it contains its moments as transcended and unseparated; the differentiation within it—as in every concept—is such that each of its moments is itself unity of the Qualitative and the Quantitative. The differentiation has thus become real, and produces a multitude of Measure-Relations which formally are totalities and as such are stable within themselves. The series which form the sides of these relations are the same constant order for each individual member which, as belonging to one side, is in a relation to the whole opposing series. This unity, as being mere *order*, is quite external; it shows itself as immanent specifying unity of a Measure which is for itself as distinct from its specification; but the specifying principle is not yet the free concept which alone gives immanent determination to its differentiations: so far the principle is only a substratum or material; and to effect that its differentiations shall be totalities, that is, shall contain the nature of the unchanging substratum, there is only the external quantitative determination, which at the same time manifests itself as differentiation of Quality. In this unity of the substratum with itself the Measure-determination is transcended, and its Quality is a condition determined by the Quantum, and external.—This course is the realizing and progressive determination of Measure, and this determination is also its reduction to the position of a moment.

CHAPTER III

THE BECOMING OF ESSENCE

A

ABSOLUTE INDIFFERENCE

BEING is abstract equivalence—which we will call Indifference, since for itself it must be thought of as Being—having, as yet, no kind of determinateness; pure Quantity is Indifference, as being capable of every determination, but in such a manner that determinations are external to it and it has no connexion with them immanent in itself; while that Indifference which may be called Absolute is that which mediates itself with itself into simple unity by the negation of every determinateness of Being, of Quality, Quantity, and their unity, Measure, which, so far, is immediate. Of this Indifference, determinateness is no more than a state, that is, a qualitative externality having Indifference for substratum.

Now this entity which has thus been determined to be a qualitative externality, is only a vanishing entity; and, as being thus external to Being, the Qualitative, as its own opposite, just transcends itself. In this manner determinateness is posited of the substratum only as an empty differentiation. But this empty differentiation is Indifference itself as result. And this Indifference is thus Concrete, or that which is mediated in itself through the negation of every determination of Being. Since it is this mediation it contains negation and relation; and that which we called state is its immanent and self-relating differentiation; it is, precisely, externality and its disappearance which make the unity of Being to be Indifference; they are therefore within Indifference, which now ceases to be abstract and to be substratum, and only applied to itself.

B

INDIFFERENCE AS INVERSE RELATION OF ITS FACTORS

We must now see how this determination of Indifference is posited in itself, and the latter therewith as being for itself.

1. The reduction of Measure-Relations, which so far are accounted stable, is based upon one common substratum; this substratum is their continuation into one another, and so that indivisible stability the totality of which exists in its differentiations. For this differentiation there are present the determinations of Quality and Quantity which it contains, and nothing matters except how these are posited in it. This, however, is determined by the fact that so far the substratum is result, and mediation in itself, while at the same time mediation is not yet posited in it as such; this fact makes it, first, substratum, and, with regard to determinateness, Indifference.

Hence differentiation is in it essentially so far merely quantitative and external: we have two different Quanta of one and the same substratum, which thus is their sum, and therefore is itself determined as Quantum. But Indifference is this fixed Measure and self-existent absolute limit only in relation to these differentiations, and that in such a manner that it is not Quantum in itself, and is not in any manner opposed as sum or exponent to other entities, whether sums or Indifferences. Indifference contains only abstract determinateness; and the two Quanta, in order that they may be posited as moments of it, are related as variable, indifferent, and greater or less. But being confined within the fixed limit of their sum they also are not external to each other, but negative;—and this is the qualitative determination which relates them. They are accordingly in Inverse Relation to each other. This Relation differs from the previous, formal Inverse Ratio, because the whole is here a real substratum, and it is posited of each of the two sides that in itself it is to be this whole.

According to the qualitative determinateness which was mentioned there is, further, a distinction of two Qualities of which one is cancelled by the other, although, since both are in and constitute a unity, each is inseparable from the other. The substratum itself, as Indifference, is also in itself the unity of the two Qualities: thus each of the sides of the relation contains both, and is differentiated only by a more of one Quality and a less of the other, and conversely; the one Quality is just preponderant by virtue of its Quantum in the one side, and the other Quality in the other side.

Thus each side is in itself an Inverse Relation: this Relation recurs, as formal, in the sides which are differentiated. Thus these sides themselves also continue themselves into each other according to their qualitative determinations: each Quality is related to itself in the other, and is in each of the two sides just in a different Quantum. Their quantitative difference is that Indifference according to which they continue themselves into each other, and this continuation, as homogeneity of the Qualities, is in each of the two unities.—Meanwhile the sides (of which each contains the totality of the determinations, and hence the Indifference itself) are thus also posited as stable relatively to each other.

2. Being now, as this Indifference, is the determinateness of Measure, no longer in its immediacy, but in the developed manner which has just been set forth. It is Indifference, since in itself it is the totality of the determinations of Being which have been dissolved into this unity; and also Determinate Being, as totality of posited realization in which the moments themselves are the self-existent totality of Indifference, supported by it as their unity. But unity has here been fixed only as Indifference and, therefore, only as *in itself*, and the moments are not yet determined as being *for themselves*, that is, as cancelling each other in and through themselves into unity; and so we have the indifference of these to themselves, as developed determinateness.

This stability, which has thus turned out inseparable, must now be further considered. It is immanent in all its determinations, and in them remains in unity with itself and undisturbed by them; but (α) since it is totality in itself, every determinateness which is cancelled in it persists in it, but only as arising in it baselessly. There is no connexion between the "in-itself" of Indifference and this its Determinate Being; every determinateness shows itself immediately in it, and in each one it is in its totality; thus the differentiation between all is at first posited as transcended, that is, as quantitative, and for that very reason as no self-repulsion; they are not posited as self-determining, but only as being, and as becoming, determinate externally.

(β) The two moments are in inverse quantitative relation —an oscillation along magnitude which is not determined by Indifference (which is, precisely, the indifference of this oscillation), but hereby is determined only externally. There is here reference to an Other which lies beyond and contains the determining. In this aspect the *absolute* as Indifference has the second deficiency of the quantitative form, which is that the determinateness of the differentiation is not determined by it; and it has the first deficiency, too, in the fact that the differentiations only *emerge* in it,—that is, its positing is immediate and not its own self-mediation.

(γ) The quantitative determinateness of the moments which now are sides of the relation, constitutes this manner of their persistence; their Determinate Being is saved from the transition of the Qualitative by this indifference. They have a persistence, however, which is different from this their Determinate Being, and is in itself, in the fact that they are the Indifference in themselves, and that each is the unity of the two Qualities into which the qualitative moment splits itself. The difference of the two sides reduces itself to this, that the one Quality is posited in the one side with a more and in the other with a less, and the other correspondingly and conversely. Thus each side is in itself the totality of Indifference.—Each of the two Qualities being taken individually remains that sum which the Indifference is: it continues itself from one side into the other, and is not restricted by the quantitative limit which is posited of it in this process. Here the determinations come into immediate opposition which develops itself into contradiction, as must now be shown.

3. For each Quality enters into relation with the other within each side, and that in such a manner that this relation is to be only a quantitative difference: this determination has already been reached. If both the Qualities are stable—as when, for instance, they are taken as independent sensible substances—then the whole determinateness of Indifference

collapses; their unity and totality would be empty names. But in fact they are also determined in such a manner that they are comprehended within one unity; they are inseparable, and each has meaning and reality only in this one qualitative relation to the other. And now each extends as far only as the other, just because their quantitativity is of this qualitative nature. If they were different as Quanta, then one would by so much fail to coincide with the other, and in this overplus it would have an indifferent existence lacking to the other. But in their qualitative relation each is only in so far as the other is.—From this it follows that they are in equipoise, that one would increase or decrease by as much as the other did, and that it would increase or decrease in the same proportion.

Hence neither Quality can achieve a "more," and no quantitative difference can arise, from the basis of their qualitative relation. The "more" by which one of the related moments exceeded the other would be a foundationless determination, that is, this "more" would only again be the other itself; but both being thus equal both would have vanished, since their existence was to be based only upon the inequality of their Quantum.—Two factors are desiderated, and each vanishes when it is supposed to exceed the other as much as when it is supposed to be equal to it. This disappearance manifests itself thus: a disturbance of the equilibrium emanates from the quantitative point of view, and one factor is taken to be greater than the other; thus the cancellation of the Quality of the other, and its foundationlessness, are posited; the first becomes preponderant, so that the other decreases with accelerated speed and is overwhelmed by it. The first thus becomes the sole stable entity: but now we no longer have two specific entities and two factors, but only the one whole.

This unity, thus posited as the totality of determination, which totality is itself here determined as Indifference, is universal contradiction. It must, therefore, be posited as being determined as stability which is for self, just because it is this self-cancelling contradiction. This stability has for result and as its truth unity which no longer is merely indifferent, but immanent in itself, negative and absolute. This is Essence.

Observation

The relation of a whole which is supposed to have its determinateness in the magnitudinal difference between factors which are qualitatively determined relatively to one another, is used in the elliptical movement of the celestial bodies. In the first instance this example shows only two Qualities in inverse relation to each other, not two sides, of which each is the unity of both and its inverse relation. And since the empirical basis is so firm, the consequence is overlooked to which the theory applied to it leads, which is, to destroy the basic fact or else (if, as is proper, it is retained) to demonstrate the emptiness of the theory as against it. But this consequence is ignored, which permits the fact and the theory which contradicts it quietly to subsist side by side.—The simple fact is that in the elliptic movement of the celestial bodies their velocity increases as they approach their perihelion, and decreases as they approach their aphelion. The quantitative part of this fact has been determined exactly by untiring diligence of observation; it has further been reduced to its simple law and formula, and thus every demand which could legitimately be made of theory has been met. But to reflective understanding this did not seem enough. In order that the phenomenon and its law may have a so-called explanation, a centripetal and a centrifugal force are assumed, as qualitative moments of movement along the curved line. Their qualitative difference consists in opposition of direction, and their quantitative difference in the fact that they are determined as unequal and that one is to increase as the other decreases, and conversely, and, further, their relation suddenly is reversed: after centripetal force has been waxing and centrifugal waning for a time, a point arises where centripetal force wanes while centrifugal force waxes.[1] But this idea is contradicted by the relation to each other of their respective determinatenesses. These are essentially qualitative, and make it quite impossible to separate the forces: each has meaning only with respect to the other; therefore, in so far as one exceeded the other, in so far also it would have no relation to it, and would be non-existent.—When the assumption is made that one is definitely greater than the other when it is related to it as greater to less, then what was said above comes about: it gains absolute preponderance, and the other vanishes; the latter is posited as that which vanishes and is unfounded; and the fact that the disappearance is gradual, and that what it loses in magnitude accrues to the other, does not alter this determination. One perishes with the other, since what it is is only in so far as the other is. And here is a very simple consideration: if, for example, as is asserted, the centripetal force of the body as it approaches the perihelion is to increase, and its centrifugal force to decrease proportionally, then the latter would no longer be able to save it from the former and to remove it from the attracting body; on the contrary, since the centripetal is supposed to be preponderant, the centrifugal is overcome, and the body approaches the attracting body with accelerated velocity. And, conversely, if centrifugal force dominates at infinite proximity to the aphelion, it is equally contradictory that it should now be overcome at the aphelion by the weaker force.—Further it is clear that the cause of this inversion must be a foreign force; which means that the velocity of movement, at times accelerated and at times retarded, cannot be understood nor, as it is called, explained from the determination of these factors which is here assumed, although these were assumed for the very purpose of explaining this difference. The conclusion which follows from the disappearance of one or the other direction, and hence of elliptical movement altogether, is ignored and concealed for the sake of the established fact that this movement persists and passes from accelerated into retarded velocity. When it is assumed that a weaker centripetal force is transmuted at the aphelion into a force which preponderates against centrifugal force, and when a converse assumption is made for the perihelion, then this assumption partly contains what was developed above, that each of the sides of the inverse relation is in itself the whole inverse relation; for the side of movement from aphelion to perihelion (where centripetal force is supposed to be preponderant) is supposed also to contain centrifugal force, though decreasing as the other increases; and the same inverse relation is supposed to subsist, in the side of retarded movement, between an increasingly preponderating centrifugal force, and centripetal force, so that neither has vanished on either side, but each merely decreases progressively until the time arrives when the reversal occurs and it begins to preponderate over the other. Consequently what recurs on each side is nothing but the flaw in this inverse relation, namely that either each force is taken independently by itself, so that, as in the parallelogram of forces, the unity of the concept and the nature of the fact is transcended in the merely *external* co-operation of the forces in one movement, or else, each being qualitatively related to the other through the concept, neither can attain indifferent and independent persistence as against the other,—this was to be the contribution of the "overplus." The intensive (or so-called dynamic) form does not change anything, since it itself has its determinateness in Quantum, and so can manifest only as much force as is opposed to it by the opposite force; and this is the measure of its existence. But partly this transmutation of preponderance into its opposite contains the alternation of positive and negative, which are qualitative determinations: the increase of one is balanced by a loss of the other. The inseparable qualitative connexion of this qualitative opposition is disjointed into chronological succession by the theory; but the theory still fails thereby to furnish the explanation of this alternation, and also, above all, of this method of disjointing. Here the illusion of unity, which was present while one increased as the other decreased, wholly vanishes: a merely external succession is proclaimed, which contradicts only the consequence of this connexion, by which, when one has established preponderance, the other must vanish.

---

[1] The following sentence is added here in the first edition: "I have considered this subject in an earlier treatise, and have demonstrated the idleness of this distinction and the explanations which are based upon it."

This same relation has been applied to Attractive and Repulsive Force in order to render intelligible the varying density of bodies; and the inverse ratio of sensibility and irritability too has been drawn upon to explain from the inequality of these factors of *life* the various determinations of the whole, of health, as also the variety of living species. But the confusion and the absurdity in which this explanation—designed to become a natural philosophical basis for physiology, nosology, and finally zoology—became involved through an uncritical use of these conceptual determinations, led here to the result that this formalism was quickly renounced, though it is still continued to its full extent in the science, especially, of physical astronomy.

Absolute Indifference may appear to be the fundamental determination of Spinoza's Substance; and in this regard we may here observe that it is so in fact in this respect that, in both, all determinations of Being, as in general all further concrete differentiation of Thought and Extension and so forth, are posited as having vanished. Indeed it is indifferent, if we are not to proceed beyond abstraction, what was the appearance, while it existed, of that which has perished in this abyss. But Substance, as Indifference, is partly connected with the need of determining and with a respect to this process; it is not supposed to remain the Substance of Spinoza, whose one determination is that negation which claims that everything is absorbed in it. With Spinoza differentiation—the attributes, Thought and Extension, the modes too, the affections, and all other determinations—is introduced quite empirically; and it is understanding—itself a mode—in which this differentiation takes place. The attributes in relation to Substance and to each other have this sole determinateness, that they wholly express Substance, and that their content—the order of things as extended, and as thoughts—is the same. But when Substance is determined as Indifference, reflection arrives at differentiation; the latter is now *posited* as that which, with Spinoza, it is in itself, namely, as external and therefore, more precisely, as quantitative. Thus, in differentiation, Indifference remains immanent in itself like Substance,—but abstractly and only in itself; differentiation is not immanent for it, but, on the contrary, being quantitative, it is the opposite of Immanence, and quantitative Indifference is rather the self-externality of Unity. Here then differentiation is not taken as qualitative, and Substance is not determined as the self-differentiating, as Subject. What follows next, with regard to the category of Indifference itself, is that in it the distinction between quantitative and qualitative determination falls asunder, as appeared in the development of Indifference: it is the dissolution of Measure, in which both moments were posited immediately as one.

C

TRANSITION TO ESSENCE

Absolute Indifference is the last determination of Being before it becomes Essence; but it does not attain to Essence. Indifference shows itself as still belonging to the sphere of Being, for, being determined as indifferent, it has differentiation as quantitative and external. This is its Determinate Being; and now it is in this contradictory state, that it is determined as against its Determinate Being only as that *which is in itself*, and is not thought of as the Absolute *which is for itself*. Or again it is external reflection, which does not progress beyond the position where the specific entities in themselves or in the Absolute are one and the same, and their difference is indifferent and not a difference in itself. What here is lacking is this, that this reflection is not the external reflection of thinking and subjective consciousness, but is the determination of the differentiations themselves of this unity to transcend themselves; which unity then turns out to be absolute negativity and indifference to itself and to its own indifference as much as to other-being.

But this self-cancellation of the determination of Indifference has already been reached: this determination has shown itself in the development of its positedness in every direction as contradiction. It is in itself that totality in which every determination of Being is transcended and contained; thus it is the basis, but so far is only in the one-sided determination of Being-in-Self, which implies that differentiations, quantitative difference, and the inverse relation of factors are external to it. It is the contradiction of itself and of its determinateness, of its determination as it is in itself and of its posited determinateness; it is, accordingly, negative totality, of which every determinateness has transcended itself of itself, and has thereby transcended this its fundamental onesidedness, its Being-in-Self. It is now posited as that which Indifference in fact is, and is simple and infinite negative self-relation, self-incompatibility and self-repulsion. Determining and being determined are here neither a transition, nor external variation, nor an emergence of external determinations in it, but the self-relation of Indifference, which is the negativity of itself and of its Being-in-Self.

The determinations, being thus repelled somethings, are not masters of themselves and do not emerge in independence or externality, but, as moments, belong first to unity *which is in itself*, and have not been dismissed from it but have in it their supporting substratum and sole filling;—and next are those determinations which are immanent in unity *which is for itself*, and have being only by virtue of its self-repulsion. They are not *existents* (as in the whole sphere of Being), but are henceforth only that which is posited; they have just this determination and meaning, that they are referred to their unity, and hence each to its other and to negation;—and as such they bear the mark of this their relativity.

And now Being in general, and the being or immediacy which belongs to each differentiated determinateness, has vanished together with Being-in-Self; and the unity is Being, immediate and presupposed totality, and is this simple self-relation only through the mediation of the transcending of this presupposition, while this presupposedness and immediate Being itself is only a moment of its repulsion; the original independence and self-identity exist only as the resulting infinite coincidence with itself.—Being is thus determined to be Essence;—Being, as through transcendence of Being simple Being with itself.